FAST RECTIFICATION FOR SPACE-BORNE SAR DIGITAL IMAGES
WITH NO GROUND CONTROL POINT

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KEY WORDS: SAR Image, Pixel Spacing, The Ground Correction, Affine Transformation

ABSTRACT:
In this paper, a fast geometric rectification method for space-borne SAR (Synthetic Aperture Radar) digital image in slant range with no ground control point is introduced, which is based on the imaging principle of SAR and the law of geometric distortion in slant range image. The main idea is at first to resample a slant range SAR image to a ground range image with a definite ground spacing because the ground spacing corresponding to each pixel in slant is different caused by the different view angle. The coordinate system of the resampled image can be recognized as a plane rectangular coordinate system formed by the flight direction and the ground range direction. For the plain areas, the ground range image can be transformed into the image in a projected geographic coordinate system by four corner points whose ground coordinates can be calculated by the imaging equation. The rectified image is obtained with high accuracy; While for the areas with fluctuation topography, a higher precision rectification can be made based on the ground range image for it maintains the information of the original SAR slant range image (ground range and flight height).

1. INTRODUCTION
Space-Borne SAR can work without the limitation of the time, the weather and the boundary of nations. What’s more, it has the ability of some penetrating, which makes the SAR image be widely used in the construction of the economy and the national defence. Many applications such as the military reconnaissance, the disguise identification, the assessment of destroy in battlefield and the estimation for nature disaster need the rectified SAR image for position and identification. Recently, the methods of rectification for SAR image are mainly the following three: polynomial rectification method, digital differential rectification method and analog image rectification method. Polynomial rectification method is based on the geometric transformation of two-dimension images. This method can be used for the rectification of the plain areas, but the precision is lower because the principle of SAR imaging is not taken into account. Therefore, the polynomial rectification method is mainly used in non-mapping applications; The digital differential rectification method transform the SAR original image to orthophoto pixel by pixel using SAR imaging model and DEM (Digital Elevation Model). Its precision is higher than that of the polynomial rectification method. So it is mainly used in mapping applications, but the rate is lower because of iterative calculation. The analog image rectification method is very complex and is not always successful.

The fast geometric rectification method introduced in this paper is based on the study of SAR imaging principle and the law of geometric distortion in slant range image. The results of the experiment indicate that the time spent in the fast geometric rectification is only one tenth of that spent in the digital differential rectification and their precision is almost equal.

2. SAR IMAGING MODEL
F.Leberl imaging model is used in the paper, it includes two condition equations [Franz Leber, 1978].

\[ (X - Xs)^2 + (Y - Ys)^2 + (Z - Zs)^2 = (D_{s0} + y \cdot My)^2 \]  

where,

\( (X, Y, Z) \) = The object space coordinates of the ground point P
\( (Xs, Ys, Zs) \) = The object space coordinates of the center of the antenna’s instantaneous positions (They are the polynomial functions of the flying time \( T \)).

They can be expressed as follows:

\[
\begin{align*}
Xs &= x_0 + Xs_0 \cdot T + \hat{X}s_0 \cdot T^2 + \cdots \\
Ys &= Ys_0 + Ys_0 \cdot T + \hat{Y}s_0 \cdot T^2 + \cdots \\
Zs &= Zs_0 + Zs_0 \cdot T + \hat{Z}s_0 \cdot T^2 + \cdots \\
\end{align*}
\]

\( T = x \cdot dT \)

In the formula:

\( Xs_0, Ys_0, Zs_0 \) = he object space coordinates of the SAR antenna center corresponding to the origin of the image.

\( Xs_0, Ys_0, Zs_0 \) = he spacecraft velocity vector corresponding to the image origin

\( \hat{X}s_0, \hat{Y}s_0, \hat{Z}s_0 \) = he spacecraft acceleration vector corresponding to the image origin.

\( X \) = he plain coordinate of the pixel in azimuth of the SAR image.
$y$ = the plain coordinate of the pixel in range of the SAR image
$T$ = the flying time corresponding to the coordinate $x$ (relative to image origin)
$dT$ = the flying time that each line in azimuth spends
$My$ = the pixel spacing in the slant range of the SAR image
$D0$ = the close slant range

2. **Zero Doppler Condition**

\[
\dot{X}_s (X - X_s) + \dot{Y}_s (Y - Y_s) + \dot{Z}_s (Z - Z_s) = 0
\]  \tag{3}

where,
\[
\dot{X}_s = \frac{\partial X_s}{\partial T} = \ddot{X}_s 0 + 2 \cdot \dot{X}_s 0 \cdot T + \ldots \\
\dot{Y}_s = \frac{\partial Y_s}{\partial T} = \ddot{Y}_s 0 + 2 \cdot \dot{Y}_s 0 \cdot T + \ldots \\
\dot{Z}_s = \frac{\partial Z_s}{\partial T} = \ddot{Z}_s 0 + 2 \cdot \dot{Z}_s 0 \cdot T + \ldots
\]


The pixel spacing of SAR image in azimuth direction is often different from that in range direction. The azimuth pixel spacing $M_x$ is equal to the ground spacing in azimuth direction; while any range pixel spacing $M_i$ is not equal to another in ground range. (See Figure 1) $\theta_i$ presents the view angle of pixel $i$; $D_{si}$ presents the slant range corresponding to the pixel $i$; $D0$ is the close slant range; $H$ is the elevation of the center antenna. Therefore:

\[
\sin \theta_i = \frac{M_y}{M_i} \quad \cos \theta_i = \frac{H}{D_{si}} \quad H = Zs
\]

\[
\therefore \sin^2 \theta_i + \cos^2 \theta_i = 1
\]

\[
\therefore \left( \frac{H}{D_{si}} \right)^2 + \left( \frac{M_y}{M_i} \right)^2 = 1
\]

Thus,
\[
M_i = \frac{M_y D_{si}}{\sqrt{D_{si}^2 - H^2}} = \frac{M_y}{\sqrt{1 - \left( \frac{H}{D_{si}} \right)^2}} \quad \tag{4}
\]

What’s more,
\[
M_i = M_y \cdot \frac{(D0 + y_i \cdot M_y)}{\sqrt{(D0 + y_i \cdot M_y)^2 - H^2}} \quad \tag{5}
\]

We can see that $M_y$ is a constant and $M_i$ will decrease if $\theta$ increases from formula 4, that is, the ground resolution increases if $\theta$ increases.

Figure 2 shows the relationship between the slant range images and the ground range image in range spacing.

4. **The Law of the Ground Rectification**

1. Planning the range spacing and resampling in each line of the image

The velocity and the altitude of the satellite relative to the earth’s surface do not change very much in a certain period. So we can resume $H = \overline{H} - Z0$, $\overline{H}$ presents the mean height of the satellite, $Z0$ presents the height of the planning plain.

The ground spacing $M_i$ of each pixel in ground range can be calculated from formula 5. Then each pixel spacing of a scan line of the slant range image was resampled to a ground spacing $M_i$ (using the nearest point method or the linear interpolation method).

Figure 3 shows:


\[ M_i = M_y \times D_{Si} \sqrt{D_{Si}^2 - H^2} \]

\[ S_j = \sum_{k=1}^{i} M_k \]

Corresponding,

\[ j = \frac{S_j}{M} \quad (6) \]

5. EXPERIMENT

5.1 The Datum of the Experiment

In this experiment, the ERS-1 data obtained in March, 1996 is used, which covers some plain area in our country. The data includes the satellite antenna’s station vectors and speed vectors of five times in the Earth Centered Rotating System, long semi-axis of the reference ellipsoid, short semi-axis of the reference ellipsoid, spacing \( M_y \) in slant range, spacing \( M_x \) in azimuth, Zero-doppler range time (two-way) of first range pixel (millisecond), the line number of the image and the pixel number in range, the time of first line, the time of the last line and the geodetic coordinates of the four corner, etc.

5.2 The Step of the Experiment

1) Firstly, according to the relationship between the reference coordinate system — — Earth Centered Rotating System and the geodetic coordinate system, the station vectors at the five given moments in the Earth Centered Rotating System are transformed to the vectors in geodetic coordinate system. And the satellite’s mean height above the earth surface of the five given moments are calculated. Secondly, according to the relationship between the geodetic coordinates and the TM projection Coordinate System, the station vectors at the five given moments are transformed to the vectors in the TM projected System.

2) Imitate the Station Vector-Time polynomials of formula 2 using the five station vectors. And calculate TM Coordinates of the four ground points corresponding to the four corners on the image using F.Leberl imaging model.

3) Ascertain the scope of the ground using the coordinates calculated by the step 2. Then rectify the image by using the indirect-method with F.Leberl imaging equation.

4) The original slant range image is resampled to the ground range image by determining the ground spacing \( M \sim 20 \text{ meters} \).

5) The ground range image is rectified by affine transformation. Then the final rectified image is derived by the fast rectification method.

6) We compared the difference between the coordinates calculated by the fast rectification method and that calculated by F.Leberl imaging equation.

5.3 Results

The original slant range image is shown in figure 4-a; The resampled ground range image is shown in figure 4-b; The final rectified image by the fast rectification method is shown in figure 4-c. (The pixel spacing of the rectified image: 20 meters /pixel; The elevation of the planning plain: 10 meters).

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<th>600</th>
<th>1400</th>
<th>2200</th>
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<td>-0.14</td>
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<td>-1.18</td>
</tr>
<tr>
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</tr>
<tr>
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<tr>
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<td>-0.81</td>
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<td>-0.75</td>
<td>-0.99</td>
<td>-1.01</td>
<td>-0.74</td>
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The Least difference: 0.011068
The Largest difference: 1.670942
The Mean Square difference: 0.687616

Table 1. The Difference of Coordinate Y (Unit: Pixel)
The difference between the coordinates calculated by the fast rectification method and that calculated by F.Leberl imaging equation method are list in table 1 (the difference of the coordinate Y) and table 2 (the difference of the coordinate X).

<table>
<thead>
<tr>
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<th>800</th>
<th>1400</th>
<th>2200</th>
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<tr>
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<tr>
<td>5000</td>
<td>0.12</td>
<td>0.16</td>
<td>0.12</td>
<td>0.23</td>
<td>0.18</td>
</tr>
<tr>
<td>6000</td>
<td>0.04</td>
<td>0.11</td>
<td>0.10</td>
<td>0.24</td>
<td>0.24</td>
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<tr>
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</tr>
<tr>
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<td>-0.13</td>
<td>-0.02</td>
<td>0.32</td>
<td>0.57</td>
</tr>
</tbody>
</table>

The Least difference:0.000143
The Largest difference:0.565479
The Mean Square difference:0.215431

Table 2. The Difference of Coordinate X (Unit: Pixel)

The speed of the fast rectification is higher than that of the rectification by F.Leberl equation. What’s more, the precision of the fast rectification is similar to the method by F.Leberl equation. Because the fast rectification of the SAR image has a large number of applications in the real time surveillance for nature ravage and other applications, the fast rectification method introduced in the paper must be widely used in the future. Further more, the ground range image generated in the course can be used for a more precision rectification because its distortion derived from the fluctuation of the topography can be corrected with DEM.

6. CONCLUSION

ACKNOWLEDGEMENT

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REFERENCE

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