NEURAL FUZZY NETWORK-SUPPORTED GENERATION OF LINGUISTIC KNOWLEDGE FOR UNDERSTANDING OF SPATIAL DECISION MAKING

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ABSTRACT

This paper discusses the understanding of linguistic knowledge based on two aspects: importance and uncertainty for which people use linguistic knowledge to simulate the process of spatial decision making. The linguistic knowledge described in this paper is represented as an If-Then rule base, which is generated from spatial information (GIS data or remote sensing data) through a proposed adaptive neural fuzzy network learning. Our goal is to use these If-Then rules instead of numerical values in the analytical models when uncertainty is involved in data. Such a method is expected to solve realistic problems in natural resource analysis and management.

1 INTRODUCTION

A spatial decision needs a good understanding of the spatial reality that the decision process is involved in the knowledge reasoning in terms of the location, shape and properties of spatial objects. In reality the spatial knowledge usually are expressed and operated with linguistic descriptions such as "in-front/back", "near/far" and "very suitable/suitable", etc., which are specified with a degree of ambiguity. Conventional GIS maps the continuous reality into a set of discrete symbols when the partitions are defined by specialists based on pragmatic considerations of what appeared to the specific application. However, the understanding of the spatial decision expressed as behavior or knowledge of specialists is often characterized by their perceptions, attitudes, motivations and personalities. Strictly speaking, it is difficult for knowledge in the form of numerical values to simulate a continuous spatial phenomenon. Hence, when we work with vague or imprecise knowledge, this does not allow for estimation with an exact numerical value. A more realistic approach is to use a linguistic assessment instead of numerical values. This approach allows a representation of the specialists' understanding of spatial reality in a more direct and adequate linguistic rule form, whether or not they are unable to express their knowledge with precision.

Artificial neural fuzzy networks have been attracting growing interest in the past decade and have been successful in many application domains (Smith 1984, Openshaw 1993, Wang 1994). This technique has the potential to provide an exciting advance over traditional analytical methods for solving intractable problems. The method allows the source information with uncertainty, or during the process of analysis allows using linguistic terms to probe into the essence of problems and their mutual relations.

In this paper we discuss the linguistic knowledge based on two aspects: importance and uncertainty. The knowledge is presented in the form of fuzzy If-Then rules, which are generated from spatial information through a proposed adaptive neural network based on a fuzzy inference system rather than directly collected from experiential knowledge. Our goal is to understand a spatial decision when it is made by the linguistic knowledge. To make explanation, a case study for spatial classification of land suitability is given in this paper. The results show that the understanding is very useful to help us to be acquainted with the process of spatial decision.

2 DEGREE OF BELIEF OF LINGUISTIC RULES

Human knowledge is imprecise in nature, it is usually the case that the knowledge base is a collection of rules which, for the most part, are neither totally certain nor totally consistent (Zadeh, 1983a). To deal with imprecise knowledge, fuzzy set theory is often used in applications.

There are three types of rules with uncertain information. The first type is the rule with a certain antecedent but an uncertain consequent. For example, *IF slope is less than 5% THEN arable suitability is very good*. For this type, the rule has a certain information 5%, but the uncertainty is to what degree we believe that "less than 5% "is very good. The second type is the rule with an uncertain antecedent but a certain consequent. For example, *IF slope is slight THEN arable suitability is class 1*. For this type, the rule has a certain information 'class 1', but the uncertainty is to what degree we believe that slope is slight. The third type is the rule with both uncertain antecedent and consequent. For example, *IF slope is slight THEN arable suitability is good*. This type, uncertainty happens in both antecedent and consequent to what degree we believe arable suitability is good given from that slope is slight.

When using linguistic rules to reason about decision making, these rules are to be concerned with a degree of belief. Because we cannot be completely certain that some facts are true or that certain relations hold each fact. The CF, a number in the internal [-1,1], indicates the certainty with each fact or relations of facts is believed. The number can be assigned in two ways. One is experiential value within [-1,1]. Another is computational value fixed as 1 or -1 in terms of the fact or relations is true or false, which indicates absolute knowledge. Positive and negative CFs indicates a predominance of confirming or opposing evidence, respectively. Here the discussion only focuses on the later.

Single premises rule with uncertainty. The general form can be in this construct: **IF A THEN C**. Such construct is applicable to decisions involving properties of spatial objects and situations in which there are more than one correct decision. The basic element is the premise. For a particular problem, a value identifies the state of the premise. Such a value is termed evidence and is discrete or continuous. To evaluate this kind of rules, the following explanations are given.

For the first rule type, which has a certain antecedent but an uncertain consequent, CF(A) = 1, because the fact of antecedent is true. The consequent of rule (**C**) is an uncertain linguistic term. If using a fuzzy subset describe the term, $MF(C) = m_C(a)$, where a is a variable associated with antecedent of rule (**A**). The degree of belief of rule can be calculated as:

$$B(r) = CF(A) \times MF(C) \tag{1}$$

For the second rule type, which has an uncertain antecedent but a certain consequent, always CF(C) =1, because the consequent of the rule is positive. The uncertain antecedent is supposed describing in fuzzy subset $MF(A) = m_A(a)$. Then the degree of belief of rule can be calculated as:

$$B(r) = MF(A) \times CF(C) = MF(A) \tag{2}$$

The third type, which has both uncertain antecedent and consequent, There should be two fuzzy subsets describing the both uncertain antecedent and consequent. Then the degree of belief of rule can be calculated as:

$$B(r) = MF(A) \times MF(C) \tag{3}$$

Considering a general formulation for calculating the degree of belief in the rule of single premises, then

$$B(r) = MF(A) \times CF(A) \times CF(C) \times MF(C) \tag{4}$$

Multiple premises with uncertainty. Rules are usually a more complex. Premises are joined through some operators of fuzzy logic. The rules may be represented by **IF A and B THEN C** or **IF A or B THEN C**. Suppose the uncertain premises **A** and **B** are described by different fuzzy subsets. The common method to derive the degree of belief of rules is to use the Minimum and Maximum operators when considering premises joined by fuzzy logical operators ‘ AND’ and ‘ OR’ , respectively. In the general case, the degree of belief of rules is

$$B(r) = Min(MF(A) \times CF(A), MF(B) \times CF(B), ...) \times MF(C) \times CF(C). \tag{5}$$

$$\text{or } B(r) = Max(MF(A) \times CF(A), MF(B) \times CF(B), ...) \times MF(C) \times CF(C). \tag{6}$$

Besides Minimum operator of fuzzy logic can be used, another common operator "Product" also can be applied for linguistic connection ‘ AND’ . The formulation of believable degree of rules is

$$B(r) = MF(A) \times CF(A) \times MF(B) \times CF(B) \times ... \times MF(C) \times CF(C). \tag{7}$$

3 NEURAL FUZZY NETWORK SUPPORTED GENERATION OF LINGUISTIC RULES

To simulate the process of a spatial decision by linguistic knowledge, an integrated model of neural fuzzy network and GIS was proposed for extracting linguistic rules. The approach is that using linguistic rules construct a neural network, and using steepest decent algorithm train these rules through the operations of fuzzification, rule inference and defuzzification in the network. The detailed description of the methodology is given in (Zheng and Kainz, 1999; Zheng et al., 1999). The general form of extracted rules is presented by a rule system

IF A and B and ..THEN Class C
…

in which the rules that have the same consequence are defined as a rule group. In this rule system the antecedent of rules always consists of multiple uncertain premises (A, B..) and the consequent is a certain class (C). Information from the antecedent of multiple premises may be collected to determine a certain conclusion. For instance, if consider two uncertain premises: soil and slope conditions to determine the suitability of arable land by a fuzzy rule system, the information of premises could be collected from two maps: soil and slope, and the consequent must be a certain class of suitability. Because the information of soil and slope is not completely accurate, the suitability of arable land is uncertain even if the conclusion is certain (belongs to a certain class).

4 UNDERSTANDING OF RULE IMPORTANCE

The solution of a decision problem is often derived from a fuzzy rule system, which contains two or more fuzzy rule groups. In the system, each rule group produces an inference result corresponding to its consequence. The rule importance is a measure about a degree of importance the individual rule has in a rule group. The measurement is realized by a set of data pairs and explained as following:

1) A sample data point may be regarded as a data triple with $(A; x, y; D)$. A is a set of attributes, which could be elevation, slope, distance, properties and so on; x and y are the spatial location; D is the desired decision result.
2) Consider the 2-D space N^2, the membership degree of linguistic terms in fuzzy rules corresponding to each spatial object (grid or polygon) might be described as $MF(A) = \{m_A(x, y)/(x, y)\};\ x, y \in \mathrm{N}^2$.
3) Each sample data triple produces a degree $B(r)$ of belief of individual rule in terms of equation (5) or (7).
4) The inference result of fuzzy rule group might be obtained through a sigma-count operation $Group(B) = \sum B(r)_i$ (i=1,…n; n is the number of rule group), which was proposed by Zadeh, 1983a and 1983b.
5) A sample data should support one rule in a rule group in terms of the degree of belief of rule in the group. If the data point support kth rule in a group, thus

$$if \quad B(r)_k = Max(B(r)_1, \cdots, B(r)_k, \cdots, B(r)_n), \qquad SupportRule_{number} = k. \tag{8}$$

   in which, n is number of rules in the rule group.

6) A sample data should support one rule group of the fuzzy rule system in terms of the inference results of all fuzzy rule groups. If the data support jth rule group, thus

$$If \quad Group(B)_j = Max(Group(B)_1, \cdots Group(B)_j, \cdots, Group(B)_m), \ SupportGroup_{number} = j. \tag{9}$$

   in which, n is the number of rule groups in a rule system. That the reasonable rule group a sample data triple $(A; x, y; D)$ support should be the group which meets the desired decision result (D). If not, this indicates that the sample data is not correct or the rule system is weak.

7) The importance of individual rule is measured based on all sample data. For example, the importance of kth rule in jth rule group might be measured by

$$RuleImportance_{jk} = \frac{N_{supportrule_k}}{N_{supportgroup_j}} \sum_{p=1}^{N_{supportrule_k}} B(r)_{kp}. \tag{10}$$

The rule importance only is a relative important degree of individual rule in a rule group. If one rule group only consists of one rule, the measure of rule importance loses meaning. If the value of rule importance is bigger, the knowledge expressed by the rule has a wider representative on a spatial area because of a higher data support ratio happened on the rule based on equation (10).

5 UNDERSTANDING OF RULE UNCERTAINTY

Klir (1987) distinguished two fundamentally different categories of uncertainty related to general fuzzy set theory in a non-spatial context, captured by the terms: fuzziness and ambiguity. Hootsmans (1996) extended the conceptual meaning into a spatial classification with fuzzy sets theory. In the context of fuzzy set theory the term fuzziness is a degree to which an event occurs, not whether it occurs. In a spatial context fuzziness may be regarded as a degree of making or distinguishing spatial entity as sharp and precise distinction in space and time. The expression of the value of fuzziness is a distance number to 0 and 1. This describes fuzziness with the understanding that all available possibility values deviate from the intuitively "ideal" possibilities for absolute truth and falsehood (1 and 0). The term ambiguity describes a particular relation of one fuzzy set to many fuzzy sets. The relation is a degree of difficulty when taking one of fuzzy sets as the decision result. For instance, suppose that three Ph.D candidates almost have same qualifications for applying one available fellowship. For the decision-maker, the Ph.D supervisor only can select one candidate from them. It is very clear that the supervisor feels a certain difficulty to make the decision. If consider the qualifications of the candidates as three fuzzy subsets, the process of decision hints an ambiguity problem which may be regarded as a measure about a difference value between maximum and all other possibilities.

There are many existing measures of fuzzy uncertainty that are used in applications. To simply apply these methods, this research concentrates on a few measures and makes some modifications for extending them into the measure of uncertainty of the fuzzy rule system.

5.1 Measuring fuzziness of a fuzzy rule system

In principle of fuzzy sets theory, suppose that X is any set. A fuzzy set A in X is characterized by a membership function $m_A : X \rightarrow [0,1]$. The value $m_A(x)$ represents the grade of membership of x in A. when m_A is valued in {0,1}, it is the characteristic function of a crisp (i.e., non fuzzy) set. A fact, for a crisp set, the fuzziness should be zero using any measure, as there is no unclearness about whether an element belongs to the set or not. If a set is maximally unclearness ($m_A(x) = 0.5 \forall x$), then its fuzziness should be maximum in terms of the meaning of membership grade. When a membership value approaches either 1 or 0, the unclearness about belongingness of the argument in the fuzzy set decreases. A fuzziness measure is defined by a function: $fuzziness : x(X) \rightarrow [0,1]$, which assigns to a value to fuzzy subset A of X with respect to a crisp subset C of X in the unit interval [0,1]. Klir (1987) considered the measure of fuzziness as a metric distance (Hamming or Euclidean) of a fuzzy subset A from the nearest crisp subset C.

$$\textit{Hamming distance:} \qquad x(x) = \sum_{j=1}^{m} | m_{Aj}(x) - m_{Cj}(x) | . \tag{11}$$

$$\textit{Euclidean distance} \qquad x(x) = \sqrt{\sum_{j=1}^{m} (m_{Aj}(x) - m_{Cj}(x))^2} . \tag{12}$$

For a fuzzy rule system, since the result of decision is derived from all groups, the fuzziness of the rule system should be a combination of fuzziness resulted from each rule group. Thus, if using equation (11) or (12), the parameter m is the number of rule groups, m_{Aj} is the output of jth group, and m_{Cj} is the membership value of nearest crisp subset to the jth fuzzy rule group.

The problem is often happened on the determination of the nearest crisp subset when measuring fuzziness by a metric distance. For single fuzzy subset, the membership value of the nearest crisp subset may be derived by

$$m_C(x) = \begin{cases} 0 & if \ m_A(x) \le 0.5 \\ 1 & if \ m_A(x) > 0.5 \end{cases} . \tag{13}$$

For multiple fuzzy subsets, if the minimum or maximum operator of fuzzy logic is used in a single fuzzy rule, equation (13) also may be applied. This is because the inference result of rule always is one of all fuzzy subsets. However, as using a product operator in rule inference, the inference result comes from a multiplying relation of all fuzzy subsets. Thus, the inference value could always be less then 0.5 and even proximate zero when the number of fuzzy subsets much increases. In this case, equation (13) is weak. To solve this problem, equation (14) is proposed in this paper.

$$m_C(x) = \begin{cases} 0 & if \ \prod_{i=1}^{n} m_{Ai}(x) \le 0.5^n \\ 1 & if \ \prod_{i=1}^{n} m_{Ai}(x) > 0.5^n \end{cases} . \tag{14}$$

in which n is the total number of fuzzy subsets in a rule.

Hootsman and Van der Wel (1993) proposed another method for determining the membership value of nearest crisp subsets when a fuzzy classification is finally directed to assigning the class with maximum membership value. This paper makes a little bit modification on the method and expresses it as following.

$$m_C(x) = \begin{cases} 0 & if \;\; m_{A_i}(x) < \underset{i=1}{\overset{n}{Max}}(m_{A_i}(x)) \\ 1 & if \;\; m_{A_i}(x) = \underset{i=1}{\overset{n}{Max}}(m_{A_i}(x)) \end{cases} \tag{15}$$

Equation (15) indicates if the class is assigned by maximum membership value, it should be an absolute truth-value of 1. Such a method is not only simple but also very useful to a spatial decision made by a fuzzy rule system.

5.2 Measuring ambiguity of a fuzzy rule system

Considering a simple example, if one expert makes a decision with 0.8 confidence, another expert makes the decision with 0.75 confidence, in principle of fuzzy sets theory, the decision made by first expert can be selected in terms of maximum operation. However, in fact, the second expert's confidence is close to the first one. In order to understand the similarity of the two decisions, a measure of ambiguity is introduced. In this paper, the measure of ambiguity is used to detect the similarity of multiple results derived from fuzzy rule groups. Two alternatives measure of possibilities non-specificity, known as *U*-uncertainty (Higashi and Klir, 1983) and *a*-uncertainty (Yager, 1982), which might be used to measure ambiguity. The extended representation can be described by following functions and applied for measure of ambiguity of a fuzzy rule system.

$$U\text{-uncertainty:} \quad g(x) = \sum_{i=1}^{m} (m_{A\,j}(x) - m_{A\,j+1}(x))^2 . \log i \tag{16}$$

$$a\text{-uncertainty:} \quad g(x) = 1 - \sum_{j=1}^{m} \frac{m_{A\,j}(x) - m_{A\,j+1}(x)}{j} \tag{17}$$

in which, $g(x)$ is ambiguity of rule system; m is the number of rule groups; $m_{A\,j}$ is the output of jth group; $m_{A\,j}(x) \geq m_{A\,j+1}(x)\ \forall i$ and $m_{A\,m+1}(x) = 0$. To enable the calculation of the both measures of ambiguity, a rank ordering of the output of fuzzy rule groups is necessary. The maximum ambiguity value for *U*-uncertainty is equal to $^2\log(m)$ and for *a*-uncertainty always equal to 1.

6 CASE STUDY

The case study is an extension of the research described in the paper (Zheng and Kainz,1999). In that study, an adaptive neural fuzzy network is used to extract fuzzy rules for assessment of arable land suitability in Da Yandian Township, Hefei municipality, China. Totally, 14 fuzzy rules were generated by the proposed neural fuzzy network and shown in table 1.

Table 1: Extracted linguistic rules by a neural fuzzy network (Zheng and Kainz,1999)

If	*Land economic index*	*Land use index*	*Distance to ditch*	*Distance to house*	*Distance to road*	*Organic matter content*	*Nitrogen (N) content*	*Potassium (K) content*	*depth to plough pan*	*Then*
R1:	L	L	S	S	S	M	M	L	S	Class 1
R2:	L	L	S	S	S	S	S	L	M	Class 1
R3	L	M	S	S	S	M	M	M	S	Class 1
R4	M	M	M	S	S	M	M	L	S	Class 2
R5	M	M	S	S	S	M	M	L	S	Class 2
R6	S	M	M	S	S	M	M	L	S	Class 2
R7	S	S	S	M	M	M	M	L	S	Class 3
R8	S	S	S	M	M	M	M	S	L	Class 3
R9	S	M	S	S	S	M	M	L	S	Class 3
R10	S	M	S	S	S	L	L	S	M	Class 3
R11	S	S	S	S	S	M	M	M	S	Class 4
R12	S	S	S	M	M	M	M	M	S	Class 4
R13	S	S	S	S	S	M	M	L	S	Class 4
R14	S	S	S	S	M	S	S	M	L	Class 4

Note: 'L', 'S' and 'M' correspond to linguistic terms "large", "middle", and "small", which are coded as 2, 1 and 0 in computerization. The connection symbol in *if* part is "AND".

6.1 Measure of rule importance

Two sets of data are used to find possible regularity existed between rules through the measure of rule importance. One is the training data set including 61 data points. Another is a set of test data, which involves 75 data points selected from available suitability map of arable land. Each data point in both data sets stands for one grid with 9 attributes mentioned in the first row of table 1.

As shown in table 1, 14 fuzzy rules are defined in 4 rule groups: group1 (rule 1, rule 2 rule3); group 2 (rule 4, rule 5, rule 6); group 3 (rule 7, rule 8, rule 9, rule 10) and group 4 (rule 11, rule 12, rule 13, rule 14). The measure of rule importance indicates a rank of importance of individual rule in a rule group. According to the description of measure of rule importance in section 4, after calculating the rule importance by the two sets of data, the measure results are shown in table 2 and 3, respectively.

Table2: Measure of rule importance by training data set with 61 data points

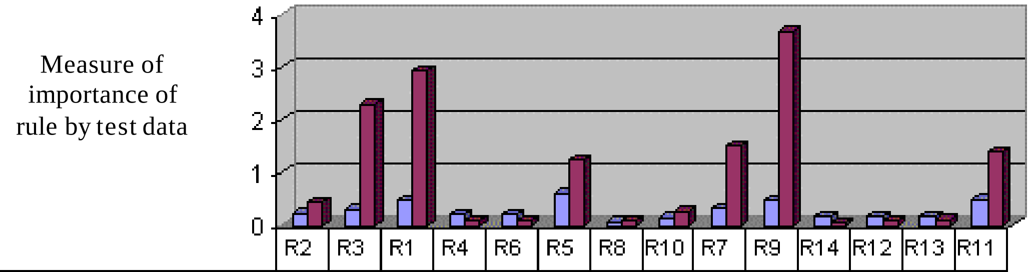
Rule	Rule code	Group	SPN_rule	SPN_group	Sum_rule output	Ratio of SPN	Importance
R2	2211111201	1	3	16	0.888908	0.1875	0.16667
R3	2011100011	1	4	16	2.724528	0.2500	0.68113
R1	2211100211	1	9	16	4.694509	0.5625	2.64066
R4	0011000212	2	2	10	1.179918	0.2000	0.23598
R6	1011000212	2	4	10	0.950985	0.4000	0.38039
R5	0011100212	2	4	10	2.595467	0.4000	1.03819
R8	1100100123	3	1	24	0.468919	0.0417	0.01954
R10	1011122103	3	4	24	1.091169	0.1667	0.18186
R7	1100100213	3	5	24	2.200669	0.2083	0.45847
R9	1011100213	3	14	24	9.905274	0.5833	5.77808
R14	1101111024	4	1	11	0.230220	0.0909	0.02093
R12	1100100014	4	1	11	0.678816	0.0909	0.06171
R13	1111100214	4	2	11	1.565720	0.1818	0.28468
R11	1111100014	4	7	11	4.767924	0.6364	3.03413

Table 3: Measure of rule importance by test data set with 75 data points

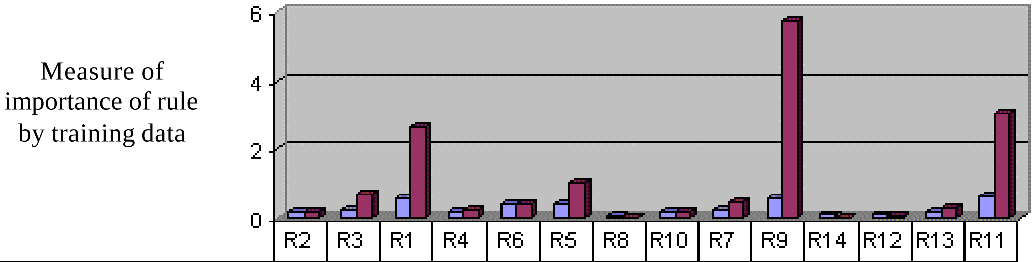
Rule	Rule code	Group	SPN_rule	SPN_group	Sum_rule output	Ratio of SPN	Importance
R2	2211111201	1	8	36	1.979377	0.2222	0.43986
R3	2011100011	1	11	36	7.492451	0.3056	2.28936
R1	2211100211	1	17	36	6.163765	0.4722	2.91067
R4	0011000212	2	1	5	0.467832	0.2000	0.09357
R6	1011000212	2	1	5	0.487621	0.2000	0.09752
R5	0011100212	2	3	5	2.059379	0.6000	1.23563
R8	1100100123	3	2	28	1.456782	0.0714	0.10406
R10	1011122103	3	4	28	1.875676	0.1429	0.26795
R7	1100100213	3	9	28	4.658344	0.3214	1.49732
R9	1011100213	3	13	6	7.876521	0.4643	3.65696
R14	1101111024	4	1	6	0.345678	0.1667	0.05761
R12	1100100014	4	1	6	0.574122	0.1667	0.09569
R13	1111100214	4	1	6	0.648675	0.1667	0.10811
R11	1111100014	4	3	11	2.792724	0.5000	1.39636

Note: SPN_rule -- Number of data points supporting individual rule; SPN_group -- Number of data points supporting rule group; Sum_rule output -- total inference values by SPN_rule; Ratio of SPN -- the ratio of SPN_rule with SPN_group.

Form both tables 2 and 3 it can be found that there are some regularity existed between the rules in a group. First, the both data sets hold a same conclusion: the ranks of rule importance are represented as group 1: R2, R3, R1; group 2: R4, R6, R5; group 3: R8, R10, R7, R9; and group 4: R14, R12, R13, R11. The second, the variation of the value of rule importance depend on the ratio of SPN_rule with SPN_group. Bigger value of the ratio more important the rule is. If two or two more rules have same ratio, the importance of rule depends on the summation of degree of belief of rule by supporting data points. The third, the summation of degree of belief of rule does not have controlling influence on the importance of rule. For example, in table 3, the summation of rule 3 (Sum_rule output) is bigger than rule 1's, but the importance of rule 3 is smaller than rule 1's. The fourth, the value of importance of rule might be different when different data sets are used to measure. Therefore, the value of importance only expresses the regularity existed between rules rather than an absolute difference between rules. This point indicates that the value of rule importance have not mathematical meaning. Above results can be got further explanations through figure 1.



	R2	R3	R1	R4	R6	R5	R8	R10	R7	R9	R14	R12	R13	R11
Support ratio of data point	0.22	0.31	0.47	0.20	0.20	0.60	0.07	0.14	0.32	0.46	0.17	0.17	0.17	0.50
Rule importance	0.44	2.29	2.91	0.09	0.10	1.24	0.10	0.27	1.50	3.66	0.06	0.10	0.11	1.40



	R2	R3	R1	R4	R6	R5	R8	R10	R7	R9	R14	R12	R13	R11
Support ratio of data point	0.19	0.25	0.56	0.20	0.40	0.40	0.04	0.17	0.21	0.58	0.09	0.09	0.18	0.64
Rule importance	0.17	0.68	2.64	0.24	0.38	1.04	0.02	0.18	0.46	5.78	0.02	0.06	0.28	3.03

Figure 1: Measure of importance of rule

6.2 Measure of Uncertainty of rule system

Measure of uncertainty includes fuzziness and ambiguity that are the products of a fuzzy rule system happened on spatial objects. Fuzziness reflects the overall deviation between the available possibility value of all rule groups and the intuitively "ideal" possibility for absolute truth and falsehood (1 and 0). In other words, the fuzziness is an uncertain degree of where the spatial pattern belongs to "ideal" class. Ambiguity, considered byanother viewpoint of uncertainty, is associated with one to many relations, i.e. situations with two or more alternatives whereby the choice between them is left unspecified. According to the description in section 5, the steps of measure might be described as following. The method is illustrated using data from whole data set of experimental area.

1) Fuzzify the attributes of each spatial object: grid (in this study, all spatial data were converted into grid files in Arc/Info) by trained membership functions in neural fuzzy network.
2) Calculate the degree of belief of rule using fuzzy values of grids based on the extracted fuzzy rule system.
3) Calculate the output of each fuzzy rule group corresponding to each grid.
4) Calculate the fuzziness and ambiguity happened on each grid by fuzzy rule system based on *Hamming distance* and a-uncertainty described in equation (16) and (17). The nearest crisp subset is determined by equation (15).

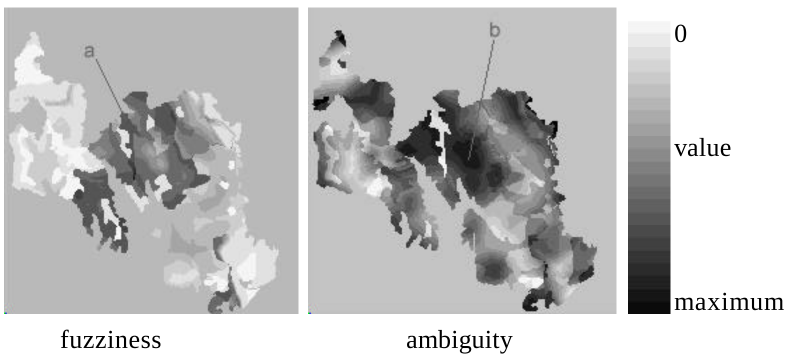


Figure 2: Fuzziness and ambiguity of fuzzy rule system

Figure 2 illustrates the measure results of fuzziness and ambiguity happened on each grid by the extracted rule system. The blacker color the bigger value of fuzziness or ambiguity is. Point *a* hints a big fuzziness value happened on it. This means besides one rule group has the biggest possibility output, other rule groups also have the big possibility output on this point based on equation (16) and (15). This indicates that the numerical value of fuzziness depends on the deviation of the possibility value of all rule groups and "ideal" truth-value. If all rule groups have a big possibility value on same place, a big fuzziness value will be assigned. The ambiguity means the difficult degree of classification due to two or more rule groups having closer possibility value on same place. For instance, point *b* has the biggest ambiguity in figure 2. This indicates if the ambiguity is bigger, the difficult degree of decision is bigger too. Related to explanations about the possibility output happened on point *a* and *b*, see (Zheng and Kainz, 1999; Zheng et al., 1999).

7 CONCLUSION

Often a set of qualitative linguistic knowledge may be put in a form of fuzzy rule and applied for decision making. Such linguistic knowledge involves many uncertain factors in the process of logic reasoning so that an uncertain result may be reached. To understand the extracted linguistic knowledge in the particular IF-THEN form, this paper discussed the measure of the degree of belief of linguistic rule, the importance of individual rule in a rule group, and the uncertainty of rule system happened on spatial objects.

From the viewpoint of fuzzy logic, the degree of belief of linguistic rule reflects a suitable degree of spatial object to individual rule. In other words, if the degree is lower, the rule can not prefect presents a kind of knowledge about the spatial object. The importance of rules is a relative comparison between rules in a fuzzy rule group. The result of comparison expresses an existed regularity, which might be explained by common natural language rather than an absolute difference in mathematics. Uncertainty of fuzzy rule system is discussed as fuzziness and ambiguity in this paper. Actually, fuzziness and ambiguity are the products of the rule system happened on spatial objects due to linguistic properties of rule system. One of them, fuzziness describes the deviation between the result from rule system and ideal result (absolute truth and falsehood: 1 and 0). The representation of fuzziness is an uncertain degree happened on the place where the spatial pattern belongs to "ideal" class. Ambiguity is considered as another form of uncertainty, which reflects the degree of difficulty to make decision on the place where there are two or two more possible values. The two kinds of the uncertainty are very helpful for the understanding of spatial decision making.

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