A Quantitative Description of Spatial Configurations

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Abstract

The description of spatial configurations plays a fundamental role in content-based retrieval and in the design of user interfaces, as this description is often the basis for the specification of query constraints. In this paper we present measures that capture the content description of spatial configurations as quantitative values that reflect characteristics of individual objects and relations among objects. These content measures are evaluated in terms of their ability to distinguish topological relations, and they are used to compare spatial configurations. Such content measures are suitable for similarity-based retrieval and indexing schemas of spatial configurations.
Keywords: spatial configurations, similarity-based retrieval, content measures, content-based retrieval

1 Introduction

The description of spatial configurations plays a fundamental role in content-based retrieval and in the design of user interfaces, as this description is often basis for the specification of query constraints. These spatial configurations consist of sets of objects that are spatially arranged. They are described by the spatial relations among objects (*i.e.*, topological relations, distance relations, and orientation relations) as well as by the geometric characteristics of objects (*e.g.*, shape, size, and density), and by attributes specifying the semantics of the spatial objects (*i.e.*, entity-type classification).

Previous studies have investigated the content description of images in the context of images retrieval (Ankerst *et al.*, 1999, Ahmad and Grosky, 1997, Park and Golshani, 1997, Petrakis and Faloustos, 1997, Flickner *et al.*, 1995, Faloustsos *et al.*, 1994). In those studies, the similarity between images relies on judgments in terms of visual descriptions, such as shape, size, texture, and color. For spatial configurations, on the other hand, the spatial arrangement of the object becomes the subject of comparison. These object arrangements are typically expressed by a set of constraints dealing with orientation (*e.g.*, north and south), topology (*e.g.*, inside, overlap), and distance (*e.g.*, 5 miles).

Many studies in the domain of image databases have compared object arrangements based on variations of *2D-strings*. 2D-strings represent configurations with a sequential structure for each encoded dimension (Costagliola *et al.*, 1992, Lee and Hsu, 1992, Lee *et al.*, 1992). Query processing with this structure is carried out as string matching, which is only possible when users specify queries by the schema of the relations according to which 2D-strings are built, and images are composed of a predefined set of objects. Another binary string representation codifies topological, orientation, and distance relations between objects that are seen as Minimum Boundary Rectangles (MBRs) (Papadias *et al.*, 1998b). Relations between MBRs are treated as interval relations in two dimensions, and a similarity function is defined as inversely proportional to the number of changes needed to make two strings equivalent. In a similar way, a 3x3 matrix was used to determine the orientation relation by calculating the proportional area in the quadrants defined by the orthogonal projection of the reference object's MBR (Goyal and Egenhofer, 2001). In general, methods based on 2D-strings and their variations handle changes in scale and translation, but they are sensitive to rotation (El-Kwae and Kabuka, 1999). Using a different approach, some studies represent configurations and queries using Attribute Relation Graphs (ARGs) (Berretti *et al.*, 2000, Papadias *et al.*, 1998b, Petrakis and Faloustos, 1997). In these graphs, spatial relations are represented quantitatively by the distance and angle between centroids, and qualitatively by the symbolic representation of spatial relations, such as the topological relations defined by Egenhofer and Franzosa. (Egenhofer, 1994, Egenhofer and Franzosa, 1991). Similarity is then defined between quantitative values as the inverse of their differences (Berretti *et al.*, 2000, Petrakis and Faloustos, 1997) and between qualitative relations as the inverse of the distance in a conceptual neighborhood structure (Papadias *et al.*, 1998a, Bruns and Egenhofer, 1996).

This work focuses on the definition of content measures that describe the object arrangement of spatial configurations. The definition of such content measures uses a simple strategy for comparing spatial configurations that should be independent of rotation and scaling and that can be used in current spatial information systems. This work uses the simplified and common representation of objects (*i.e.*, MBRs) in current spatial indexing schemas of Geographic Information Systems (GISs). Although this simplification of objects lacks details, it is broadly used for its desirable computational properties, and it can be used as an approximation in a similarity-based retrieval. This work uses a quantitative approach to characterize object arrangements in terms of topology and relative size. In this sense, it follows closely the ideas derived from Egenhofer and Shariff's work (1998) that provide metric refinement of topological relations. However, instead of defining measures to refine individual topological relations, we pursue the definition of a single and continuous content measure that distinguishes topological relations using metric refinement. Although the problem of determining spatial similarity is not a new one, to the best of our knowledge, none of the previous studies has attempted to define a content measure that continuously distinguishes spatial relations, making this single content measure suitable for comparing transitions as objects move and change their spatial relations. Furthermore, our proposed content measures can be used for content-based indexing schemas instead of complementing current indexing schemas (*e.g.*, R-Tree) with heuristics based on similarity functions (Papadias *et al.*, 1999, Papadias *et al.*, 1998b).

The organization of this paper is as follows. Section 2 defines parameters that characterize MBRs, whereas Section 3 defines the content measures that are used to describe arrangements of MBRs. Section 4 compares the behavior of these content measures in terms of their ability to distinguish topological relations. Content measures are used to compare spatial configurations in Section 5. Finally, conclusions and future research directions are presented in Section 6.

2 Characterising Objects' Arrangements

This work defines content measures of object arrangements based on basic parameters that characterise individual MBRs and pairs of MBRs. In doing so, it takes an incremental approach to characterising spatial configurations composed of an arbitrary number of MBRs; that is, the aggregation of content measures that describe a pair of MBRs determines a global characterisation of spatial configurations.

Two basic parameters allow us to characterize MBRs as single dimensional values: areas and diagonals, such that they are simplified views of MBRs.

A second set of parameters characterizes a pair of MBRs. The idea is to create parameters that reflect the relationship between MBRs rather than the properties of individual MBRs. These parameters are the area and diagonal of the MBRs created by the *union* and *intersection* of pairs of individual MBRs, and the *external* (d_e) and *internal* (d_i) *minimum distances* between MBRs' boundaries (δA and δB) (Fig. 1).

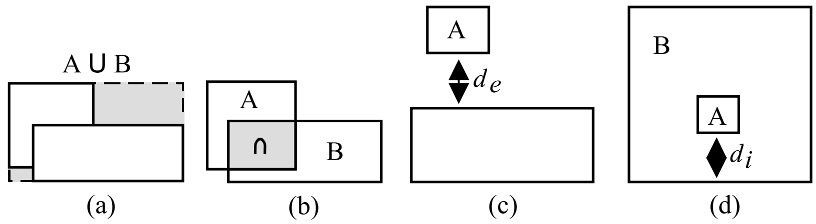


Fig. 1. *Union* **(a)**, *intersection* **(b)**, *external minimum distance* **(c)**, and *internal minimum distance* **(d)**, derived from the combination of two individual MBRs

We explain in the following section how the parameters described above are used to define content measures of objects' arrangements.

3 Defining Content Measures

Two basic ideas guide the definition of content measures of objects' arrangements:

Making metric refinement of topological relations so that we want distinguish degrees of disjointedness or overlap. In Fig. 2, for example, while both pairs of objects A-B and C-D are disjoint by the same distance *d*, the pair A-B should be considered more disjoint than the pair C-D, thus making the configurations independent of scale (Egenhofer and Shariff, 1998).

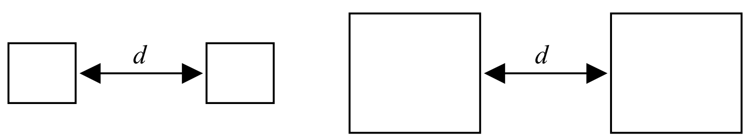


Fig. 2. Differences in the degree of disjointedness

Making explicit the influence of each object in the relation; that is, being able to distinguish between large and small objects as well as between objects as containers and objects as containments in a contains/inside relation. In Fig. 3, for example, objects A and B are disjoint, but A is more disjoint with respect to B than vice versa. In other words, the degree of disjointedness is asymmetric with respect to the size and distance between MBRs. Therefore, describing the arrangement of two objects needs two values, one in each direction of the relation.

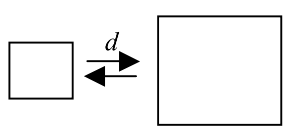


Fig. 3. Asymmetric property of topological relations

We explore three different content measures:

The first measure (F_a) takes areas of individual object MBRs and normalizes them by the area of the union of objects' MBRs, leading to values that are larger than 0 and less or equal than 1 (in the range between ~0 and 1) (Eqs. 1). The ratio between areas represents a relative size relation, while the area of the union of objects' MBRs may indicate whether or not objects share common areas.

$$F_a(A,B) = \frac{\text{area}(A)}{\text{area}(A \cup B)} \quad , \quad F_a(B,A) = \frac{\text{area}(B)}{\text{area}(A \cup B)} \tag{1}$$

Like the first measure, the second measure (F_d) uses diagonals instead of areas of MBRs (Eqs. 2), yielding to values between ~0 and 1.

$$F_d(A,B) = \frac{\text{diagonal}(A)}{\text{diagonal}(A \cup B)} \quad , \quad F_d(B,A) = \frac{\text{diagonal}(B)}{\text{diagonal}(A \cup B)} \tag{2}$$

Making further distinctions among topological relations, the last measure (F_m) uses the diagonal and area of the base MBR[1], the minimum internal or external distances (d_i and d_e, respectively) between MBRs's boundaries (indicated with the δ symbols), and the area of intersection between MBRs (Eqs. 3). The underlying idea of this function is that *distance* between objects is a basic parameter for the refinement of *disjointedness*, while *area* is for *overlapping*.

$$\begin{aligned}
F_m(A,B) &= \frac{\text{area}(A) - 2\text{area}(A \cap B)}{\text{area}(A)} + \frac{\text{distance}\,(\delta A, \delta B)}{\text{diagonal}(A)} \quad , \\
F_m(B,A) &= \frac{\text{area}(B) - 2\text{area}(A \cap B)}{\text{area}(B)} + \frac{\text{distance}\,(\delta B, \delta A)}{\text{diagonal}(B)}
\end{aligned} \tag{3}$$

where :

$$\text{distance}\,(\delta A, \delta B) = \begin{cases} d_e(\delta A, \delta B) & \text{if } A \cap B = \varnothing \\ -\,d_i(\delta A, \delta B) & \text{if } A \cap B \neq \varnothing \end{cases}$$

[1] We consider the *base* object as the first parameter and the *target* object as the second parameter of the function *F*.

4 Properties of Content Measures

The first two content measures (F_a and F_d) use the same structures, that is, areas or diagonals over both individual MBRs and the union of MBRs. Unlike the first measure, the second measure is less sensitive to orientation. For example, Fig. 4 has configurations (a) and (b) with equivalent MBRs separated by the same distance. The difference determined between configurations using an area-based measure is larger than it is using a diagonal-based measure. Although we differentiate these configurations, this difference should not be very large, since in topological terms, the MBRs are disjoint by the same distance.

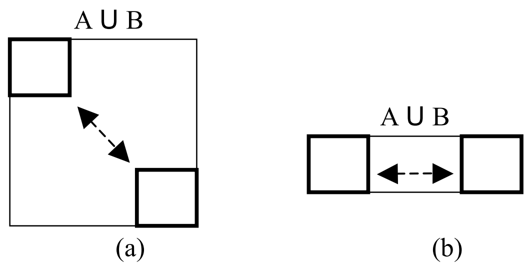


Fig. 4. *Diagonals* are less sensitive than *areas* to orientation.

The three content measures defined above are continuous functions whose values depend on the objects size and arrangement, such that they are independent of scale. Values of F_a and F_d tend to 1 when the MBRs overlap completely; whereas, these values approximate to zero as the MBRs' separate and, therefore, the area and diagonal of the MBRs' union increase. The last measure F_m ranges from negative to positive values, being equal to zero when objects overlap and the area of the *base* object inside of the *target* object is equivalent to the area outside of the *target* object.

Table 1 shows the ranges of values for each measure and each of the topological relations between regions. As table 1 indicates, these measures are asymmetric, and it is the combination of the evaluations in both directions (*e.g.*, *F*(A,B) and *F*(B,A)) that provides some indication of the type of topological relation between objects. The measures F_a and F_d distinguish 3 of the 8 relations, whereas values of F_m can differentiate all 8 topological relations.

As an example of how these proposed content measures are capable of distinguishing topological relations, consider Fig. 5, where the same two objects have different topological relations in transition states.

In Fig. 5, the values of the measures based on *areas* and *diagonals* increase as the degree of overlap increases. These measures are, however, incapable of distinguishing between *inside* and *covered_by* relations, since the MBR that contains both individual MBRs is the same. The third measure, on the other hand, changes from positive to negative values as the smaller object moves inside the larger one. Although the last measure detects differences between the *inside* and *covered_by* relations, it is unable to detect different degrees of *meet*.

The proposed measures do not address explicitly *orientation*; however, a synergy occurs when constrains that related more than one pair of objects are satisfied. Thus we can distinguish between configurations whose objects have the same relative orientation, making the content description independent of the rotation of the whole configuration. For instance, consider three objects with the same topological relation that are arranged in different locations.

Table 1. Range of values for the three content measures (according to 8 topological relations)

Content Measure	Relations	Values
F_a	Disjoint	$0 < F_a(A,B) + F_a(B,A) < 1$
	Meet	$0 < F_a(A,B) + F_a(B,A) \leq 1$
	Overlap	$0 < F_a(A,B) < 1$, $0 < F_a(B,A) < 1$
	Contain	$F_a(A,B) = 1$, $0 < F_a(B,A) < 1$
	Cover	$F_a(A,B) = 1$, $0 < F_a(B,A) < 1$
	Inside	$0 < F_a(A,B) < 1$, $F_a(B,A) = 1$
	Covered_by	$0 < F_a(A,B) < 1$, $F_a(B,A) = 1$
	Equal	$F_a(A,B) = 1$, $F_a(B,A) = 1$
F_d	Disjoint	$0 < F_d(A,B) < 1$, $0 < F_d(B,A) < 1$
	Meet	$0 \leq F_d(A,B) + F_d(B,A) \leq \sqrt{2}$
	Overlap	$0 < F_d(A,B) + F_d(B,A) < 2$
	Contain	$F_d(A,B) = 1$, $0 < F_d(B,A) < 1$
	Cover	$F_d(A,B) = 1$, $0 < F_d(B,A) < 1$
	Inside	$0 < F_d(A,B) < 1$, $F_d(B,A) = 1$
	Covered_by	$0 < F_d(A,B) < 1$, $F_d(B,A) = 1$
	Equal	$F_d(A,B) = 1$, $F_d(B,A) = 1$
F_m	Disjoint	$1 < F_m(A,B)$, $1 < F_m(B,A)$
	Meet	$F_m(A,B) = 1$, $F_m(B,A) = 1$
	Overlap	$\lvert F_m(A,B) \rvert < 1$, $\lvert F_m(B,A) \rvert < 1$
	Contain	$\lvert F_m(A,B) \rvert < 1$, $F_m(B,A) < -1$
	Cover	$\lvert F_m(A,B) \rvert < 1$, $F_m(B,A) = -1$
	Inside	$F_m(A,B) < -1$, $\lvert F_m(B,A) \rvert < 1$
	Covered_by	$F_m(A,B) = -1$, $\lvert F_m(B,A) \rvert < 1$
	Equal	$F_m(A,B) = -1$, $F_m(B,A) = -1$

In Fig. 6, both configurations are composed of the same three objects separated by the same external distances between each other. They differ only on the rotation of the whole configuration such that the MBRs of the unions of pairs of objects are equivalent in size but not in rotation. The content measures of corresponding pairs of objects are, therefore, the same. We have the same results when the configurations differ by a mirror effect; *e.g.*, when the objects are swapped with respect to a horizontal or vertical axis.

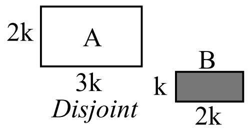
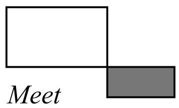
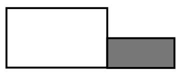
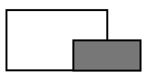
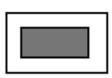
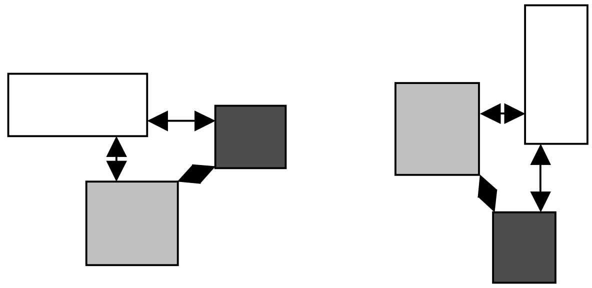
Transition State	$F_a(A,B)$, $F_a(B,A)$	$F_d(A,B)$, $F_d(B,A)$	$F_m(A,B)$, $F_m(B,A)$
 *Disjoint*	(0.33 , 0.11)	(0.54 , 0.33)	(1.28 , 1.45)
 *Meet*	(0.40 , 0.13)	(0.62 , 0.38)	(1.00 , 1.00)
 *Meet*	(0.60 , 0.20)	(0.67 , 0.42)	(1.00 , 1.00)
 *Overlap*	(0.75 , 0.25)	(0.81 , 0.50)	(0.67 , 0.00)
 *Covers, covered_by*	(1.00 , 0.33)	(1.00 , 0.62)	(0.33 , -1.00)
 *Contains, inside*	(1.00 , 0.33)	(1.00 , 0.62)	(0.19 , -1.22)

Fig. 5. Distinguishing different transition states of topological relations between two objects



(a) (b)

Fig. 6. Capturing global orientation of spatial configurations

5 Comparing Spatial Configurations

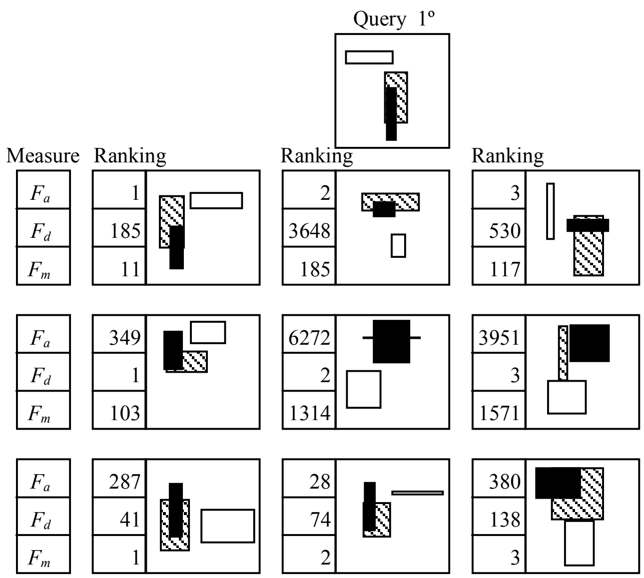
This section checks the performance of the three proposed content measures when comparing different spatial configurations. In this context, this paper concentrates just on the question how *sensible* and *consistent* the results of these measures are. By *sensible* we mean that the content measures find correct topological relations, by *consistent* we mean that all three measures yield *similar*.

The experiments use a spatial database composed of 10,000 configurations with 3 objects of different classes created randomly. The definition of a general searching algorithm for comparing configurations with an undefined number of objects can be built upon these measures and is left for future work. The number of occurrences of different types of topological relations in the database is as follows: 33550 *disjoint*, 68 *meet*, 24512 *overlap*, 28 *covers*, 907 *contains*, 28 *covered_by*, 907 *inside*, and zero *equal*. We describe configurations as points in a 6D space (*i.e.*, a space composed of two values for each pair of relations between MBRs). The similarity is determined as inversely proportional to the Euclidean distance in this 6D space.

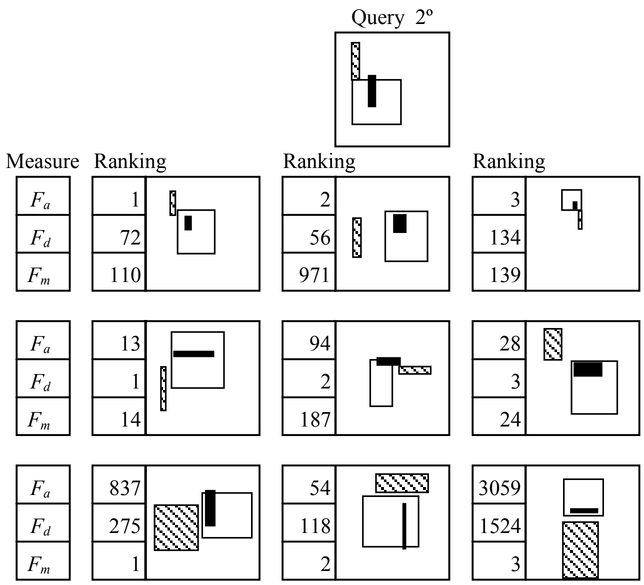
Fig. 7 shows results of two queries. The first query includes *overlap* and *disjoint* relations, the two most common relations in the database. The second query is composed of *overlap*, *meet*, and *disjoint*. Since there are few occurrences of *mee*t in the database, this constraint is highly significant to discriminate configurations in topological terms. For both queries we run all three measures and we give the 3 best matchings with their respective rankings for each of the similarity measures.

For all three content measures, the best configurations have objects whose relations closely satisfy the query's topological constraints. The worse results are obtained with the measure F_d , where we obtain configurations that have objects with different topological relations with respect to the query. A reason is that neither F_a nor F_d differentiates the topological relations as F_m does, and that the Euclidean distance combines each dimension without ensuring the satisfaction of any topological constraints.

In terms of consistency, our results indicate that content measures may result in different rankings of similarity for spatial configurations; however, results obtained from measure F_m are closer to the best results derived from F_a and F_d. Some reasons to the low consistency in the rankings given by the different measures are that the query has objects with very common topological relations in the database and that each measure captures different aspects of spatial configurations (*i.e.*, *area* versus *diagonal*, *area* versus *distance*). If there were few configurations in the database that satisfy the query constraints, there would be better chances that the three measures give the same results.



(a)



(b)

Fig. 7. Answer to first and second queries using each different measure

6 Conclusions

We have proposed three content measures to characterise spatial configurations. They are based on parameters that describe objects by MBRs' basic dimensions (*i.e.*, areas and diagonals) and objects' relations by MBRs' interaction (*i.e.*, areas and diagonals of MBRs' union and intersection, and internal and external distances between MBRs' boundaries).

Our experimental results indicate that these simple content measures are able to distinguish topological relations at different levels of detail. Therefore, these content measures allow us to quantitatively compare spatial configurations. In particular, the last measure F_m that combines basic dimension of MBRs with distances between MBRs' boundaries was seen to be the best of our content measures for comparing spatial configurations.

We left for future work the analysis of different aggregation functions that define a similarity measure. Likewise, we want to explore different weight schemas for similarity measures of topological relations. For example, considering a query composed by objects that *meet* and objects that are *disjoint*, we would like to perform the search process giving more importance to the *meet* constraints than to the *disjoint* constraints. We will also pursue the definition of a content-based indexing schema that avoids the exhaustive search over the whole database.

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