K-ORDER SPATIAL NEIGHBOURS BASED ON VORONOI DIAGRAM: DESCRIPTION, COMPUTATION AND APPLICATIONS

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ABSTRACT

Spatial neighbouring relation is a very important concept in spatial information science. It has even been regarded as the base of spatial information system. Neighouring information is indispensable for many operations in spatial information system. Therefore, it is significant to describe and acquire such in an efficient way. Most of current concepts of neighbours only involve objects sharing common boundaries. However, spatial neighbours are not restricted to only this type in practical applications. There is still a lack of systematical discussion on the definition and computation of such spatial neighbours.

In this paper, a more generalized concept of neighbours and some applications in GIS fields are systematically discussed. A model is developed for a particular type of neighbours, called k-order neighbours using Voronoi diagrams. Several efficient methods for computation of k-order neighbours are developed for different cases. Then some possible applications are discussed including spatial query and neighborhood analysis.

1 INTRODUCTION

Spatial neighbour is one of very important concept in spatial information science, and is even regarded as indispensable for spatial information system (Gold, 1989). Neighbour information needs to be maintained during the whole modelling and processing procedure of spatial data. Also, many operations for spatial query and analysis in GIS are based on this concept.

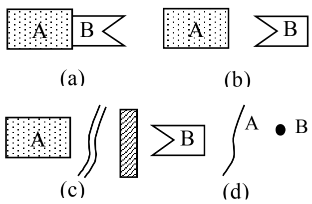


Figure 1. Various kinds of spatial neighbours

Spatial neighbours can be usually defined as objects having common boundaries. However, neighouring relations are not restricted to such a type in real geographical space. In Figure 1, spatial objects A and B in (a) are neighbours, but in (b) and (c), they are not be neighbouring because their boundaries don't touch each other. But spatial interaction still remains between these two objects and the relational information in the cases (b) and (c) is very useful in some applications of spatial query and analysis. The reality is that due to the fact that they have no common boundaries, it is difficult to define and compute the relational information.

For such spatial information, several authors have also mentioned such concept as neighbours among points using the dual graph of Voronoi diagrams, i.e., Delaunay triangular nets, in some practical applications such as generalization and spatial analysis (Okabe 1992, Broke 1996, Zhang and Murayama 2000). But such a method can be applied to only point objects. An alternative method is presented by Chen and Zhao (Chen et al., 2000, Zhao et al., 1999), using Voronoi distance based on Voronoi diagrams. In this method, two objects are k-order neighbors if there are at least k Voronoi regions in between. But there is still a lack of deep and systematical analysis of the concept and search methods of spatial neighbours. In this paper, spatial neighbours, especially k-order spatial neighbours are systematically discussed, i.e. on its definition, characteristics, computation and applications.

The remainder of this paper is organized as follows: Section 2 analyzes the current methods for defining spatial neighbours. Section 3 gives definitions and Section 4 describes the algorithms for computing k-order spatial neighours and then exploits its characteristics. Section 5 discusses some potential new applications of k order neighbours to GIS, concentrating on spatial query, analysis, and constructing neighbourhood on the Voronoi distance. Section 6 gives some conclusions.

2 CURRENT METHODS FOR DEFINTING SPATIAL NEIGHBOURS

Generally, spatial neighbouring relations can be classified into two types: one type is referred as the relation between two objects touching each other; the other is as the relation between disjoint objects.

The former can be easily and clearly defined by common parts of neighbouring objects, and can be obtained directly since they are often explicitly stored in most GIS databases with the support of topology. This type of neighbouring relations can be called topological adjacency.

The other type of neighbouring relations exists among disjoint objects. Statistically, the distribution of disjoint relation prevails much over that of other relations in geographical space (Florence, 1996). So, it is very significant to describe this type of neighbours among discrete object. But this type of neighbors are often fuzzy and complex, and related to a certain definition of distance, are also dependent on the context within which they are applied (Gahen, 1995).

Currently, there are two main methods, i.e., metrical method and topological method, to define and compute the two types of spatial neighbours including this type of neighbours among disjoint objects.

The metrical method is widely used in GIS. In this method, neighbouring relations are usually defined by distance and neighbours are defined as objects within a specified distance (Hernandez et.al. 1995; Huang and Sevonson 1993, Hong, 1995). Distance relations are quantitative and often easily got. But neighbouring relations are qualitative, so the conversion between quality and quantity has to be done in the method. In some cases, the computation of distances is complex and difficult, so the cost becomes high.

The topological method mainly refers to the fact that spatial neighbours are defined by topological concepts. In this method, spatial neighbours will have common boundaries but no interiors. The advantage of this method lies in: the high distance computation can be avoided and related operations will be simple and clear. So it is a qualitative method different from the metric method.

If topological method is extended to Voronoi diagrams, it forms another useful method, Voronoi method. Gold (1992) gives the definition of neighbors based on Voronoi diagram, i.e., two objects are neighbouring if they share common Voronoi boundaries. Chen et al. (1991) also describe this type of neighboring relations by using Voronoi 9-Intersection model for spatial relations. Furthermore, it can be distinguished into more detailed neighboring relations such as lateral neighbor, nearest neighbor and second-order neighbor. Voronoi method actually transforms the neighbouring problems of spatial objects into those of their Voronoi regions. In this way, use of the various advantages of Voronoi diagram [Aurenhammer, 1991] can be made, particularly semi-quantity nature [Edward, 1993; Gold, 1989].

3 DESCRIPTION OF K-ORDER SPATIAL NEIGHBOURS USING VORONOI DIAGRAM

3.1 Description of K-order neighbour based on Voronoi diagram

From the viewpoint of computational geometry, Voronoi diagram is essentially 'a partition of the plane into N polygonal regions, each of which is associated with a given point. The region associated with a point is the locus of points closer to that point than to any other given point (Lee and Drysdale 1981). A point associated with Voronoi regions is called generation point. Of course, these generators can be also line and area objects (Okabe, 1992).

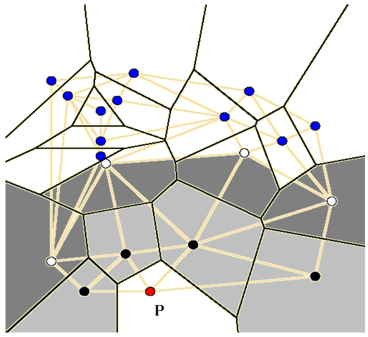


Figure 2 k-order neighbors to object P

On Voronoi diagrams, k-order neighbour can be defined by using Voronoi regions as the following:

(1) Two objects are 1-order neighbours, if two objects share boundaries of Voronoi region;

(2) An object is a k-order neighbour of a given object, if the object is an immediate neighbour of one k-1 order neighbour of the given object, only if it is not a k-2 order neighbour.

In Figure 2, the bold points are *1*-order neighbours to generator *P*, the hole points are 2-order neighbours to generator *P*.

3.2 Mathematical definitions of K-order neighbours

Mathematically, k-order neighbour can be described formally in terms of point set theory as follows.

K-order neighbour: Let P is a set of objects P_1, P_2, ……, Pn in a finite convex in IR^2, P_i, $P_j \in P$ ($i \neq j$, i, j=1,……n). K-order neighbour of Pi can be described:

(1) *1*-order neighbour:

$$N_1(P_i) = \{P_j \mid \exists x, d(x, P_j) = d(x, P_i)\} \quad \text{……… ..} \quad (1)$$

(2) k-order neighbour:

$$N_k(P_i) = \{P_j \mid \exists x, d(x, P_j) = d(x, N_{k-1}(P_i), k > 1, if\ k > 2, P_i \notin N_{k-1}(P_i)\} \quad \text{. ……………… .} \quad (2)$$

where, *d* denotes a distance function.

If the order is described as Voronoi distance (denoted by *vd)*, k-order spatial neighbour of object *Pi* can be defined as the following equation.

$$N_k(P_i) = \{P_j \mid vd(P_i, P_j) = k, k > 0\} \quad \text{………………..} \quad (3)$$

Using the concept of point set topology, k-order spatial neighbour can be classified as:

(1) Topological neighbour (*TN*): two objects have common boundaries

$$TN(P_i) = \{P_j \mid \partial P_j \cap \partial P_i \neq \varnothing\ and\ P_i^0 \cap P_j^0 = \varnothing\} \quad \text{………} \quad (4)$$

(2) Geometrical neighbour (*GN*):

$$GN_k(P_i) = \{P_j \mid P_i \cap P_j = \varnothing\ and\ vd(P_i, P_j) = k, k > 0\} \quad \text{.……} \quad (5)$$

4 COMPUTATION OF K-ORDER SPATIAL NEIGHBOURS

4.1 General principle

From the definition of k order neighbors, it can be derived that a certain relations hold among k order, k-1 order and k-2 order neighbors, as follows:

$$\bigcup_{i\geq 0} S_i^{n+1} = \bigcup_{i\geq 0}\bigcup_{j\geq 0} (S_i^{n})_j^1 - \bigcup_{i\geq 0} S_i^{n-1} \qquad \text{...........} \ (6)$$

$\sum S_i^{n+1}$ -----object S's all n+1 order neighbors;

$\sum\sum (S_i^n)_j^1$ ---all immediate neighbors to object S's all n order neighbors;

$\sum S_i^{n-1}$ ----object S's all n-1 order neighbors;

$\sum S_i^{n}$ ----object S's all n order neighbors;

S_i^n ---a n order neighbor of object S.

This forms the basis of the computation of k-order neighbors. According to this relation, the computation of k-order is composed of two parts: (1) compute and representing 1-order neighbour; (2) derive k-order neighbour.

4.2 Computing and Representing 1-order Neighbour

According to the definition, two objects will be 1-order neighbours if their Voronoi regions share common parts. Therefore, if all edges of a Voronoi region are searched once, all 1-order neighbours to the object (to which the Voronoi region belongs) will immediately be found.

The procedure for the computation can be described as the following,

Let E_V be the set of the edges of Voronoi boundaries; L_O the list of objects as generators; L_{N1} the list of 1-order neighbours; O_T the set of objects related to the current edge; and P_{N1} the pointer of each object to its 1-order neighbours in the list of L_{N1}.

Step 1: O_T = empty, L_{N1} = empty, for each object $P_{N1} = NULL$;

Step 2: Take an edge from E_V, find all objects which take the edge as Voronoi boundaries, put them into O_T;

Step 3: For each object T in O_T, record 1-order neighbours. If P_{N1} is *NULL*, allocate relative position to P_{N1} in L_{N1}; Record the others of O_T as its 1-order neighbours, insert them into L_{N1} at the position pointed by P_{N1}. If P_{N1} is not NULL, check if each object in the others of O_T recorded as 1-order neighbour to T, insert the object into the L_{N1} at the position pointed by P_{N1}; $O_T = NULL$;

Step 4: if all edges in E_V are searched, then stop; or go to step 2

4.3 Computing K-order Neighbours

Several methods for searching k-order neighbours have been designed for different cases.

4.3.1 Waved front Method

If one wants to know all k-order neighbours to a given object, the search can start with this object in a waved way. The procedure for the computation is the following.

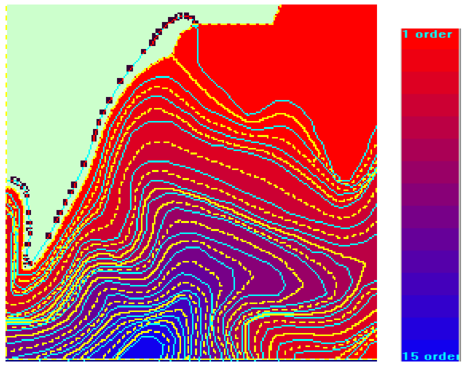


Figure 3. All k order neighbours to a given object obtained with wave front method (up to 15-order).

Let L_N be the list of found k-order neighbours to a given object T_0; T the current object; L_{N1} the list of 1-order neighbours to T; Q_N the queue of neighbours to be used for searching higher order neighbours; P the beginning position of I order objects and K a given order value of neighbours, then

Step 1: $L_N \leftarrow T_0$, Q_N = empty, I = 0, T=T_0 , P= T_0;

Step 2: IF L_{N1} is not *NULL*, I < K, THEN Check all objects in L_{N1}, and put those objects not in L_N into L_N ELSE stop.

Step 3: IF I < K and $T = P$, THEN I=I+1, move the pointer P to the end of L_N, ELSE Stop and output L_N,;

Step 4: Move the pointer T to the next object in L_N , goto step 2.

Figure 3 is an illustration of a 15-order neighbours of a contour line, computed by this method.

4.3.2 Meeting Method

If two objects are given, the computation can implemented with a meeting method in which searching starts with the two objects at the same time (Figure 4).

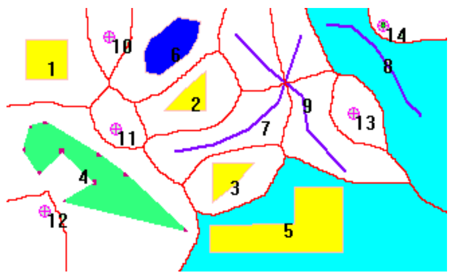


Figure 4. Computing the order of two objects using meeting method: object 4 and 14 are 3 order neighbors

Let T_0, and T_1 be the two specified objects; L_0 k-order neighbours to T_0; L_1: k-order neighbours to T_1, then

Step 1: L_0 and L_1 are set empty, i = 0;

Step 2: IF objects pointed to by P_{N1} of T_0 include $T1$, THEN K=1, stop; ELSE i =1;

Step 3: Perform the waved front method respectively for T_0 and T_1, search i-order neighbours: Search for i-order neighbours to T_0, record them into L_0, and search for i-order neighbours to T_1, record them into L_1.

Step 4: Check L_0 and L_1. IF L_0 and L_1share at least one object, then K = 2*i stop. IF there is an object in L_0 which is 1-order neighbour to an object in L_1, THEN $K = 2*i + 1$, stop; ELSE L_0 and L_1 are set empty, i = i+1, go to Step 3.

4.3.3 Directional Method

This method can be used for searching k-order neighbours to given two objects along a specified routine.

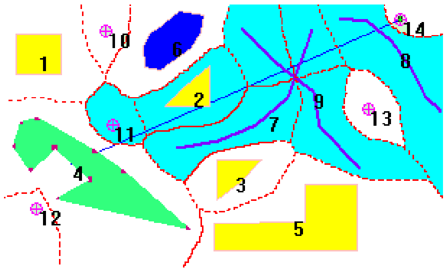


Figure 5. Computing k-order neighbors along a specified line: object 4 reaches 14 through 11,2,7,9,8.

There exist only two cases between two neighboring objects of an specific object: (a) two objects are not immediate neighbors but separated by the Voronoi region of the specific object, so the two objects are 2-order neighbors; and (b) two objects are immediate neighbors. Therefore, the searching procedure must be traced back to check if each new object in current step is one immediate neighbour of the got object. If they are not immediate neighbours, the order *K* should be increased by 1, else *K* value retains as the old order.

The procedure is implemented as follows: (a) Suppose an auxiliary straight line starting from an object to another one as the desired routine; and (b) Trace k-order neighbours along the line at the beginning of an end of the line, stop till finding another object (Figure 5).

The algorithm can be implemented in terms of the following steps.

Let K be the order to be determined between two specified objects; *T* the current object whose Voronoi boundary intersects the auxiliary line; *N* the found neighbour; *A* the source object, *B*: the destination object; *L* the straight line between *A* and *B; then*

Step 1: Select one of them as the source object *A*, then the other is the destination object *B*, construct the straight line *L* between *A* and *B*, $T = A$, K=0;

Step 2: Compute the intersection point between line *L* and *T* 's Voronoi boundary, get the edge of the Voronoi boundary intersecting the auxiliary line, then the neighbours sharing this edge with *T* can be found immediately. The found neighbours are denoted by *N*;

Step 3: IF $K > 1$, check whether N are the (k-1)-order neighbours to *T*. ELSE, THEN $K = K+1$;

Step 4: $T = N$;

Step 5: IF *N* is the destination object *B*, THEN stop and output the value of *K*, ELSE, go to step 2.

5 POTENTIAL APPLICATIONS

K-order neighbours can be applied into spatial query and analysis in GIS. Several authors have applied spatial neighbours into spatial analysis and map generalization based on Delaunay nets (Broke 1996, Zhang and Murayama 2000). Here it is attempted to discuss some potential other applications in GIS.

5.1 Nearest neighbour query

Since (k+1)-order neighbours are 'farther' to the generator than k-order neighbours and located behind k-order neighbours, there must exist an object of k-order neighbours, which is nearer to the generator than anyone of (k+1)-order neighbours. If one wants to reach one generator from (k+1)-order neighbours, he must pass through k-order neighbours to the generator. Therefore, from these two properties and Euclidean distance, one very important fact can be inferred: the *kth* nearest object to a given object must be within from 1-order neighbours to k-order neighbours to the object. For example, the nearest object to a given one must be within 1-order neighbours, it impossibly belongs to the 2-order neighbours to the given object. Or, it will violate the above two properties. So the query for kth nearest object can be implemented with the combination of k-order neighbours and Euclidean distance. It reduces the searching range to a local range, but at the same it can guarantee the completeness and accuracy of the result.

5.2 Buffering Query

Buffer zone is a kind of influence or service range of a spatial object or engineering planning item. It is one of important functions for searching for information in generalization of map and spatial analysis of GIS. Many algorithms for buffer zone construction have been presented in order to improve the efficiency of buffer analysis in GISs. Currently, the general method for searching based on buffer zone based on traditional space model is implemented in the whole geographical space so as to guarantee the completeness of searching result. The cost of time is very large. From the definition, k-order neighbours and their Voronoi regions are very similar to buffer zones, but it is not based on traditional distance. So it need integrate traditional distance to make some spatial query in a specified buffer.

The introduction of k-order spatial neighbour can make the query based on buffer zone become simpler. Furthermore it still guarantees the completeness of the result though the range is reduced from the whole geographic space to adjacent areas of some particular objects.

5.3 Neighboring tree among contours

It is necessary and important to construct and recognize the neighboring relations among contours. K order neighbors can be used to construct the relations tree based on k order neighbors among contours. The tree can be possibly used for forming contour tree and automatic elevation assignment or check. Figure 6 shows a k order relation graph of contours.

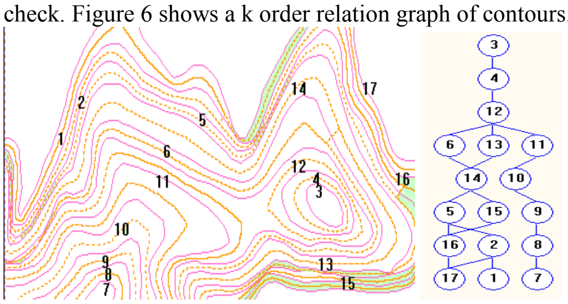


Figure 6. A k order relation graph of contours

5.4 Neighbourhood analysis and reasoning

It is important for spatial reasoning and analysis to construct a suitable neighborhood. Voronoi distance is a kind of means for measuring how far an object is from another in terms of the number of objects, so it can be used for neighborhood analysis and reasoning.

Coefficient for related nearness: Let P is a set of spatial objects in geographical space, if the largest Voronoi distance related to object Pi is n and P_j is k-order neighbour to P_i, the coefficient for related nearness of P_j related P_i (denoted by $reln\ (P_i, P_j)$) is :

$$reln\ (P_i, P_j) = k / n \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (7)$$

In the above equation, if P_j is the farthest from Pi then $reln\ (P_i, P_j)$ will equal 1, if P_j is the nearest to P_i, then $reln\ (P_i, P_j)$ will equal 0. The coefficient for related nearness is asymmetric.

Nearness predicate $near\ (P_i, P_j)$: taking the value true when object P_j is deemed to be near to object P_i. Assume a fixed parameter p ($0 < p < 1$) and define $near\ (P_i, P_j)$ as follows:

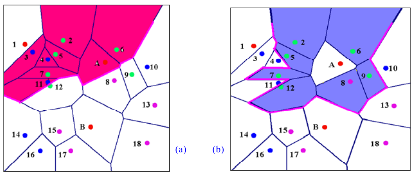
$$near(P_i, P_j) = TRUE \quad if\ and\ only\ if\ reln(P_i, P_j) > p \qquad \ldots\ldots\ldots \ (8)$$

If $near\ (P_i, P_j)$ = TRUE, we will denote it by $P_i\ near\ P_j$ and say that P_j is *near* to P_i. Note that for $P_i = P_j$, $reln(P_i, P_j) = 0$, so *near* is reflexive, that is for all spatial object P_i belonging P, $P_i\ near\ P_j$. But it does not satisfy symmetry or transitivity. However, by introducing a concept of neighbourhood based on k-order neighbors, we are able to get transitivity.

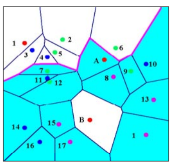
Neighbourhood relationship: Let G be a geographic space in which a adjacent predicate *near* is defined, and a set P of spatial objects is embedded in G. Define a neighbourhood relationship *nr* on G, so that for all spatial objects P_i, P_j belonging P, $P_i\ nr\ P_j$ if and only if, both of the following conditions hold:

1) $P_i\ near\ P_j$;

2) For all P_k, if $P_i\ near\ P_k$ then $Pj\ near\ P_k$.



(a) (b)



(c)

Object	Neighborhood objects
1(a)	2-7, 11, A
A(b)	2, 5-9,12
B(c)	7-18, A

Figure 7: Neighbourhood analysis based on Voronoi distance

Using this Neighbourhood relationship, one can get neighbourhood of P_i as follows,

K-order nearness based neighbourhood: Let G be a geographic space in which a adjacent predicate *near* is defined, and a set P of spatial objects is embedded in G, then for any a object P_i, the neighbourhood of P_i is defined to be the set of all objects neighbour to P_i, denoted by U_{Pi},

$$U_{p_i} = \{P_k \mid P_k \in P, and\ P_i\ nr\ P_k\} \qquad \ldots\ldots\ldots\ldots.. \ (9)$$

The concept of neighbourhood based on k-order nearness U_{Pi} has the properties similar to point-set topological neighbourhood. The result can be further used for the analysis of spatial distribution and determination of graph structure. Figure 7 gives all neighbourhood objects of object *1*, *A* and *B* when *reln* is specified no greater than 0.5.

6 CONCLUSIONS

Spatial neighbour is the foundation of modelling any spatial information system such as GIS. The raster model defines neighbourhood by the neighbour between cells, but it can not directly recognize adjacent relations between spatial objects. The vector model defines neighbourhood by complicated line intersection computation. Voronoi spatial model, on the other hand, has many good properties and is very promising in the application of GIS community. Voronoi neighbours imply both qualitative and quantitative properties. The introduction of Voronoi-based K-order provides a hybrid model for neighbour relations. This study provides a new attempt for spatial query and analysis.

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