STEREO PLANE MATCHING TECHNIQUE

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Commission III

KEY WORDS: Stereo Matching, Surface Modeling, Projective Transformation, Homography

ABSTRACT:

This paper presents a new type of stereo matching algorithm called "Stereo Plane Matching". Stereo plane matching adopts least square method under the constraint that all points in an area specified by a polygon should lie on a common plane. This technique utilizes the fact that correspondence between stereo images becomes 2-D projective transformation (homography) within an area where a common plane is projected. Three corresponding point pairs on stereo image pair are used for parameterization of geometry of a plane, which allows computation of homography within the polygon. The most powerful feature of this technique is that it can impose geometrical constraint in planar direction or position. For example, the target plane can be fixed in horizontal or vertical direction, as well as in other specified direction in 3-D space. Another feature is that it can be applied to unrectified stereo pair, so far as its orientation parameters are known. Experimental results show that this algorithm can measure oblique roof or vertical wall of a building.

1. INTRODUCTION

Existing stereo matching techniques automatically measure points or line features, but most of them cannot set constraint in 3-D shape of target objects, which includes one of the most simple 3-D structure, "Plane". One approach of stereo matching with planar constraint can be realized by image registration by homography (Szeliski, 1994). This approach uses the fact that correspondence between stereo images becomes 2-D projective transformation (homography) within an area where a common plane is projected. This approach can impose constraint on the direction of planes parallel to specified plane (Oda et.al.,1997), but other type of constraint, such as parallel to line direction, or going through a specific point, cannot be attained.

In this paper, a new type of stereo matching algorithm called "Stereo Plane Matching" is presented. Stereo plane matching optimizes square difference of pixel value between stereo image pair under constraint that all pixels in specified area should be on a common plane. The most powerful feature of this technique is that it can impose geometrical constraint in planar direction or position. For example, a target plane can be fixed in horizontal or vertical direction, as well as in other specified direction in 3-D space. Another eminent feature is that stereo plane matching can directly measure unrectified stereo pair under the condition that its orientation parameters are known.

Stereo plane matching utilizes the fact that correspondence between stereo images becomes 2-D projective transformation (homography) within a region where a common plane is projected. Three corresponding point pairs, called "control pairs", parameterize the target plane geometry. It is a type of homography with epipolar constraint, and computed by use of three control pairs and epipoles.

This paper first defines the target problem which stereo plane matching technique handles, and presents the basic strategy of stereo plane matching where an evaluation function for least square method is formularized. The following sections show details of stereo plane matching, including how to parameterize a plane in stereo plane matching, how to derive homography between stereo images, how to execute least square method and how to impose constraint on plane position and direction. Some experimental results are also presented to show that this method can measure oblique roof and vertical wall in 3-D city space.

2. PROBLEM DEFINITION AND BASIC STRATEGY

2.1 Problem Definition

Suppose that object A consisted of planer faces is photographed in image I_1 and I_2, and polygon A in I_1 is measured as the contour of face A of object A as show in Figure 1. The target problem is how to determine three-dimensional coordinates of polygon A automatically by stereo matching, under the constraint that all the points are co-planar, and the assumption that interior and exterior orientation parameters of the stereo pair are known. Interior and exterior orientation parameters can give the following sets of coordinate transformation functions:

1) Transformation functions from 3D coordinates $\mathbf{P}(X, Y, Z)$ to image coordinates $\mathbf{p}(x, y)$ which is projection of $\mathbf{P}$:

$$\mathbf{p} = \mathbf{F}(\mathbf{P}) \quad (1)$$

In the case of stereo images I_1 and I_2, we have two functions, $\mathbf{F_1}(\mathbf{P})$ for I_1, and $\mathbf{F}_2(\mathbf{P})$ for I_2.

2) Transformation functions from Image coordinates $\mathbf{p}(x, y)$ to 3D coordinates $\mathbf{P}(X, Y, Z)$, under the condition that one of the coordinates (X, Y, Z) is known:

$$\begin{aligned} \mathbf{P} &= \mathbf{Gx}(\mathbf{p}, X) \\ \mathbf{P} &= \mathbf{Gy}(\mathbf{p}, Y) \\ \mathbf{P} &= \mathbf{Gz}(\mathbf{p}, Z) \end{aligned} \quad (2)$$

Similar to function (1), we have two sets of the functions, $\mathbf{Gx_1}, \mathbf{Gy_1}, \mathbf{Gz_1}$ for I_1, and $\mathbf{Gx_2}, \mathbf{Gy_2}, \mathbf{Gz_2}$ for I_2.

3) Transformation functions from a stereo pair of image coordinates $\mathbf{p_1}(x_1, y_1)$ and $\mathbf{p_2}(x_2, y_2)$ to 3D coordinates $\mathbf{P}(X, Y, Z)$:

$$\mathbf{P} = \mathbf{G_{12}}(\mathbf{p_1}, \mathbf{p_2}) \quad (3)$$

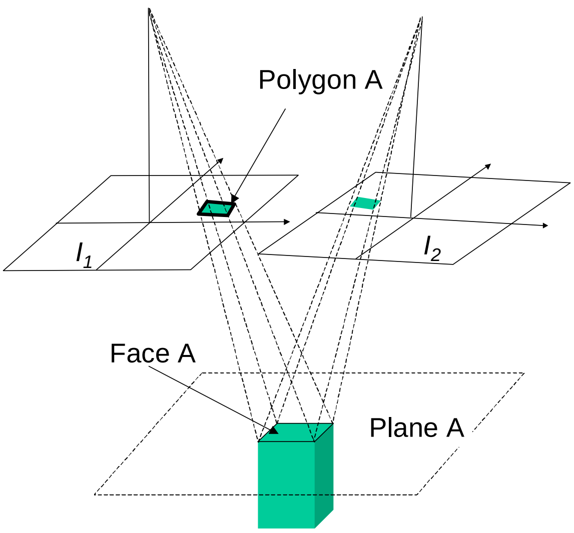


Figure 1. The target problem

2.2 Basic Strategy

Assuming that plane A can be described by a set of parameters $\mathbf{D} = [D_1\ D_2\ D_3]$. The matching function $\mathbf{H}$, which gives matching point $\mathbf{p_{2i}}$ for a given point $\mathbf{p_{1i}}$ within a polygon A, depends on parameter $\mathbf{D}$:

$$\mathbf{p_{2i}} = \mathbf{H}(\mathbf{p_{1i}}, \mathbf{D}) \quad (4)$$

To determinate $\mathbf{D}$ by stereo matching, least square method can be applied to minimize the following evaluation function $\chi(\mathbf{D})$:

$$\chi(\mathbf{D}) = \sum e_i(\mathbf{D})^2 \quad (5)$$

$$e_i(\mathbf{D}) = I_2(\mathbf{H}(\mathbf{p_{1i}}, \mathbf{D})) - I_1(\mathbf{p_{1i}}) \quad (6)$$

where summation in equation (5) is computed for all pixels in polygon A.

3. DEFINITION OF PARAMETER D

Suppose that $\mathbf{Q_1}, \mathbf{Q_2}, \mathbf{Q_3}$ are projected onto $\mathbf{p_{1(1)}}, \mathbf{p_{1(2)}}, \mathbf{p_{1(3)}}$ in I_1 and $\mathbf{p_{2(1)}}, \mathbf{p_{2(2)}}, \mathbf{p_{2(3)}}$ in I_2, respectively. Also suppose that points $\mathbf{p'_{2(1)}}\ \mathbf{p''_{2(1)}}$ are on the epipolar line of $\mathbf{p_{1(1)}}$, points $\mathbf{p'_{2(2)}}$ and $\mathbf{p''_{2(2)}}$ are on the epipolar line of $\mathbf{p_{1(2)}}$, and points $\mathbf{p'_{2(3)}}$ and $\mathbf{p''_{2(3)}}$ on the epipolar line $\mathbf{p_{1(3)}}$. With a set of parameters $\mathbf{D} = [D_1\ D_2\ D_3]$, $\mathbf{p'_{2(1)}}$, $\mathbf{p''_{2(1)}}$, $\mathbf{p'_{2(2)}}$, $\mathbf{p''_{2(2)}}$, $\mathbf{p'_{2(3)}}$ and $\mathbf{p''_{2(3)}}$, $\mathbf{p_{2(1)}}$, $\mathbf{p_{2(2)}}$ and $\mathbf{p_{2(3)}}$ can be described by the following equations:

$$\begin{aligned} \mathbf{p_{2(1)}} &= (1 - D_1) \cdot \mathbf{p'_{2(1)}} + D_1 \cdot \mathbf{p''_{2(1)}} \\ \mathbf{p_{2(2)}} &= (1 - D_2) \cdot \mathbf{p'_{2(2)}} + D_2 \cdot \mathbf{p''_{2(2)}} \\ \mathbf{p_{2(3)}} &= (1 - D_3) \cdot \mathbf{p'_{2(3)}} + D_3 \cdot \mathbf{p''_{2(3)}} \end{aligned} \quad (7)$$

A set of $\mathbf{D}$ determines three corresponding point pairs and three 3-D points on plane A can be computed by equation (3). Since three non-colinear points determine a unique plane, $\mathbf{D}$ determines geometry of plane A.

Here we call pairs of corresponding points "control pairs".

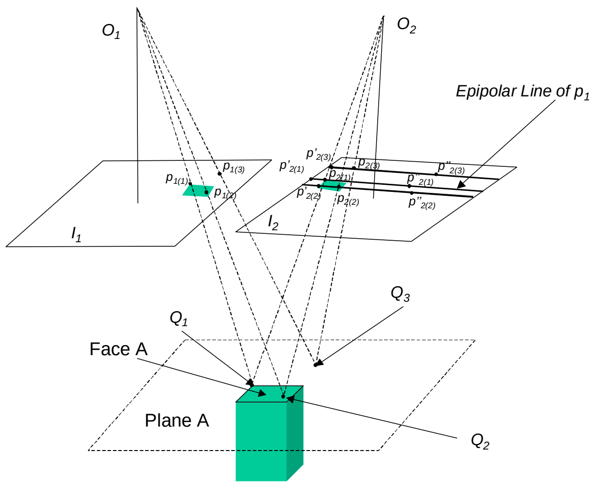


Figure 2. Definition of Planar Parameter D

4. DERIVATION OF FUNCTION H

It is well known that correspondence between $\mathbf{p_1} = (x_1, y_1)$ on I_1 and $\mathbf{p_2} = (x_2, y_2)$ on I_2, where Point $\mathbf{Q}$ on plane A is projected, can be described by projective transformation (or 2-D homography). Therefore, the function H in equation (4) is projective transformation. Projective transformation has eight independent parameters. Projective transformation between homogeneous coordinates $(x_1, y_1, 1)$ and $(x_2, y_2, 1)$ can be expressed with the following equation:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \cong \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} \tag{8}$$

where $\cong$ means equality up to scale factor, that is:

$$[X, Y, Z] \cong [X', Y', Z'] \Leftrightarrow X:Y:Z = X':Y':Z' \tag{9}$$

The matrix on the right side of equation (8) is called homography matrix.

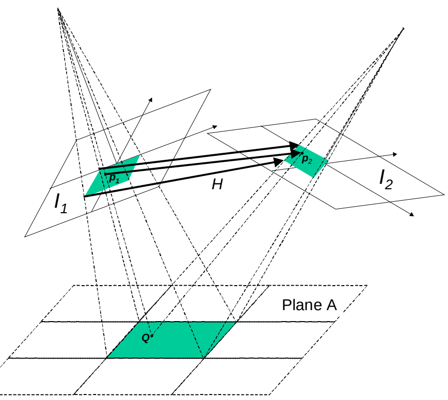


Figure 3. Homography H for Plane A

Two linear equations can be derived by substituting a pair of corresponding points between I_1 and I_2 into equation (8). This means that 8 parameters of the projective transformation can be determined with four pairs of corresponding points. Three of those can be given by control pairs that are used in the definition of the parameter $\mathbf{D}$ in equation (7). Next we show that epipolar geometry provides the fourth pair of corresponding points; epipoles.

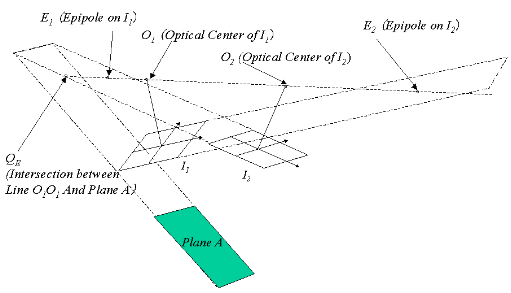


Figure 4. Epipoles

Epipoles $\mathbf{E_2}$ and $\mathbf{E_2}$ are intersection points between image planes and the straight line $\mathbf{O_1O_2}$ which goes through two optical center $\mathbf{O_1}$ and $\mathbf{O_2}$ as shown in Figure 4. Note that epipoles are the projection of the point $\mathbf{Q_E}$, where $\mathbf{O_1O_2}$ intersects with plane A. In other words, these epipoles are also corresponding points for plane A.

Epipoles are known to have the properties below.

1) Epipoles are independent from geometry of plane A and determined only by epipolar geometry. Epipoles can be computed by substituting optical centers for $\mathbf{P}$ in equation (1):

$$\begin{aligned} \mathbf{E_1} &= \mathbf{F_1}(\mathbf{O_2}) \\ \mathbf{E_2} &= \mathbf{F_2}(\mathbf{O_1}) \end{aligned} \tag{10}$$

2) All epipolar lines go through epipoles.

3) When an image plane is parallel to $\mathbf{O_1O_2}$, epipolar lines are also parallel and epipole is at infinite distance.

4.1 Determination of H with D and Epipoles

From three control pairs and a pair of epipoles gives eight linear equations for eight unknown parameters of homography matrix. These equations can be written in matrix style as follows:

$$\mathbf{M} \cdot \mathbf{B} = \mathbf{V} \tag{11}$$

where $\mathbf{B} = {}^t[a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8]$, $\mathbf{M}$ is a 8×8 matrix, and $\mathbf{V}$ is 8 dimensional vector.

Homogeneous expression of projective transformation allows handling epipoles in infinite distance. For example, when all epipolar lines are parallel to x axis both in I_1 and I_2, homogeneous coordinates of epipoles are $[1, 0, 0]$. Substitution of this coordinates to both side of equation (8) leads to:

$$\begin{aligned} a_4 &= 0 \\ a_7 &= 0 \end{aligned} \tag{12}$$

5. OPTIMIZATION WITH LEAST SQUARE METHOD

We have adopted Gauss-Newton method for non-linear least square optimization. This method revises target parameters $\mathbf{D}$ with $\Delta\mathbf{D}$:

$$D + \Delta D \Rightarrow D \tag{13}$$

$\Delta\mathbf{D}$ can be computed by the following equation:

$$\Delta\mathbf{D} = -({}^tJ \cdot J)^{-1} \cdot {}^tJ \cdot \mathbf{e} \tag{14}$$

$$\mathbf{e} = {}^t\left[e_1\ e_2\ \dots\ e_n\right] \tag{15}$$

$$J = {}^t\left[{}^t\left(\frac{\partial e_1}{\partial \mathbf{D}}\right)\ {}^t\left(\frac{\partial e_2}{\partial \mathbf{D}}\right)\ \dots\ {}^t\left(\frac{\partial e_n}{\partial \mathbf{D}}\right)\right] \tag{16}$$

where weights of all points are assumed to be equal and n is number of samples.

This computation is repeated until the evaluation function $\chi(D)$ converges on the minimum value.

Equations (15) and (16) shows that computation of e_i and its partial differentiation can realize least square optimization. The value e_i can be easily computed by equation (6). The following discussion gives how partial differentiation can be computed.

Denoting $\mathbf{p_{1i}} = (x_i, y_i)$ and $\mathbf{p_{2i}} = (x_i', y_i')$, and differentiating equation (6), we have:

$$\frac{\partial e_i}{\partial D_j} = \frac{\partial}{\partial x_i'}(I_2 - I_1) \cdot \frac{\partial}{\partial D_j}x_i' + \frac{\partial}{\partial y_i'}(I_2 - I_1) \cdot \frac{\partial}{\partial D_j}y_i'$$

$$= \frac{\partial I_2}{\partial x_i'} \cdot \frac{\partial}{\partial \mathbf{B}}x_i' \cdot \frac{\partial \mathbf{B}}{\partial D_j} + \frac{\partial I_2}{\partial y_i'} \cdot \frac{\partial}{\partial \mathbf{B}}y_i' \cdot \frac{\partial \mathbf{B}}{\partial D_j} \tag{17}$$

Partial differentiation of I_2 in x and y direction can be numerically computed with pixel values of I_2.

From Equation (8), $\frac{\partial}{\partial \mathbf{B}}x_i'$ and $\frac{\partial}{\partial \mathbf{B}}y_i'$ can be described as following:

$$\frac{\partial}{\partial \mathbf{B}}x_i' = \begin{bmatrix} x_i/G \\ y_i/G \\ 1/G \\ 0 \\ 0 \\ 0 \\ -x_i' \cdot x_i/G \\ -x_i' \cdot y_i/G \end{bmatrix} \qquad \frac{\partial}{\partial \mathbf{B}}y_i' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ x_i/G \\ y_i/G \\ 1/G \\ -y_i' \cdot x_i/G \\ -y_i' \cdot y_i/G \end{bmatrix} \tag{18}$$

$$G = a_7 x_i + a_8 y_i + 1$$

Differentiation of both side of equation (11) by D_j gives:

$$\frac{\partial M}{\partial D_j} \cdot B + M \cdot \frac{\partial B}{\partial D_j} = \frac{\partial V}{\partial D_j} \tag{19}$$

This leads:

$$\frac{\partial B}{\partial D_j} = \mathbf{M}^{-1} \cdot \left(\frac{\partial V}{\partial D_j} - \frac{\partial M}{\partial D_j} \cdot B\right) \tag{20}$$

$\frac{\partial M}{\partial D_j}$ and $\frac{\partial V}{\partial D_j}$ can be computed with equations (7) and (8). Thus all items in equation (17) can be calculated.

6. STEREO PLANE MATCHING UNDER VARIOUS CONSTRAINTS

One particular merit of stereo plane matching is that various constraints can be implemented by fixing control pairs.

6.1 Fixation of One/Two Points on Plane in 3-D Space

Fixation of one or two points on the plane in 3-D space means fixation of one or two of control pairs. One-point fixation can be implemented simply by setting projection pair of this point as $\mathbf{p_{1(3)}}$ and $\mathbf{p_{2(3)}}$. Fixation of two points can be also implemented by setting projection pairs of these points as $\mathbf{p_{1(3)}}$-$\mathbf{p_{2(3)}}$ and $\mathbf{p_{1(2)}}$-$\mathbf{p_{2(2)}}$

6.2 Constraint in the Direction of Plane

There are two types of constraint in the direction of a plane. One is the specification of direction parallel to the plane, and the other is the specification of direction perpendicular to the plane.

6.2.1 Constraint in direction parallel to the target plane: In this case, vanishing points in the specified direction can be used as a fixed control pair as show in Figure 5. This type of constraint decrease the degree of freedom of $\mathbf{D}$ to two. For example, vertical walls of buildings in an aerial photo have a common vanishing point: the vertical point. The Vertical points in both images can be the control pair that is common in all vertical planes.

Vanishing points in direction $\mathbf{v}$ can be calculated by the following equation:

$$\begin{aligned} \mathbf{Ev_1} &= \mathbf{F_1}(\mathbf{O_1} + \mathbf{v}) \\ \mathbf{Ev_2} &= \mathbf{F_2}(\mathbf{O_2} + \mathbf{v}) \end{aligned} \tag{21}$$

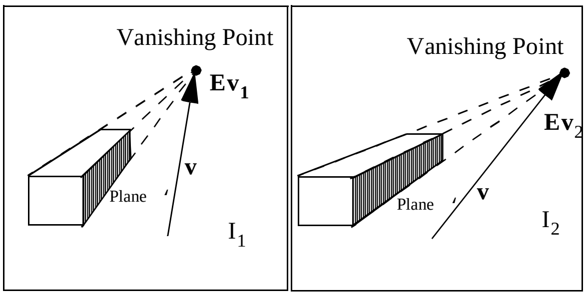


Figure 5. Matching between Vanishing Points in Direction v

6.2.2 Constraint in direction perpendicular to the target plane: in this case, two control pairs can be fixed by two pairs of vanishing points in directions that are perpendicular to the specified direction. Thus degree of freedom of parameter $\mathbf{D}$ comes down to one. However, this implementation does not work when straight line $\mathbf{O_1O_2}$ is parallel to the target plane, since an epipole and all vanishing points align in a common line and M in the equation (11) becomes a singular matrix.

There is a more robust way to calculate homography between stereo images by the following theorem:

Theorem 1: Suppose that plane Π_0 and Π_1 are parallel and $\mathbf{H}_{\Pi_0}$ and $\mathbf{H}_{\Pi_1}$ are their homography matrices between stereo images. Then, homography matrix for any plane parallel to Π_0 and Π_1 can be described in following form:

$$\mathbf{H}(D) = (1 - D) \cdot \mathbf{H}_{\Pi_0} + D \cdot \mathbf{H}_{\Pi_1} \qquad (22)$$

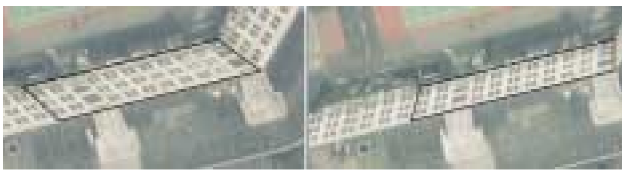
Appendix A gives the detailed proof of this theorem.

The equation (22) can compute homography matrix for any planes parallel to Π_0 and Π_1. The optimum parameter D is the value that minimizes the evaluation function $\chi(D)$ and can be computed simply by 1-D search of D.

7. EXPERIMENTS AND RESULTS

Experiments has been performed with a pair of stereo images with a scale of 1/4000, which cover the former building of Institute of Industrial Science (IIS), Tokyo University. Stereo plane matching has been implemented on softcopy plotter, “Geo-Plotter”, which has been developed by Asia Air Survey Co., Ltd. (Sakamoto et al. 2000).

(1)The polygon which traced the wall in the left image



(2)Result of stereo plane matching

Figure 6. Test with a Vertical Wall

7.1 A Test with a Vertical Wall

To demonstrate the effect of constraint in direction of the plane, a wall of the old IIS building was plotted as polygon, whose height was initially fixed to a certain height. Figure 6(1) shows the initial state just after an operator traced the wall in the left image, where the polygon in right image was not fit on the wall. After matching operation, all points on polygon in the right image came to the exact matching position on the wall as shown in Figure 6 (2).

(1)The polygon which traced the oblique roof in the left image

(2)Result of 1-D search based on theorem 1

(3)Result of stereo plane matching without constraint

Figure 7. Test with an Oblique Plane

7.2 A Test with an Oblique roof

To demonstrate measurement ability for general oblique plane, a part of roof of the old IIS building was traced as a polygon. Figure 7 (1) shows the initial state just after tracing the roof in the left image. After that, stereo matching based on theorem 1 searched the horizontal plane which gave the best match for the polygon as shown in Figure 7 (2). This gave good initial estimation of parameter $\mathbf{D}$. Figure 7 (3) shows that stereo plane matching without constraint polygon computed better result than simple 1-D search.

8. CONCLUSION

We have proposed a stereo plane matching technique to impose constraint on automatic stereo matching process. This technique is considered as least square matching under the constraint that all points in a specified area are fit on a common plane. Parameterization with control pairs including epipoles enables implementation of geometrical constraint in planar direction or position. This technique can measure planar surface of man-made structure, which includes vertical walls or oblique roofs. We plan to utilize this technology for developing user interface for softcopy mapping system, where an operator can measure planar surface easily without adjusting height.

REFERENCE

Oda, K., Kano, H., and Kanade, T., 1997. Generalized disparity and its applications for multi-stereo camera calibration. *Optical 3-D measurement Techniques IV*, pp.109-116.

Sakamoto, M., Uchida, O., Doihara, T., Oda, K., Lu, W., Obata, M., 2000. Geo-plotter - a softcopy mapping system for low cost digital mapping process. *IAPRS*, Amsterdam, The Netherlands, XXXII, Part B4, pp.889-892.

Szeliski, R., 1994. Image Mosaicing for Tele-Reality Applications, DEC CRL 94/1, April.

APPENDIX A: PROOF OF THEOREM 1

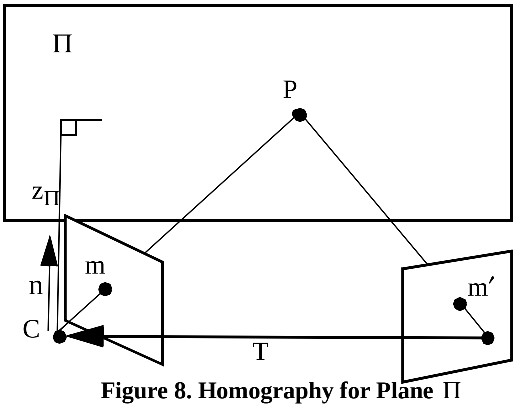


Figure 8. Homography for Plane Π

Homography matrix for plane Π is determined by geometric relationship between the stereo cameras (C and C') and the plane Π:

$$H_\Pi = \alpha \cdot M'\left(R + \frac{1}{z_\Pi} T \cdot {}^t n_\Pi\right) M^{-1} \tag{23}$$

where M and M' are matrices of interior orientation parameters of C and C', R is the rotation matrix between C and C', T is the translation vector from C' to C in C''s coordinate system, n_Π is a normal vector to the plane Π in C's coordinate system, z_Π is the depth from the camera C to plane Π in the direction of n_Π, and α is a scale factor which adjust the lower right element of the matrix to 1. The matrix of interior orientation M is a 3 x 3 matrix:

$$M = \begin{bmatrix} F/s_x & 0 & u \\ 0 & F/s_y & v \\ 0 & 0 & 1 \end{bmatrix} \tag{24}$$

where F is focal length, s_x is the size of pixel in x direction, s_y is the size of pixel in y direction, and (u, v) is coordinates of the principal point.

Assume that two homography matrices H_{Π_0} and H_{Π_1} for two planes Π_0 and Π_1 are given:

$$H_{\Pi_0} = \alpha_0 \cdot M'\left(R + \frac{1}{z_{\Pi_0}} T \cdot {}^t n_{\Pi_0}\right) M^{-1}$$
$$H_{\Pi_1} = \alpha_1 \cdot M'\left(R + \frac{1}{z_{\Pi_1}} T \cdot {}^t n_{\Pi_1}\right) M^{-1} \tag{25}$$

If $n_{\Pi_0} = n_{\Pi_1} = n$ (i.e., two planes are parallel) and $\gamma = \alpha_0 / \alpha_1$, we can define another homography matrix H_{Π_β} by a linear combination of H_{Π_0} and H_{Π_1} with parameter β:

$$\begin{aligned} H_{\Pi_\beta} &= (1-\beta)H_{\Pi_0} + \beta \cdot \gamma \cdot H_{\Pi_1} \\ &= \alpha_0 \cdot M'\left(R + \frac{\beta z_{\Pi_0} + (1-\beta) z_{\Pi_1}}{z_{\Pi_0} z_{\Pi_1}} T \cdot {}^t n\right) M^{-1} \end{aligned} \tag{26}$$

Now another homography matrix $H(D)$ with parameter D:

$$H(D) = (1-D) \cdot H_{\Pi_0} + D \cdot H_{\Pi_1} \tag{27}$$

Equation (27) can be deformed into:

$$\begin{aligned} H(D) &= \alpha((1-\beta)H_{\Pi_0} + \beta \cdot \gamma \cdot H_{\Pi_1}) \\ \alpha &= ((1-D)\gamma + D)/\gamma \\ \beta &= D/((1-D)\gamma + D) \end{aligned} \tag{28}$$

Comparison between equation (28) and equation (26) reveals that $H(D)$ is a homography matrix for a plane parallel to Π_1.