APPLICATION OF GENERALIZED RIDGE ESTIMATE IN COMPUTING THE EXTERIOR ORIENTATION ELEMENTS OF SATELLITIC LINEAR ARRAY SCANNER IMAGERY

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ABSTRACT:

With the development of space technology, space photogrammetry is becoming a direction of photogrammetry. Satellitic linear array scanner imagery, as chief space imagery, is used increasingly. It is used widely in geospatial data production and updating. Because linear array scanner imagery is dynamic, its exterior orientation elements change along with the movement of satellite and normal equation that we use when its exterior orientation elements are computed is ill-conditioned. First, this paper analyses previous methods of solving this problem. Then, generalized ridge estimate is presented to do it. The method of educing the parameters of generalized ridge estimate is explained in the end. Experiments evince this method is very simple, steady and effective.

1. INTRODUCTION

1.1 Mathematical Model

Solid scanner that uses linear array device as received device usually scans by push-broom sensor. Because its projection method is row-central, we can establish collinearity equation for each row of the image. If the direction of linear array is horizontal and that of flight is vertical, the collinearity equations can be written as follows:

$$\left.\begin{aligned} x &= -f\,\frac{a_1(X-X_s)+b_1(Y-Y_s)+c_1(Z-Z_s)}{a_3(X-X_s)+b_3(Y-Y_s)+c_3(Z-Z_s)} \\ 0 &= -f\,\frac{a_2(X-X_s)+b_2(Y-Y_s)+c_2(Z-Z_s)}{a_3(X-X_s)+b_3(Y-Y_s)+c_3(Z-Z_s)} \end{aligned}\right\} \quad (1)$$

The exterior orientation elements of linear array scanner imagery is a function of time. They can be described approximately by multinomial of time considering satellitic posture is often stable. Supposing the origin of photo coordinate system lies in midpoint of the center scanner line, we can consider that the exterior orientation elements of every scanner line is changed with "y". If the exterior orientation elements of every scanner line are linear functions, it can be written as follows:

$$\begin{cases} \varphi = \varphi^0 + k_\varphi y \\ \omega = \omega^0 + k_\omega y \\ \kappa = \kappa^0 + k_\kappa y \\ X_s = X_s^{\ 0} + k_{X_s} y \\ Y_s = Y_s^{\ 0} + k_{Y_s} y \\ Z_s = Z_s^{\ 0} + k_{Z_s} y \end{cases} \quad (2)$$

Where $\varphi^0, \omega^0, \kappa^0, X_s^0, Y_s^0, Z_s^0$ =exterior orientation elementsof the center scanner line of a image
$k_\varphi, k_\omega, k_\kappa, k_{X_s}, k_{Y_s}, k_{Z_s}$ =variation rates of the exterior orientation elements
$\varphi, \omega, \kappa, X_s, Y_s, Z_s$ =exterior orientation elements of line "y".

If we linearize equations (1) considering ground coordinates and image coordinates of control points have been known ($dX = dY = dZ = dx = dy = 0$), finally, the error equations of exterior orientation can be written as follows:

$$\left.\begin{aligned} &V_{F_x} = a_{11}d\varphi + a_{12}d\omega + a_{13}d\kappa + a_{14}dX_s + a_{15}dY_s \\ &+a_{16}dZ_s + a_{11}ydK_\varphi + a_{12}ydk_\omega + a_{13}ydk_\kappa + \\ &a_{14}ydk_{X_s} + a_{15}ydK_{Y_s} + a_{16}ydK_{Z_s} + F_x^{\ 0} \\ &V_{F_y} = a_{21}d\varphi + a_{22}d\omega + a_{23}d\kappa + a_{24}dX_s + a_{25}dY_s \\ &+a_{26}dZ_s + a_{21}ydK_\varphi + a_{22}ydk_\omega + a_{23}ydk_\kappa + \\ &a_{24}ydk_{X_s} + a_{25}ydK_{Y_s} + a_{26}ydK_{Z_s} + F_y^{\ 0} \end{aligned}\right\} \quad (3)$$

Equations (3) are observation equations that the exterior orientation elements of linear array scanner imagery are computed. There are 12 unknowns in Equations (3), so we will need not less than 6 control points if we compute the exterior orientation elements of linear array scanner imagery and their variation rates.

Many experiments shows there is a strong relativity among unknowns of Equations (3). The correlation is also called multi-collinearity. Especially in SPOT image, the correlation is much in evidence. The correlation causes the coefficient matrix of normal equation is closer to singular. Normal equation is ill-conditioned. In the end, the exterior orientation elements are computed imprecisely or the exterior orientation elements can not be computed at all.

1.2 Current Status of Research

Four methods are often used to overcome the correlation and compute the exterior orientation elements. Adding fictitious error equation is one of methods. It is effective, because it adds the independence of each parameter. But this method adds workload, moreover, it is based on more data, such as some orbit parameters, satellitic photo data, and so on. Incorporating very correlative elements is the other method. In the condition of closer to vertically photograph, because among the exterior orientation elements exist a relation of multi-collinearity, this method is OK. But its precision is hardly assured when camera is not closer to photograph vertically. The third method is the plan that line elements and angle elements are computed by separate iterativeness. Because it can often weaken the correlation, its application has more valuable than others. But this method isn't rigorous in theory. The precision and iterative numbers of its result depend on the precision of its initial value. The last method is application of ridge estimate. The ridge estimate is also entitled ordinary ridge estimate. Because it uses biased estimate to compute the exterior orientation elements, it will overcome the correlation. But this method must compute the ridge parameter. The ridge trace method is usually used to decide its ridge parameter. But this method is short of rigorous theoretical basis and has a certain of subjectivity. Because it need draw, its application isn't convenient. If we use the double "H" formula to compute, its result is not precise. The double "H" formula is used in follow experiments. Generalized ridge estimate is presented to do it in this condition.

2. THEORETICAL BASIS AND TASK STEPS

2.1 Theoretical Basis

Generalized ridge estimate is called GRE for short. It was presented by Hoerl and Kennard in 1970. It is a generalize form of ridge estimate. In China, professor Zheng Zhaobao has made use of generalized ridge estimate to do experiments of computing inner and exterior orientation elements of aerial image at the same time.

When the relation of multi-collinearity exists, LS estimate is not optimum estimate can be proved.

Function model is given as follows:

$$\left.\begin{aligned} &L = AX + \Delta_n, rk(A) = t \\ &E(\Delta_n) = 0 \\ &Cov(\Delta_n) = \sigma_0^2 I_m \end{aligned}\right\} \tag{4}$$

Veracity is scaled by Mean Square Error. Mean Square Error is called MSE for short. Here $\hat{X}$ indicates a certain of estimate value, its Mean Square Error is defined as

$$MSE(\hat{X}) = E\left\|\hat{X} - X\right\|^2 = E((\hat{X} - X)^T(\hat{X} - X))$$

Which can also be written as

$$MSE(\hat{X}) = tr(Cov(\hat{X})) + \left\|E(\hat{X}) - X\right\|^2 \tag{5}$$

Mean Square Error scales the difference of between a estimate value and its true value. A good estimate should have a smaller MSE.

$$\hat{X} = [\hat{x}_1 \, \hat{x}_2 \cdots \hat{x}_t]^t$$

Precede formula is supposed. So the first item of equation (5) can be written as

$$tr(Cov(\hat{X})) = \sum_{i=1}^{t} Var(\hat{x}_i)$$

The first item of equation (5) is the sum of variance of each component. The second item of equation (5) can be written as

$$\left\|E(\hat{X}) - X\right\|^2 = \sum_{i=1}^{t}(E(\hat{x}_i) - x_i)^2$$

It is the square sum of deviation of each component. A good estimate should has a smaller mean square error, namely, both "tr(Cov($\hat{X}$))" and " $\left\|E(\hat{X}) - X\right\|^2$ " are smaller.

If " $\hat{X}$ " is a LS estimate of "X", formula can be deduced as follows:

$$\hat{X} = (A^T A)^{-1} A^T L$$

Because " $\hat{X}$ " is a unbiased estimate of "X", the second item of equation (5) can be written as

$$\left\|E(\hat{X}) - X\right\|^2 = \left\|X - X\right\|^2 = 0$$

The following equation has already been known.

$$Cov(\hat{X}) = \hat{\sigma}_0^2 (A^T A)^{-1}$$

So equation (5) can be written as follows:

$$MSE(\hat{X}) = \hat{\sigma}_0^2 \, tr((A^T A)^{-1})$$

The mean square error can be deduced as

$$MSE(\hat{X}) = \hat{\sigma}_0^2 \sum_{i=1}^{t} 1/\lambda_i \tag{6}$$

Here λ_i (i=1,2,…t) is a eigenvalue of $A^T A$. Because the relation of multi-collinearity exists, normal matrix " $A^T A$ " is ill-conditioned, closer to singular, and its least eigenvalue is much closer to zero. So MSE becomes very big and LS is not a optimum estimate.

Because ridge estimate uses " $A^T A+\kappa I$ " to take the place of " $A^T A$ ", the least eigenvalue of normal matrix will increase from " $\min\lambda_i$ " to " $\min\lambda_i+\kappa$ (κ>0)". So MSE will decrease. In fact ridge estimate is realized by increasing the second item of equation (5) properly in exchange for the first item of equation (5) being decreased more. If here " κ" is a number that it is constant, bigger than zero, the estimate will go by the name of ordinary ridge estimate. But if " κ" is a vector that it is constant ,bigger than zero vector, the estimate will go by the name of generalized ridge estimate.

$\tilde{X}$ denotes generalized ridge estimate value and it can be estimated as,

$$\tilde{X}=(A^T PA+\kappa I)^{-1}A^T PL \tag{7}$$

$$\kappa = \mathrm{diag}(\kappa_1,\ \kappa_2,\ \cdots \kappa_t)$$

Where P is a weight matrix. Because here generalized ridge parameters κ_i (i=1,2,…t) are not always equal to each other, MSE of generalized ridge estimate will become smaller than that of ordinary ridge estimate in theory.

That ridge parameters are implemented is a important problem. MSE is the smallest when " κ" is the best. By experiments, that ridge parameters are computed is very effective as follows:

$$\kappa_i=\sigma^2\Big/\hat{\alpha}_i^{\,2} \tag{8}$$

Where σ^2 represents unit-weight reference variance. Eq. (9) then leads to the a posteriori reference variance estimate.

$$\sigma^2=V^T PV\big/(n-t) \tag{9}$$

$$\hat{\alpha}_i=\hat{X}\,Q_i^T \tag{10}$$

where Q_i = a eigenvector of matrix $A^T PA$

$\hat{X}$ = a vector that is computed by LS estimate.

The initial values of exterior orientation elements have little influence on the results that are computed by this method.

2.2 Process of the Method

The process that exterior orientation elements are computed by GRE can be divided into several steps:

Step 1: Computation of the initial values of exterior orientation elements.

Step 2: Computation of the estimate value " $\hat{X}$ " by LS estimate. First, designed matrix "A", vector "L" and weight matrix "P" are constructed. Second, matrix " $A^T PA$ (N)" and " $A^T PL$ (U)" are computed. In the end, the estimate value " $\hat{X}$ " is computed by LS estimate.

Step 3: Computation of general ridge parameters " κ". First, matrix "Q", consists of eigenvector of matrix " $A^T PA$ ", is computed. Second, vector " $\hat{\alpha}$ " is computed by Eq. (10). Third, vector "V" is computed by Eq. (3). Fourth, " σ^2 " is computed by Eq. (9). In the end, general ridge parameters " κ " is computed by Eq. (8).

Step 4: Computation of the estimate value " $\tilde{X}$ " by GRE.

Step 5: The values of exterior orientation elements are updated by adding " $\tilde{X}$ ".

Step 6: We judge whether " $\tilde{X}$ " is smaller than threshold value. If " $\tilde{X}$ " is smaller than threshold value, the values of exterior orientation elements here are the last results. If " $\tilde{X}$ " is bigger than threshold value, computation will go on processing iteratively by entering step 2. Iterative computation will not be over until " $\tilde{X}$ " is smaller than threshold value.

2.3 Extra Illustrations

Eq. (7) is only a form of General ridge estimate. General ridge estimate is usually defined as follows:

$$\hat{X}_K=(A^T PA+\phi K\phi^T)^{-1}A^T PL \tag{11}$$

where ϕ = a orthogonal matrices.

$$\phi^T A^T PA\phi=\Lambda=diag(\lambda_1\cdots\lambda_t)$$
$$K=diag(k_1\cdots k_t)$$

where k_i (i=1,2,…t) = generalized ridge parameter.

3. EXPERIMENTS AND CONCLUSIONS

3.1 Experiments

Two SPOT images are used by experiments. They go separately by the name of image 01 and image 02. Ground-coordinates of control points are measured from 1:50,000 scale maps of the same area. Each image has nine directional points and twenty points that are used to check. It is comparative measure that we use different methods to compute the exterior orientation elements and compute coordinates of image points, then compute mean square error of coordinates of image points.

Table 1 is gained when initial values of exterior orientation elements are good. But Table 2 is not. Little square estimate is called LS for short. Method that line elements and angle elements are computed by separate iterativeness is called LA . Ordinary Ridge Estimate is called OR and generalized ridge estimate is called GR for short. Mean square errors of directional points are called V_{dx} and V_{dy}. Mean square errors of points that are used to check are called V_{cx} and V_{cy}.

3.2 Conclusions

Table 1 shows that precision of exterior orientation elements are computed by GR is the best when initial values of exterior orientation elements are good. Table 2 shows that precision of exterior orientation elements are computed by GR is the best and what is computed by LA is the worst when initial values of exterior orientation elements are not good.. Both of tables show precision of exterior orientation elements are computed by OR is better and what is computed by LS is worse. Experiments evince that exterior orientation elements are computed by GR is very effective and almost isn't influence of initial values of exterior orientation elements.

Unit：Micron

Image	Method	V_{dx}	V_{dy}	V_{cx}	V_{cy}
01	LS	26	28	40	37
01	LA	35	31	37	31
01	OR	26	29	38	34
01	GR	29	30	31	32
02	LS	408	138	232	51
02	LA	19	32	43	32
02	OR	15	27	52	30
02	GR	16	31	45	29

Table 1. Mean square errors of points when initial values of exterior orientation elements are good

Unit：Micron

Image	Method	V_{dx}	V_{dy}	V_{cx}	V_{cy}
01	LS	26	27	52	39
01	LA	97	32	77	31
01	OR	26	28	52	35
01	GR	27	32	47	32
02	LS	10	26	51	27
02	LA	127	30	112	33
02	OR	12	25	52	27
02	GR	16	28	44	26

Table 2. Mean square errors of points when initial values of exterior orientation elements are not good

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