

# DETECTING AND IMAGING OF MOVING TARGETS OF CLUSTER SAR SYSTEM

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## ABSTRACT:

Based upon the sparse array composed of cluster SAR satellites, this paper analyzes the space-time and space-frequency characteristic of the received signal in detail, and proposes an algorithm for the moving targets detection and imaging based on array signal processing and Wigner-Ville Distribution (WVD). The results of theoretical analysis and simulation show that the cluster SAR system can achieve the targets SAR image even if the targets is being moving slowly.

**Keyword:** cluster SAR system moving targets detection imaging

## 1. INTRODUCTION

In the moving targets detection and imaging with synthetic aperture radar, the signal processing consists of detection and imaging of moving targets [1-4]. In the detection process, in order to detect the moving targets and improve detection probability, the echoes from stationary targets (clutters) must effectively be suppressed. By strongly clutter suppression, removing clutter component and retaining useful signal component of the received signal, the moving targets can just be detected. In the imaging process, since the motion states of moving targets is unknown and the imaging reference function for stationary targets and moving targets is difference, the Doppler parameters must be picked up from the received signal after suppression clutter. The precision of the Doppler rate bears upon the image quality, and the Doppler centroid bears upon the target position. Consequently, the moving targets detection and imaging on the stationary background can be implemented by the following two steps:

- 1) Improving as the moving targets signal to clutter and noise ratio as possible by clutter suppressed.
- 2) Estimating the Doppler parameters for moving targets the received signal after suppression clutter if the moving targets have been detected presence.

The cluster SAR system makes up of many SAR satellites, if every SAR satellite is regarded as a sensor, it is equivalence to a sparse antenna array, and the received signal by cluster SAR system is provided with have space-time characteristic, and also with have space-frequency characteristic by transformed into frequency domain. By taking advantage of the space-frequency characteristic of the received signal, the cluster SAR system can achieve the moving targets imaging based on the array signal processing and the Wigner-Ville Distribution (WVD).

## 2. THE SIDE-LOOK IMAGING MODE OF CLUSTER SAR SYSTEM

With reference to figure 1, let  $N_c$  small SAR satellites ( $S_1, S_2, \dots, S_{N_c}$ ) cluster together, its whole function and characteristic in flight can be represented by a virtual satellite  $S_v$ , the small satellites cluster distribute along a circle whose centre and diameter are respectively  $S_v$  and  $D_d$ , the plane

of small satellites cluster parallels with ground (without regard to the satellite dynamics as well as controlling). When the virtual satellite  $S_v$  with velocity  $V$  moves along the earth orbit, the small satellites cluster also move with same velocity  $V$  along their respective orbits. The relative geometrical relations among the small satellites as well as the small satellites cluster and virtual satellite  $S_v$  keep on invariability in the flying.

The signal for cluster SAR system is transmitted by single SAR, and all SAR receive the echo. The transmitting signal SAR is being in side-look imaging mode, and other SAR satellites illuminate the same scene on the ground by adjusting range and azimuth beam.

## 3. MODE OF THE RECEIVED SIGNAL

For further analysis an orthogonal earth fixed coordinate system  $X, Y, Z$  is established in Fig.1, the  $X$ -axis is pointing to the flight path of virtual satellite. Suppose at time  $t = t_0$  the targets  $T$  on scene is being the centre of azimuth beam. The range vector between the transmitting signal SAR and the target  $T$  is  $R$ , and the velocity and acceleration

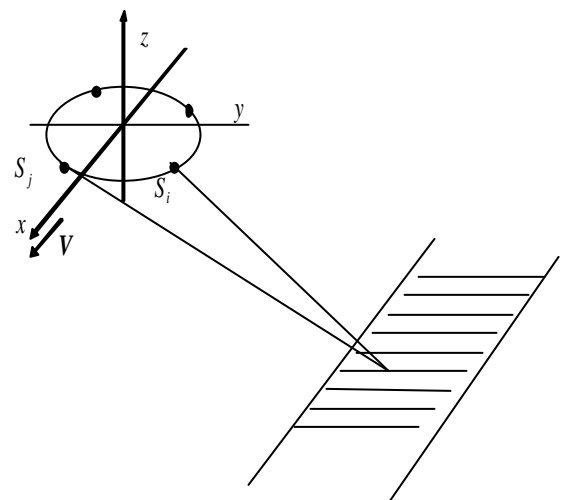


Fig.1 Cluster SAR system (side-look)

vectors of the transmitting signal SAR relative to the target  $\mathbf{T}$  respectively are  $\mathbf{V}$  and  $\mathbf{A}$ . When the target in the ground is being moving, the range vector between the transmitting signal SAR and target  $\mathbf{T}$  at time  $t$  is according to

$$\mathbf{R}_T(t) = \mathbf{R} + (\mathbf{V} - \mathbf{v})(t - t_0) + \frac{1}{2}(\mathbf{A} - \mathbf{a})(t - t_0)^2 \quad (1)$$

where  $\mathbf{v}$  and  $\mathbf{a}$  respectively are velocity and acceleration vectors of the target. Due to the signal of the cluster SAR is transmitted by single SAR and all SAR receive echo, the range vector between the receiving echo SAR and the target  $\mathbf{T}$  is according to

$$\mathbf{R}_R(t) = \mathbf{R}_T(t) - \mathbf{r} \quad (2)$$

where  $\mathbf{r}$  is the position difference vector between the transmitting signal SAR and the receiving echo SAR. When the system parameters for cluster SAR system is constant, (1) and (2) show that the range vectors are depend upon the parameters  $\mathbf{v}$ ,  $\mathbf{a}$ ,  $t_0$  and  $\mathbf{R}$ , and the  $\mathbf{R}$  can be regarded as known after range compression. If the motion parameters of the target is denoted as  $\mathbf{J}$ , the range vectors can be denoted as  $\mathbf{R}_T(t, \mathbf{J})$  and  $\mathbf{R}_R(t, \mathbf{J})$ . The total range that electromagnetic wave arrives to the receiving echo SAR is  $R(t, \mathbf{J}) = |\mathbf{R}_T(t, \mathbf{J})| + |\mathbf{R}_R(t, \mathbf{J})|$ . To substitute  $R(t, \mathbf{J})$  by (1) and (2), under the conditions that  $\mathbf{r}$  is small and the azimuth spectrum shift among different SAR can be neglected, the range  $R(t, \mathbf{J})$  can be presented by

$$R(t, \mathbf{J}) = 2R_T(t, \mathbf{J}) - \mathbf{u}(t, \mathbf{J}) \cdot \mathbf{r} \quad (3)$$

with  $\mathbf{u}(t, \mathbf{J}) = \mathbf{R}_T(t, \mathbf{J}) / R_T(t, \mathbf{J})$  is unit direction vector of the transmitting signal SAR relative to the target at time  $t$ .

Neglected the scatter coefficient difference in the different SAR and the width of sinc function, the received signal after range compression is given by

$$s(t, \mathbf{t}, \mathbf{J}) = D \mathbf{d}(\mathbf{t} - \frac{R(t, \mathbf{J})}{c}) \exp(-\frac{j2pR(t, \mathbf{J})}{I}) \quad (4)$$

where  $t$  is the time in azimuth dimension,  $\mathbf{t}$  is the time in range dimension,  $D$  depends upon the scatter coefficient of targets and the gain of transmitting and receiving antennas. The received signal by cluster SAR system in time domain after compensating range migration and matched each other in range dimension is given by

$$\mathbf{S}(t, \mathbf{J}) = \exp(-j2bR_T(t, \mathbf{J})) \mathbf{a}(\mathbf{u}(t, \mathbf{J})) \quad (5)$$

with  $\mathbf{b} = 2p/I$ ,

$\mathbf{a}(\mathbf{u}(t, \mathbf{J})) = D_m \exp(j\mathbf{b}\mathbf{u}(t, \mathbf{J}) \cdot \mathbf{r}_m)_{m=1}^N$  is the target array response vector. By the range formula, the phase history of the target for the transmitting signal SAR is  $\Phi(s, \mathbf{J}) = 4pR_T(s, \mathbf{J})/I$ , correspondingly, the

instantaneous frequency is according to

$$\begin{aligned} f(t, \mathbf{J}) &= -\frac{1}{2p} \frac{\partial \Phi(t, \mathbf{J})}{\partial t} \\ &= -\frac{2}{I} \mathbf{u}(t, \mathbf{J}) \cdot \mathbf{V}(t, \mathbf{J}) \end{aligned} \quad (6)$$

where  $\mathbf{V}(t, \mathbf{J}) = (\mathbf{V} - \mathbf{v}) + (\mathbf{A} - \mathbf{a})(t - t_0)$  is the instantaneous velocity vector of the radar relative to target. Because the relative position between radar and target is changing with time  $t$  and the radial direction of vector  $\mathbf{u}(t, \mathbf{J})$  is also changing, and the vector  $\mathbf{V}(t, \mathbf{J})$  is linear function of the time, thus its motion locus is radial in three dimensions space and the radial direction is changeless during the azimuth processing time. If the radial direction of velocity vector  $\mathbf{V}(t, \mathbf{J})$  is denoted as  $\mathbf{V}_c(\mathbf{J})$ , the formula (6) can be expressed by

$$f(t, \mathbf{J}) = -\frac{2}{I} |\mathbf{V}(t, \mathbf{J})| \cos \angle(\mathbf{u}(t, \mathbf{J}), \mathbf{V}_c(\mathbf{J})) \quad (7)$$

the instantaneous Doppler frequency depends upon parameter  $\mathbf{J}$  and time  $t$ , if two targets have the same Doppler frequency and different parameter  $\mathbf{J}$ , by (6) and (7), it can be proved  $\mathbf{u}(t, \mathbf{J}_1) \neq \mathbf{u}(t, \mathbf{J}_2)$ .

$\mathbf{S}(t, \mathbf{J})$  is composed of a common chirp term and a slow variety array response vector, its Fourier transform is given by

$$\mathbf{S}(f, \mathbf{J}) = \int \exp(-j2bR_T(t, \mathbf{J})) \mathbf{a}(\mathbf{u}(t, \mathbf{J})) e^{-j2pft} dt \quad (8)$$

When the time-bandwidth product is enough large, by the stationary phase theory, applied Taylor expansion to  $-(j2bR_T(t, \mathbf{J}) + j2pft)$  and neglected the high order term, (8) can be expressed as

$$\mathbf{S}(f, \mathbf{J}) = e^{-j2pft_0} e^{-j2bR_T(t_0, \mathbf{J})} \int \exp(-j\mathbf{b} \frac{\partial^2 R_T(t, \mathbf{J})}{\partial t^2} \Big|_{t=t_0} t^2) dt \quad (9)$$

By  $\mathbf{d}(t) = \lim_{c \rightarrow 0} (1/c \sqrt{j\mathbf{p}}) e^{j\mathbf{t}^2/c^2}$  and the formula  $(1/c \sqrt{j\mathbf{p}}) \int x(t) e^{j(t-t_0)^2/c^2} dt \approx x(t_0)$  [1], (9) can be expressed as

$$\mathbf{S}(f, \mathbf{J}) \approx K \exp(-j2pft_0) \mathbf{S}(t_0, \mathbf{J}) \quad (10)$$

with  $K = \sqrt{j\mathbf{p} / (-\mathbf{b} \frac{\partial^2}{\partial t^2} R_T(t, \mathbf{J}) \Big|_{t=t_0})}$ . When the

frequency  $f$  is constant, (10) can be expressed as

$$\mathbf{S}(f, \mathbf{J}) \approx c\mathbf{a}(\mathbf{u}(f, \mathbf{J})) \quad (11)$$

By the locked relation between the Doppler frequency and the azimuth time, if two targets have same frequency  $f$  and different parameter  $\mathbf{J}$ , then they have different  $\mathbf{u}(f, \mathbf{J})$ .

#### 4. THE MOVING TARGETS DETECTION BASED ON ARRAY SIGNAL PROCESSING AND WVD

In the cluster SAR system, neglected the target scatter coefficient difference in different SAR, the received signal from stationary targets without range migration is given by

$$\mathbf{C}(t, \mathbf{J}) = \mathbf{s}(t_0) \mathbf{S}(t, \mathbf{J}) \quad (12)$$

where  $\mathbf{s}(t_0)$  is the target back scattered coefficient, it depend upon the range and azimuth position between target and radar. When the range  $|\mathbf{R}|$  is constant,  $\mathbf{s}(t_0)$  is function of time  $t_0$ , it corresponds to swath on the ground parallel with the fly path, and is the scatter function of all targets on the swath. If the received signal is denoted as  $\mathbf{S}_0(t) = \mathbf{S}(t, \mathbf{0})$  at time  $t_0 = 0$  when the stationary targets in the ground is located in the azimuth beam centre of the SAR transmitting signal, then the received signal by cluster SAR system can be denoted as follows

$$\mathbf{C}(t) = \mathbf{S}_0(t) \otimes \mathbf{s}(t) \quad (13)$$

where  $\otimes$  denotes the convolution. If the cluster SAR system is regarded as time-invariable vector filter and the target scattered coefficient  $\mathbf{s}(t)$  as input signal, then  $\mathbf{S}_0(f)$  is the system transfer function. When  $\mathbf{s}(t)$  is the realization of the gauss stationary process with spectral density matrix  $\mathbf{R}_s(f)$ , the clutter  $\mathbf{C}(t)$  is also stationary random process, its spectral density matrix is given by

$$\mathbf{R}_C(f) = \mathbf{S}_0(f) \mathbf{R}_s(f) \mathbf{S}_0^H(f) \quad (14)$$

When there are moving targets in clutter and noise, the received signal by cluster SAR system can be presented by

$$\mathbf{Z}(t) = \mathbf{S}(t, \mathbf{J}) + \mathbf{Q}(t) \quad (15)$$

where  $\mathbf{Q}(t) = \mathbf{C}(t) + \mathbf{N}(t)$  is clutter plus noise. When  $\mathbf{N}(t)$  is stationary process with spectral density matrix  $\mathbf{R}_N(f)$ , the  $\mathbf{Q}(t)$  is also stationary process with spectral density matrix  $\mathbf{R}_Q(f) = \mathbf{R}_C(f) + \mathbf{R}_N(f)$ .

Applied Fourier transform to signal  $\mathbf{Z}(t)$

$$\mathbf{Z}(f) = \mathbf{S}(f, \mathbf{J}) + \mathbf{Q}(f) \quad (16)$$

As mentioned, the array response in cluster SAR system is depend upon the parameter  $\mathbf{J}$  and frequency  $f$ , if taken maximum achievable SCNR (signal to clutter plus noise ratio) as optimization criterion, the optimal beam weights of (16) is evaluated as follows

$$\mathbf{W}(f, \mathbf{J}) = \mathbf{R}_Q^{-1}(f) \mathbf{S}(f, \mathbf{J}) \quad (17)$$

Applied Fourier transform to the received signal by cluster SAR system and multiplied the optimal beam weights for matched filter; the output signal after filtering is given by

$$z(\mathbf{J}) = \mathbf{W}^H(f, \mathbf{J}) \mathbf{Z}(f) \quad (18)$$

If there are moving targets signal in the received signal, there will also be moving targets signal component in the each frequency component after filtering, the moving targets detection is given by

$$T(\mathbf{Z}, \mathbf{J}) = \left| \int \mathbf{S}^H(f, \mathbf{J}) \mathbf{R}_Q^{-1}(f) \mathbf{Z}(f) df \right|^2 \quad (19)$$

The moving targets detection in frequency domain consists of the clutter suppression and coherent summation, if the two processing are separated for signal processing, then the clutter suppression and coherent summation can be carried out in the different domain. The output signal after clutter suppression is given by

$$\hat{\mathbf{Z}}(f, \mathbf{J}) = \mathbf{R}_Q^{-1}(f) \mathbf{Z}(f) \quad (20)$$

The inverse Fourier transform of (20) is given by

$$\hat{\mathbf{Z}}(t, \mathbf{J}) = F^{-1}[\mathbf{R}_Q^{-1}(f) \mathbf{Z}(f)] \quad (21)$$

with  $\hat{\mathbf{Z}}(t, \mathbf{J}) = (\hat{z}_1(t, \mathbf{J}), \hat{z}_2(t, \mathbf{J}), \Lambda, \hat{z}_{N_c}(t, \mathbf{J}))^T$ ,

$\hat{z}_k(t, \mathbf{J})$  is the received signal after clutter suppression by the  $k$  th SAR. Thus the moving targets detection is transformed into the follow

$$\begin{aligned} T(z, \mathbf{J}) &= \left| \int \mathbf{S}^H(t, \mathbf{J}) \hat{\mathbf{Z}}(t, \mathbf{J}) dt \right| \\ &= \left| \int \hat{z}_1(t, \mathbf{J}) s_1^*(t, \mathbf{J}) dt + \int \hat{z}_2(t, \mathbf{J}) s_2^*(t, \mathbf{J}) dt + \Lambda + \int \hat{z}_{N_c}(t, \mathbf{J}) s_{N_c}^*(t, \mathbf{J}) dt \right| \end{aligned} \quad (22)$$

The moving targets detection can be carried out respectively by each SAR. By the conservation of the inner product of the WVD<sup>[4-7]</sup>, the optimal detection with the  $k$  th SAR based on WVD is given by

$$\begin{aligned} T(z, \mathbf{J}) &= \left| \int_{-\infty}^{+\infty} \hat{z}_k(t, \mathbf{J}) s_k^*(t, \mathbf{J}) dt \right|^2 \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_{\hat{z}_k}(t, f) W_{s_k}^*(t, f) dt df \end{aligned} \quad (23)$$

By respectively calculating WVD of  $\hat{z}_k(t, \mathbf{J})$  and  $s_k^*(t, \mathbf{J})$ , then the moving targets can be detected by compared their inner product to the threshold. Because the signal power concentrates on the instantaneous frequency curve at  $t-f$  plane, the double integral in (31) can be turned into curve integral.

The echoes from the moving targets is chirp signal, the Doppler centroid  $f_{DC}$  and the Doppler rate  $f_{DR}$  depend upon the parameter  $\mathbf{J}$ , it corresponds to the parameter of the maximum power curve at  $t-f$  plane. Therefore, the detection processing in (23) can be turned into calculating the WVD of  $\hat{z}_k(t, \mathbf{J})$ , then calculating the curve integral of  $W_{\hat{z}_k}(t, f)$  by changing the parameters  $f_{DC}$  and  $f_{DR}$ . When the curve integral for maximum power is more than the threshold, the parameter of beeline is regard as Doppler parameters for moving targets. When the SCNR of the  $k$  th SAR is not low, the above detection and estimating processing can be given by the following steps

- (1) Calculating the inverse Fourier transform of  $z_k(f, \mathbf{J})$  and obtain the time domain signal  $\hat{z}_k(t, \mathbf{J})$ .

(2) Calculating WVD of  $\hat{z}_k(t, \mathbf{J})$ .

(3) Estimating the instantaneous frequency  $f(t)$  by the following formula.

$$f(t) = \frac{1}{2\mathbf{p}} \frac{\partial \mathbf{j}}{\partial t} = \frac{\int_{-\infty}^{+\infty} fW(t, f)df}{\int_{-\infty}^{+\infty} W(t, f)df} \quad (24)$$

(3) Estimating the instantaneous phase by the following formula.

$$\mathbf{f}(t) = 2\mathbf{p} \int_{t_0}^t f(\mathbf{t})d\mathbf{t} + \mathbf{f}(t_0) \quad (25)$$

(5) Constituting the reference function as follows

$$s_k(t) = \exp(-j\mathbf{f}(t)) \quad (26)$$

The above steps are effective when the SCNR is enough large, and the SCNR can be enhanced by suppressed and smoothed the cross term of the WVD.

In the clutter suppression, it is necessary to estimate the spectral density matrix of the clutter and noise, the matrix  $\mathbf{R}_Q(f)$  can be estimated by the following formula

$$\hat{\mathbf{R}}_Q(f) = \frac{1}{N} \sum_{n=1}^N \mathbf{Z}_n(f) \mathbf{Z}_n^H(f) \quad (27)$$

where  $\mathbf{Z}_n(f)$  denotes the azimuth spectrum correspond to the  $n$  range cell,  $N$  is the numbers of range cells for the matrix estimation. After clutter suppression, the azimuth spectrum is composed of the moving targets spectrum component and the clutter spectrum component remained, the SCNR has been improved. In order to reduce calculating load, the azimuth spectrum after suppression clutter can be used for the moving targets elementary detection, the elementary detection is given by

$$T(\mathbf{Z}, \mathbf{J}) = \int e^T |\mathbf{R}_Q^{-1}(f) \mathbf{Z}(f)|^2 df \quad (28)$$

with  $e = (1, 1, \Lambda, 1)^T$ . By compared the value of (28) to the threshold, it is possible to elementary judge whether there is moving targets.

### 5. THE IMAGING OF MOVING TARGETS

By the foregone analysis, under the conditions that the range migration can be neglected, the detection and parameters estimating can be carried out based on WVD of the received signal after suppression clutter. For the spaceborne SAR, the range migration is dependent upon range resolution, the more low range resolution is, and the more small range migration is. Therefore, in the moving targets detection and imaging processing, the range migration can be neglected by degrading the range resolution according to the rough motion state of the moving targets that will be imaged. Having detected the presence of the moving targets, the range resolution can be gotten back and the range migration can be compensated by the phase modulated law. By the relation of range and phase shift, the range migration compensating formula is given by

$$d(t) = \frac{1}{4\mathbf{p}} \mathbf{f}(t) \quad (29)$$

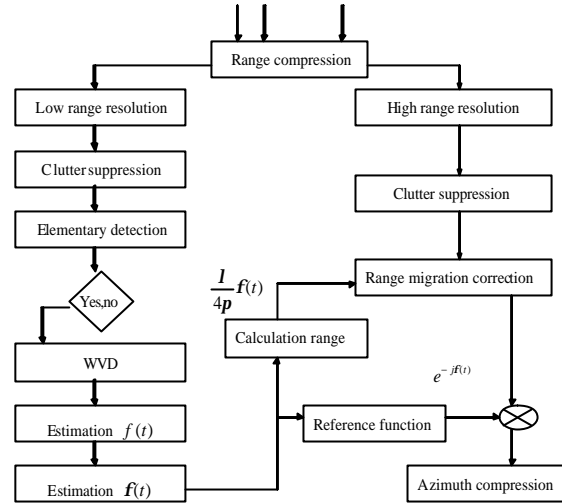


Fig.2 the imaging processing flow chart of moving target

where  $\mathbf{f}(t)$  is the instantaneous phase estimated according to (25). The processing flow chart of the moving targets detection and imaging by cluster SAR system is showed in figure 2.

### 6. SIMULATION

Suppose resolution for the single SAR of cluster SAR system is  $5\text{m} \times 5\text{m}$ , the cluster SAR system makes up of four SAR satellites, these satellites distributed along the circle with diameter for  $240\text{m}$  according to equal angle, the plane of cluster SAR satellites parallels with the ground. The satellite  $S_1$  is located at the positive half  $x$ -axis and transmit signal, four SAR simultaneously receive echo from targets, the range and azimuth dimensions respectively get 8192 and 4096 samples.

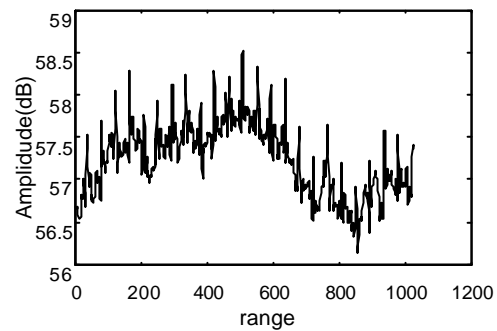


Fig.3 azimuth spectrum summation before suppression clutter

The background clutter power is normalized, the clutter to noise power ratio is  $-25\text{dB}$ , the moving targets to background clutter mean power ratio is  $0\text{dB}$ , and the moving targets takes up four resolution cells and moves at  $8\text{km/h}$ . The azimuth spectrum summation according to range cell is presented in figure 3, the moving targets can not be detected, corresponding to the figure 3, figure 4 shows the azimuth summation after suppression clutter, it is the result of elementary detection by formula (28). As shown by the figure 4, the clutter has been suppressed commendably and the

moving targets can be detected. The instantaneous frequency estimated by WVD of the received signal after suppression clutter is presented in figure 5. Figure 6 shows the simulation SAR image by the SAR transmitting signal, the moving targets image produced by the reference function  $\varrho_6$  is presented in figure 7. The result of the simulation shows that the cluster SAR system can obtain the image of the slow moving targets.

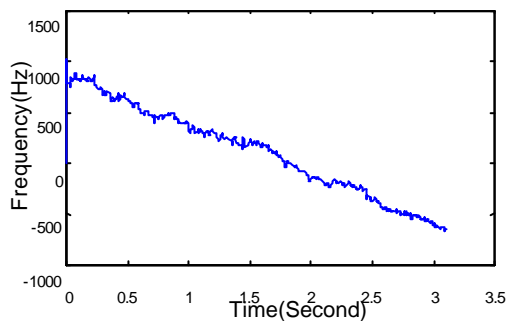


Fig 5 the moving target instantaneous frequency

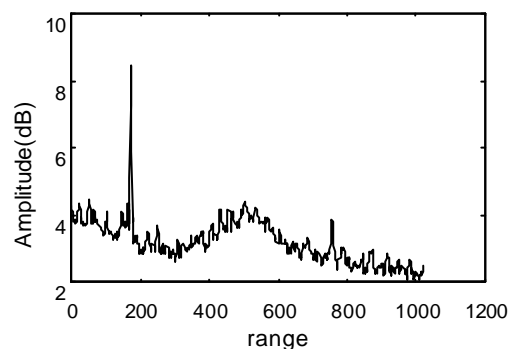


Fig.4 azimuth spectrum summation after clutter suppression

## 7. CONCLUSIONS

The cluster SAR system makes up of many SAR satellites, compared with the single SAR, the echo from target contains the Doppler information and the arriving angle information. In this paper, we analyze the echo characteristic in time domain and frequency in detail, and have proved theoretically that the clutter can be suppressed by cluster SAR system. On the basis of theoretical analysis, by taking advantage of the array response vector difference between stationary targets and moving targets, we propose an algorithm for the moving targets detection and imaging based on the array signal processing and WVD, and this algorithm is validated by simulation. The results of theoretical analysis and simulation show that the cluster SAR system can achieve the targets SAR image even if the targets is being moving slowly.

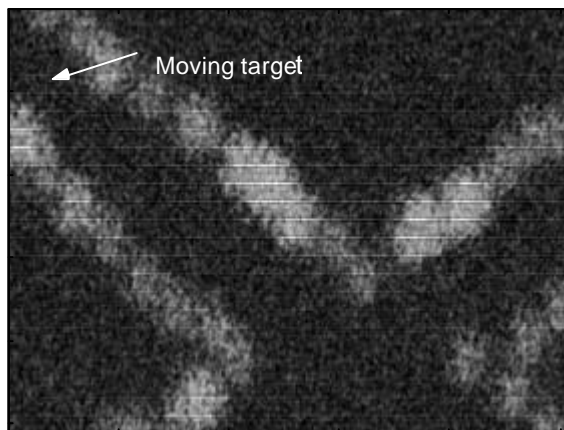


Fig.6 the simulation SAR imaging

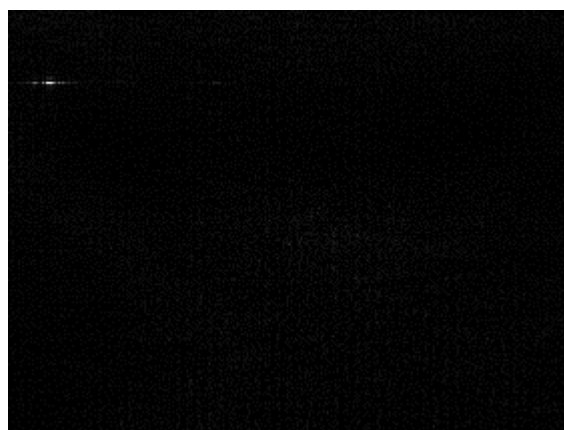


Fig.7 the moving target imaging

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