RIGOROUS QUALITY ASSESSMENT OF 3D OBJECT RECONSTRUCTION FOR AN ARBITRARY CONFIGURATION OF CONTROL POINTS

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ABSTRACT

3D object reconstruction that is based on an overdetermined set of control points is considered in this contribution. The crucial difficulty of quality assessment of 3D reconstruction is nonlinearity of the fundamental equations that describes relationship between coordinates of corresponding points in image and reality. Solution of this difficulty is the main objective of this contribution. Probability distribution of measurement errors is not approximated by normal distribution, but by a special distribution that allows exact and efficient Bayesian solution of the original nonlinear problem. Bayesian approach to nonlinear estimation results in true probability distribution of estimated parameters that enables rigorous quality assessment of position of the reconstructed object.

1 INTRODUCTION AND NOTATION

A 3D object has to be reconstructed from several images captured by a multiple view camera system. Multiple view reconstruction has been extensively studied by many authors. For example, (Hartley and Zisserman, 2000) approached this problem from the geometrical point of view. In the other hand, statistical analysis of the linear least-squares solution is stressed in (Förstner, 1998).

The presented approach offers direct solution of the reconstruction problem in its original nonlinear form without distinguishing relative and absolute orientation.

To state mutual position of all the cameras a set of control points is utilized. The geometry of such a multiple view camera system can be described by the following equation:

$$\mathbf{x}_i = \mathbf{x}_{0,i} + s_{ij}\,\mathbf{R}_j\,\mathbf{x}'_{ij}\ , \qquad i \in I\ ,\ j \in J \tag{1}$$

where

- $\mathbf{x}_i$ ... vector of coordinates of i-th control point in ground coordinate system (world coordinates), $\mathbf{x}_i = [x_i, y_i, z_i] \in \mathbb{R}^3\ ,\ i \in I := \{1, \dots, n\}$
- $\mathbf{x}_{0,j}$ ... world coordinates of projection center of j-th camera, $j \in J := \{1, \dots, m\}$
- $\mathbf{x}'_{ij}$ ... vector of coordinates of i-th control point in the image coordinate system of j-th camera, $\mathbf{x}'_{ij} = [x'_{ij}, y'_{ij}, -z'_j] \in \mathbb{R}^3\ ,\ j \in J_i \subseteq J$
- z'_j ... focal length of j-th camera, $j \in J$
- s_{ij} ... scale factor of i-th control point in the image of j-th camera, $i \in I\ ,\ j \in J_i$
- $\mathbf{R}_j$ ... orthonormal rotation matrix that describes orientation of j-th camera in the ground coordinate system, $j \in J$

Note that quantities $\mathbf{x}'_{ij}$, s_{ij} need not be necessarily accessible for all pairs $[i, j]$, $i \in I$, $j \in J$ since some control points can be visible from only a subset of all the cameras. Set of images where i-th control point is displayed is denoted $J_i\ ,\ |J_i| \geq 2$.

Parameters $\mathbf{x}_{0,j}$, $\mathbf{R}_j$ for all $j \in J$ have to be estimated to state the mutual position of the all cameras. To consider orthonormality of matrices $\mathbf{R}_j$ another set of equations has to be added to (1):

$$\mathbf{R}_j^T\,\mathbf{R}_j = \mathbf{I}\ ,\ \forall j \in J\ . \tag{2}$$

Here I stands for identity matrix of 3-rd order.

Coordinates $\tilde{\mathbf{x}} = [\tilde{x}, \tilde{y}, \tilde{z}]$ of a point on the object being reconstructed can be computed from its image coordinates $\tilde{\mathbf{x}}'_j = [\tilde{x}'_j, \tilde{y}'_j, -z'_j]$ of j-th camera as follows.

$$\tilde{\mathbf{x}} = \mathbf{x}_{0,j} + \tilde{s}_j\,\mathbf{R}_j\,\tilde{\mathbf{x}}'_j\ , \qquad j \in \tilde{J} \subseteq J\ ,\ |\tilde{J}| \geq 2\ . \tag{3}$$

System of equations (3) for unknowns $\tilde{\mathbf{x}}$, $\{\tilde{s}_j \,|\, j \in \tilde{J}\}$ is overdetermined even for $|\tilde{J}| = 2$ (stereo rig). It is possible to solve it together with system (1) and estimate all the unknown parameters $\tilde{\mathbf{x}}$, $\{\tilde{s}_j \,|\, j \in \tilde{J}\}$, $\{\mathbf{x}_{0,j} \,|\, j \in J\}$, $\{\mathbf{R}_j \,|\, j \in J\}$, $s_{ij} \,|\, i \in I,\ j \in J_i\}$, $\{z'_j \,|\, j \in J\}$ simultaneously. Note that focal lengths z'_j, $j \in J$ need not be included to the unknown parameters and rather be treated as constants if they were determined sufficiently precisely in advance. The case of unknown focal lengths is considered in this contribution since it is more general and more suitable for practice.

Reasonable solution of system of equations (1), (2), (3) offers Bayesian approach. This approach results in probability density function (pdf) of the estimated parameters called a posterior pdf. The dispensable parameters $\{\tilde{s}_j \,|\, j \in \tilde{J}\}$, $\{s_{ij} \,|\, i \in I,\ j \in J_i\}$, $\{z'_j \,|\, j \in J\}$, $\{\mathbf{x}_{0,j} \,|\, j \in J\}$, $\{\mathbf{R}_j \,|\, j \in J\}$ can be easily eliminated from the posterior pdf by integrating over them. The final result is then given as a pdf of coordinates of the object point $\tilde{\mathbf{x}}$ that depends only on the measured coordinates $\mathbf{x}'_{ij}$, $\mathbf{x}_i$, $\tilde{\mathbf{x}}'_j$.

2 PROBLEM FORMULATION

Probability distribution of object point $\tilde{\mathbf{x}}$ has to be determined for given $\tilde{\mathbf{x}}'_j$, $j \in \tilde{J}$ on the basis of equations (1), (2), (3). Bayesian solution of equations (1), (2), (3) can be clearly expressed after modification of them to another equivalent system of equations.

$$\left.\begin{array}{rcll} \mathbf{x}_i &=& \mathbf{x}_{0,j} + s_{ij}\,\mathbf{x}_{ij}\,, & i \in I,\, j \in J_i, \\ \mathbf{x}'_{ij} &=& \mathbf{R}_j^T\,\mathbf{x}_{ij}\,, & i \in I,\, j \in J_i, \\ \mathbf{I} &=& \mathbf{R}_j^T\,\mathbf{R}_j\,, & j \in J, \\ \tilde{\mathbf{x}}'_j &=& \frac{1}{\tilde{s}_j}\,\mathbf{R}_j^T\,(\tilde{\mathbf{x}} - \mathbf{x}_{0,j})\,, & j \in \tilde{J}\,. \end{array}\right\} \quad (4)$$

In this form measured quantities $\mathbf{x}_i$, $\mathbf{x}'_{ij}$, $\tilde{\mathbf{x}}'_j$ are separated from unknown parameters $\{\tilde{s}_j \,|\, j \in \tilde{J}\}$, $\{s_{ij} \,|\, i \in I,\, j \in J_i\}$, $\{\mathbf{x}_{ij} \,|\, i \in I,\, j \in J_i\}$, $\{\mathbf{x}_{0,j} \,|\, j \in J\}$, $\{\mathbf{R}_j \,|\, j \in J\}$, $\tilde{\mathbf{x}}$. The number of unknown parameters was extended due to addition of auxiliary parameter $\mathbf{x}_{ij} := [x_{ij}, y_{ij}, -z'_j]$ (coordinates in a fictitious image perpendicular to the (x, y) plane of the ground coordinate system), but it does not matter since the all dispensable parameters will be eliminated by integration as mentioned above.

To make the Bayesian approach clear let us group the measured quantities together into vector

$$\boldsymbol{\eta} := \Big[[\mathbf{x}_i \,|\, i \in I],\, \big[[x'_{ij},\, y'_{ij}] \,|\, i \in I,\, j \in J_i\big]\,,\, \Big[[\tilde{x}'_j,\, \tilde{y}'_j] \,|\, j \in \tilde{J}\Big]\Big]$$

and, similarly, denote the unknown parameters by vector

$$\boldsymbol{\theta} := \Big[\tilde{\mathbf{x}}\,, [\mathbf{R}_j \,|\, j \in J],\, [\mathbf{x}_{0,j} \,|\, j \in J]\,, [\tilde{s}_j \,|\, j \in \tilde{J}],\, [z'_j \,|\, j \in J], \\ [[x_{ij},\, y_{ij}] \,|\, i \in I,\, j \in J_i]\,,\, [s_{ij} \,|\, i \in I,\, j \in J_i]\Big]$$

Furthermore, let us denote vector of directly measured values $\hat{\boldsymbol{\eta}}$ and measurement errors by vector $\boldsymbol{\varepsilon}$ so that

$$\boldsymbol{\eta} = \hat{\boldsymbol{\eta}} + \boldsymbol{\varepsilon}\,. \quad (5)$$

As a last step before application of Bayesian approach to equations (4) a prior information about values of parameters being estimated has to be expressed. In our case of equations (4) conditions of orthonormality of matrix $\mathbf{R}_j$, namely (2) have to be considered. Furthermore, the fact that focal length z'_j of j-th camera is common for all the points in j-th image has to be expressed as another constraints for unknown parameters. These constraints can be simply obtained from the second and fourth row of (4) where focal length is included as the third coordinate of points $\mathbf{x}'_{ij}$, $\tilde{\mathbf{x}}'_j$.

$$\left.\begin{array}{rcll} -z'_j &=& \mathbf{r}_{3j}^T\,[x'_{ij},\, y'_{ij},\, -z'_j]^T, & j \in J,\, i \in I_j, \\ -z'_j &=& \frac{1}{\tilde{s}_j}\,\mathbf{r}_{3j}^T\,(\tilde{\mathbf{x}} - \mathbf{x}_{0,j}), & j \in \tilde{J}\,, \end{array}\right\} \quad (6)$$

where

- I_j ... set of indices of such control points that are visible from j-th camera,
- $\mathbf{r}_{3j}$ ... third column of matrix $\mathbf{R}_j$.

Actually, system of equations (4) with constraints (6) represents nonlinear regression equations with constraints for unknown parameters.

Let us denote $\boldsymbol{\Theta}$ set of values of unknown parameters defined by constraints (2), (6). Then system of equations (4) can be written in concise form

$$\boldsymbol{\eta} = \mathbf{a}(\boldsymbol{\theta})\,,\ \boldsymbol{\theta} \in \boldsymbol{\Theta}\,. \quad (7)$$

A prior information about unknown parameters can be formulated in a more detailed way than pure constraints (2), (6). A prior probability distribution of random vector $\boldsymbol{\theta}$ can be introduced. Let us denote its pdf

$$p : \boldsymbol{\Theta} \to \mathbb{R} : \boldsymbol{\theta} \mapsto p(\boldsymbol{\theta})\,.$$

3 BAYESIAN SOLUTION

The requested pdf of unknown parameters is conditional pdf

$$g(\,.\,|\,\hat{\boldsymbol{\eta}}) : \boldsymbol{\Theta} \to \mathbb{R} : \boldsymbol{\theta} \mapsto g(\boldsymbol{\theta}\,|\,\hat{\boldsymbol{\eta}})\,.$$

It can be easily expressed by direct application of Bayes theorem (see e.g. (Koch, 1990)).

$$g(\boldsymbol{\theta}\,|\,\hat{\boldsymbol{\eta}}) = \frac{f(\hat{\boldsymbol{\eta}}\,|\,\boldsymbol{\theta})\,p(\boldsymbol{\theta})}{\displaystyle\int_{\boldsymbol{\Theta}} f(\hat{\boldsymbol{\eta}}\,|\,\mathbf{t})\,p(\mathbf{t})\,d\mathbf{t}}\,. \quad (8)$$

Probability distribution of observational errors of quantities $\{\mathbf{x}_i \,|\, i \in I\}$, $\{[x'_{ij},\, y'_{ij}] \,|\, i \in I,\, j \in J_i\}$ is supposed to be normal. The remaining observations $\{[\tilde{x}'_j,\, \tilde{y}'_j] \,|\, j \in \tilde{J}\}$ are supposed to be independent and identically distributed with conditional pdf $h(\,.\,|\boldsymbol{\theta})$. Then the joint pdf f is given by

$$f(\hat{\boldsymbol{\eta}}\,|\,\boldsymbol{\theta}) = \prod_{k=1}^{l} f_k(a_k(\boldsymbol{\theta}) - \hat{\eta}_k) \prod_{j \in \tilde{J}} h(\tilde{x}'_j|\boldsymbol{\theta})\,h(\tilde{y}'_j|\boldsymbol{\theta})\,, \quad (9)$$

where

- f_k ... normal pdf
$$f_k(\epsilon_k) = \frac{1}{\sqrt{2\,\pi}\sigma_k}\,\exp\left(-\frac{1}{2}\left(\frac{\epsilon_k}{\sigma_k}\right)^2\right),$$
- σ_k ... standard deviation of k-th coordinate of control points,
- l ... number of coordinates of control points,
$$l := 3\,n + 2\sum_{i=1}^{n} |J_i|,$$
- a_k ... k-th function of mapping $\mathbf{a}$ from (7),
- $\hat{\eta}_k$ ... k-th element of vector $\hat{\boldsymbol{\eta}}$.

A prior pdf p will be chosen as noninformative on the set $\boldsymbol{\Theta}$. A proper set of unknowns has to be designed before to assign them uniform pdf.

Probability distribution of of coordinates of an object point $\tilde{\mathbf{x}}$ can be obtained from (8) by integrating over the dispensable variables. Let make a vector from the dispensable variables and denote it $\mathbf{u}$, so that $\boldsymbol{\theta} = [\tilde{\mathbf{x}}'^T, \mathbf{u}^T]$. Then the required pdf is marginal pdf $g(\tilde{\mathbf{x}} \,|\, \hat{\boldsymbol{\eta}})$

$$g(\tilde{\mathbf{x}} \,|\, \hat{\boldsymbol{\eta}}) = \int_{\mathcal{U}} g(\tilde{\mathbf{x}}, \mathbf{u} \,|\, \hat{\boldsymbol{\eta}}) \, d\mathbf{u} \, . \qquad (10)$$

Set $\mathcal{U}$ is defined by the constraints for unknowns (2), (6). It means that $\mathcal{U}$ depends on the all unknown parameters except s_{ij}, i.e. $\{\tilde{s}_j \,|\, j \in \tilde{J}\}$, $\{[x_{ij}, y_{ij}] \,|\, i \in I, j \in J_i\}$, $\{z'_j \,|\, j \in J\}$, $\{\mathbf{x}_{0,j} \,|\, j \in J\}$, $\{\mathbf{R}_j \,|\, j \in J\}$, $\tilde{\mathbf{x}}$. To respect this fact, integration has to be performed in specific order. Therefore variables $\{s_{ij} \,|\, i \in I, j \in J\}$ will be eliminated first. Then integration over z'_j would follow.

Focal length z'_j is constrained by (6), so that coordinates x_{ij}, y_{ij} have to be incorporated to integration as well. Constraints (6) for fixed $j \in J$, $\{\tilde{s}_j \,|\, j \in \tilde{J}\}$, $\{\mathbf{x}_{0,j} \,|\, j \in J\}$, $\{\mathbf{R}_j \,|\, j \in J\}$, $\tilde{\mathbf{x}}$ represent a hyper-plane $\mathcal{V}_j$ in vector space V_j generated by vectors $\left[\, [\, [x_{ij}, y_{ij}] \,|\, i \in I], z'_j \,\right]$. This hyper-plane can be parameterized by a number of another unknown variables which is lower than dimension of of vector space V_j. Integration over hyper-plane $\mathcal{V}_j$ has to be performed with the aid of the new parameters. These new parameters will be assigned by uniform probability distribution. This integration procedure has to be recalled in cycle for all $j \in J$.

Integration over $\mathbf{x}_{0,j}$ can be done directly. For elimination of $\tilde{s}_j$ another variable $t_j = \frac{1}{\tilde{s}_j}$ will be introduced.

For integration over the last set of unknowns - elements of rotation matrix $\mathbf{R}_j$ - special transformation is used. It can be easily showed that an arbitrary orthonormal matrix $\mathbf{R}$ can be expressed by means of a quaternion (see e.g. (Ward, 1997), p. 28).

$$q = q_0 + \hat{i}\, q_1 + \hat{j}\, q_2 + \hat{k}\, q_3 \, ,$$

$$|q|^2 := \sum_{i=0}^{3} q_i^2 = 1 \, , \qquad (11)$$

$$\mathbf{R} = \qquad (12)$$

$$\begin{bmatrix} \begin{vmatrix} q_0^2+q_1^2, & 1 \\ q_2^2+q_3^2, & 1 \end{vmatrix} & 2\begin{vmatrix} q_0, & q_1 \\ q_2, & q_3 \end{vmatrix} & 2\begin{vmatrix} q_0, & -q_1 \\ q_3, & q_2 \end{vmatrix} \\ 2\begin{vmatrix} q_0, & -q_1 \\ q_2, & q_3 \end{vmatrix} & \begin{vmatrix} q_0^2+q_2^2, & 1 \\ q_1^2+q_3^2, & 1 \end{vmatrix} & 2\begin{vmatrix} q_2, & q_1 \\ q_0, & q_3 \end{vmatrix} \\ 2\begin{vmatrix} q_1, & q_2 \\ q_0, & q_3 \end{vmatrix} & 2\begin{vmatrix} q_0, & -q_2 \\ q_3, & q_1 \end{vmatrix} & \begin{vmatrix} q_0^2+q_3^2, & 1 \\ q_1^2+q_2^2, & 1 \end{vmatrix} \end{bmatrix}$$

This matrix is named Rodrigues matrix and such a parameterization of rotation is called Cayley parameterization. (see (Gruen and Huang, 2001), p. 37, (Sansò, 1973)).

This parameterization enables to replace constraints (2) by single constraint (11). Equation (11) describes surface of four-dimensional sphere. Integration over spherical surface is traditionally performed with the aid of parameterization

$$\left.\begin{array}{rcl} q_0 &=& \cos(\varphi_1)\, \cos(\varphi_2)\, \cos(\varphi_3) \\ q_1 &=& \sin(\varphi_1)\, \cos(\varphi_2)\, \cos(\varphi_3) \\ q_2 &=& \sin(\varphi_2)\, \cos(\varphi_3) \\ q_3 &=& \sin(\varphi_3) \end{array}\right\} \qquad (13)$$

where each angle φ_i, $i \in \{1, 2, 3\}$ revolve in circle, i.e.

$$[\,\varphi_1, \varphi_2, \varphi_3\,] \in (0, 2\pi) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \, .$$

The two successive transformations (12), (13) cause vanishing the constraints for unknowns and the integration can be finished.

The resulting pdf $g(\tilde{\mathbf{x}} \,|\, \hat{\boldsymbol{\eta}})$ is in form of very complicated expression. It can be significantly simplified by means of special pdf h introduced in (9). Function h is designed in such a way that integral in the denominator of Bayes theorem (8) can be evaluated analytically.

A posterior pdf $g(\tilde{\mathbf{x}} \,|\, \hat{\boldsymbol{\eta}})$ is explicitly given in closed form that is not so simple to write it down here but it is enough convenient for effective evaluation by computer.

4 CONCLUSION

Probability density function $g(\tilde{\mathbf{x}} \,|\, \hat{\boldsymbol{\eta}})$ of coordinates of an object point that is analytically tractable was presented in this contribution. This convenient feature of $g(\tilde{\mathbf{x}} \,|\, \hat{\boldsymbol{\eta}})$ provides a photogrammetrist with an effective possibility to reliably determine accuracy of position of a point on the reconstructed object for arbitrary configuration of cameras and control points.

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