Spatial Relations Between Area Objects Under Metric Spaces

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Abstract

The well-known 9-intersection model has been widely used to describe spatial relations in GIS databases. However, the model cannot effectively distinguish the difference of spatial relations between two disjoint area objects with only a short distance between each other from those of with a relatively long distance because the model is only based on the topology property of the point sets without considering their metric property. In this paper, we begin with reviewing the 9-intersection model, and then propose a new approach to improve it by combining the point set topology with metric property. The method proposed in this paper can partly eliminate the limitation of the model. In our method the representation of spatial relations between area objects includes three steps: (1) to find and locate spatial objects, (2) to make a approximate classification for spatial relations between a selected reference object and a target one using the 9-intersection model, and (3) to identify the spatial relations in more detail by applying metric parameters.
Keywords: topological relations, metric parameters, spatial query, spatial analysis

1 Introduction

Spatial data stored in spatial databases within GIS (Geographic Information System) depict the positional information of spatial entities associated with their spatial relations. Accordingly, the requried information for end-users such as urban planners, environment managers, and government decision-makers can be extracted from spatial databases by means of spatial query and spatial analysis operations. It has been extensively recognized that the spatial relation theory lays the foundation for developing effective spatial databases. In the past decade, spatial relations have been deeply studied by researchers such as Egenhofer and Franzosa (1991, 1995), Clementini *et al.* (1994), Egenhofer and Herring (1991), and Winter (2001). Presently, one of the most widely used models of spatial relations is the point set topology-based 9-intersection model proposed by Egenhofer and Herring (1991). It can be pointed out that the 9-intersection model can only provide a description of coarse classification of spatial relations (Egenhofer 1997). The model has been refined in various manners such as by Chen *et al*. (2001) who used Voronoi diagrams of spatial objects to reform the model. In addition, numerous works have been done on how to optimize spatial query under the 9-intersection model (Egenhofer 1997). Shariff and Egenhofer (1998) further developed the model in natural-language spatial relations between linear and area objects by introducing metric parameters. The focus of this paper, however, is on the spatial relations between area objects via combining point set topology with metric properties in order to advance the precision of spatial analysis. First, we examine the 9-intersection model in the applications with spatial analysis in section 2.

2 Review of the 9-Intersection Model

2.1 Current Description of Spatial Relations

A formalization of topological relations has been investigated through point set topology since the latest of 1980s (Egenhofer and Franzosa 1991). A so-called 4-intersection model, which can describe full topological relations between two simple spatial objects (i.e. simple points, lines, and areas), was proposed. In that model, a spatial object is considered the sets which consist of its two topological components, i.e., boundary and interior. The intersection of the two components will contain a 2×2 matrix, which is defined as

$$R_4(A,B) = \begin{bmatrix} \partial A \cap \partial B & \partial A \cap B^0 \\ A^0 \cap \partial B & A^0 \cap B^0 \end{bmatrix} \tag{1}$$

where, ∂A and ∂B denote the boundary set of spatial objects *A* and *B*, respectively, while A^0 and B^0 - the interiors. There are 2 types of topological relations for point/point, 3-for point/line, 3-for point/area, 16-for line/line, 13-for line/area, and 8-for area/area relations, which can be identified by the 4-intersection model. The topological relations, however, between lines and lines, and lines and areas can not be distinguished by the 4-intersection model because of having the same description matrixes for some different topological relations. Later, Egenhofer and Herring (1991) extended the 4-intersection model, leading to a 9-intersection model

$$R_9(A,B)=\begin{bmatrix} \partial A\cap\partial B & \partial A\cap B^0 & \partial A\cap B^- \\ A^0\cap\partial B & A^0\cap B^0 & A^0\cap B^- \\ A^-\cap\partial B & A^-\cap B^0 & A^-\cap B^- \end{bmatrix} \tag{2}$$

where, A^0, ∂A and A^- mean the interior, boundary and exterior of A, respectively. The exterior of A is normally defined as the complement of the matrix (2). Several limitations associated with the 4-intersection model are mastered in the 9-intersection model. For instance, 2 kinds of topological relations for points/points, 3-for points/lines, 3-for points/areas, 23-for lines/lines, 19-for lines/areas and 8-for areas/areas can be identified by the 9-intersection model. In addition, 7-types of topological relations for lines/lines and 6-for lines/areas can be distinguished from one another under 9-intersection model but the 4-intersection model. However, both models cannot deal with those spatial relations such as between points and points, points and areas, areas and areas because the intersection sets are point, line and area sets, respectively. Therefore, a so-called dimension extended method was proposed by Clementini *et al* (1993), in which 0D, 1D and 2D were used to identify spatial relations when the intersection sets are point, line and area sets, respectively.

2.2 Limitations of the 9-Intersection Model

2.2.1 Immensity of Spatial Object Complement

In the 9-intersection model, the complement of an object is defined as the entire space except it. Therefore, such an expression $A^-\cap B^- = -\Phi$ meets the condition of that the quantities of spatial relations described by 9-intersection model are decreased to a high degree. Furthermore, each element takes a value from a binary set $\{\Phi, -\Phi\}$. Such a function changes infinite cases of topological relations into definite classification descriptions. Although the description is beneficial to computer storage and representation, it cannot provide detailed information. In addition, the description cannot effectively used to perform spatial analysis on a deeper-level since it is only a coarse description of spatial relations. For example, even though both topological relations between A and B in Figure 1 are described as ‘disjoint’ based on 9-intersection model, the topological relation in Figure 1 (*a*), to some degree, is more similar to ‘touch’ than ‘disjoint’.

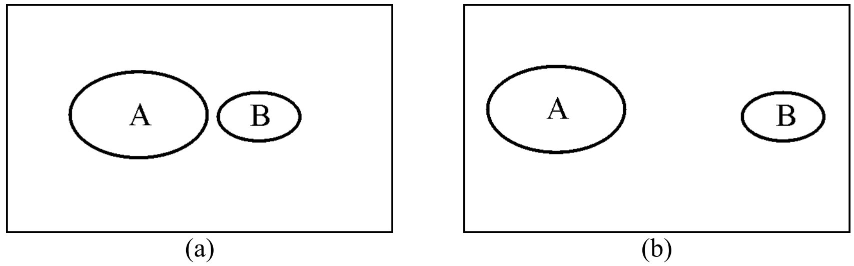


Fig. 1. Two cases of spatial relations 'disjoint'

2.2.2 Sensitivity of Boundary Data Errors

The 9-intersection model is a discrete description of topological relations because the model is based on intersection values of interior, boundary, and exterior of A, and those of B. Based on the model, it is quite easy to implement simple spatial query. Since the 9-intersection model is sensitive to the boundary and interior of the point sets, it is susceptible to error or uncertainty of spatial data. With a tiny variation of spatial data, there is a vermiculation for the contents of point set boundary and its interior in sequence. For instance, the topological relation between A and B in Figure 2 *(a)* is 'overlap'. However, due to the effect of errors or uncertainties in the boundary data on the relations, what we can not deny is whether their relations are 'touch' or 'disjoint'. It may be more likely the 'touch' case in a real world. Such uncertain topological relations can be determined by semantics of spatial entities, but it does not work well in many cases.

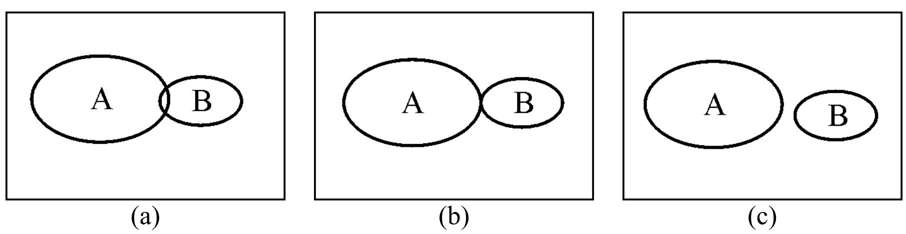


Fig. 2. Effect of uncertainty on topological relations, **(a)** overlap, **(b)** touch, **(c)** disjoint

2.2.3 Difficulty in Representing Dynamic Topological Relations

With the relative movement of spatial objects, their spatial relations will change. Accordingly, spatial relations can be used to detect spatial changes. In geography, the variations of spatial objects in geometry may be abstracted as shift, evolvement, split, mergence, death, and so on, among which the shift and evolvement are two kinds of the most common changes, such as soil erosion, and desert shift. Those spatial variations appear commonly continuous. Correspondingly, the spatial relations also change little by little. Since the 9-

intersection model cannot express such a quantitative change, it cannot carry out spatial analysis unless there is a larger geometrical variation for spatial objects. In Figure 3, based on the 9-intersection model, the topological relations between A and B are invariant in t_1, t_2, t_3, and t_4 with geometrical variation of spatial object B.

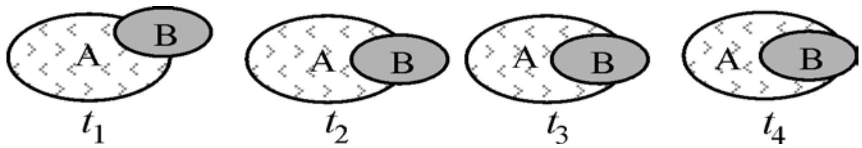


Fig. 3. Quantitative variations of spatial relations for two changeable objects

3 Metrical Analysis of Topological Relations

According to Eq.2, there are 8 types of topological relations between area objects, as shown in Figure 4. Based on topology, the division model can only identify the spatial relations between two area objects. It cannot show the differences of the spatial relations in geometry, i.e., the model cannot describe topological relations in more details. In order to overcome that drawback, we introduce geometrical metric parameters of point sets, such as the distance, area and circumference.

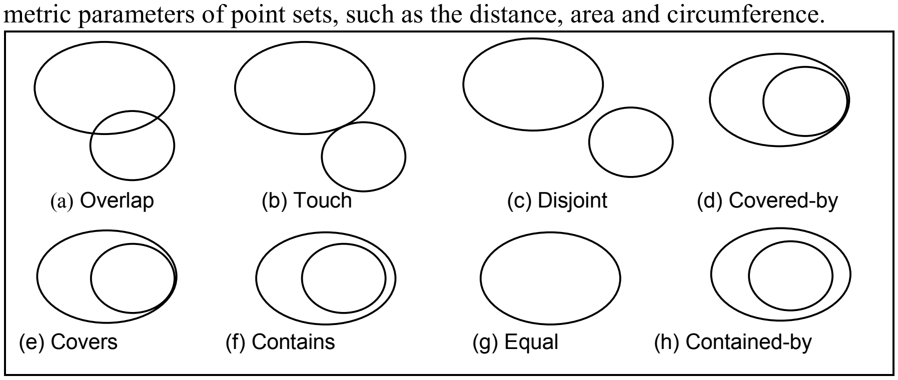


Fig. 4. Eight topological relations between two area objects

3.1 Segmentation Parameters of Point Sets

Since topological relations between A and B are represented by mutual effects of the interior, boundary, and exterior of A and those of B, the degree of the mutual effects can be expressed by segmentation ratio of the point sets. We consider one of the area objects the reference for the query and spatial analysis and the remained object is regarded as target. For instance, let A be the reference object and B the target. For those parameters with subscript of (b) in Table 1, their definitions are:

- II_b denotes the segmentation degree of the interior of *B* by the interior of *A*, and $area\ (B^0) = area\ (B)$, defined as

$$II_b = area\ (A^0 \cap B^0) / area\ (B) \tag{3}$$

- BI_b denotes the segmentation degree of the boundary of *B* by the interior of *A*, defined as

$$BI_b = length\ (A^0 \cap \partial B) / length\ (\partial B) \tag{4}$$

- EI_b denotes the segmentation degree of the exterior of *B* by the interior of *A*, defined as

$$EI_b = area\ (A^0 \cap B^-) / area\ (B^-) \tag{5}$$

- IB_b denotes the segmentation degree of the interior of *B* by the boundary of *A*, defined as

$$IB_b = area\ (\partial A \cap B^0) / area\ (B) \tag{6}$$

- BB_b denotes the segmentation degree of the boundary of *B* by the boundary of *A*, defined as

$$BB_b = length\ (\partial A \cap \partial B) / length\ (\partial B) \tag{7}$$

- EB_b denotes the segmentation degree of the exterior of *B* by the boundary of *A*, defined as

$$EB_b = area\ (\partial A \cap B^-) / area\ (B^-) \tag{8}$$

- IE_b denotes the segmentation degree of the interior of *B* by the exterior of *A*, defined as

$$IE_b = area\ (A^- \cap B^0) / area\ (B) \tag{9}$$

- BE_b denotes the segmentation degree of the boundary of *B* by the exterior of *A*, defined as

$$BE_b = length\ (A^- \cap \partial B) / length\ (\partial B) \tag{10}$$

- EE_b denotes the segmentation degree of the exterior of *B* by the exterior of *A*, defined as

$$EE_b = area\ (A^- \cap B^-) / area\ (B^-) \tag{11}$$

where, *area* ( ) and *length* ( ) are the area and length operators, respectively. From analyzing the 9 segmentation parameters presented above, the following relations can be found: (i) the value of IB_b is equal to that of II_b; (ii) since the external range is too large, EB_b and EI_b trend to 0; (iii) IE_b are identically equal to $(1 - II_b)$; (iv) since overlapping area between A^- and B^- is too large, *i.e.*, $area\,(A^{-} \cap B^{-}) / area\,(B^{-}) \approx 1$. Therefore, those of IB_b, EB_b, EI_b, and IE_b are not granted to consideration. Other segmentation parameters like II_b, BI_b, BB_b, and BE_b can be used to describe the topological relations. For example, in Figure 4 II_b are (*a*), (*d*), (*g*), (*h*), BI_b - (*a*), (*f*), BB_b - (*a*), (*b*), (*d*), (*e*), (*g*), and BE_b - (*a*), (*b*), (*d*).

Table 1. Ratio parameters of a point set between area objects

	A^0	∂A	A^-	B^0	∂B	B^-
A^0				II_b	BI_b	EI_b
∂A		*Null*		IB_b	BB_b	EB_b
A^-				IE_b	BE_b	EE_b
B^0	II_a	BI_a	EI_a			
∂B	IB_a	BB_a	EB_a		*Null*	
B^-	IE_a	BE_a	EE_a			

Admittedly, BI_b is equal to 1 for case (*f*) in Figure 4, and both II_b and BB_b are equal to 1 for case (*g*). In addition, when spatial relations are classified roughly based on topological property of point sets, the values of BI_b and BB_b are equivalent for case (*a*), while both BE_b and BB_b for cases (*a*), (*b*), and (*d*) satisfy $BE_b = 1 - BB_b$. Therefore, we only need calculate two segmentation ratio parameters, II_b and BB_b.

3.2 Nearness Parameters of Point Sets

Nearness parameters of point sets are used to describe the detachment degree of the point sets between two area objects. They are classified as the inner nearness metric (IC_b) and the outer nearness metric (EC_b).

- IC_b denotes the degree of the inner nearness for the boundary of *B* to that of *A*, defined as

$$IC_b = 1 - area\,[B\Theta\; dist(\partial B, \partial A)] / area\,(B) \tag{12}$$

- EC_b denotes the degree of the outer nearness for the boundary of *B* to that of *A*, defined as

$$EC_b = area\,[B \oplus dist\,(\partial B, \partial A)] / area\,(B) - 1 \tag{13}$$

where, Θ and $\oplus$ are the erosion and dilation operators respectively. In Figure 5, $dist\,(\partial B, \partial A)$ is the distance of the boundary of B to that of A, while $dist\,(p, q)$ is the distance between two points located in the boundaries of A and B respectively. We have

$$\begin{aligned} dist(\partial B, \partial A) &= \min(dist\,(p, \partial A) \;\mid p \in \partial B) \\ &= \min(dist\,(p, q) \quad \mid p \in \partial B, q \in \partial A) \end{aligned} \tag{14}$$

Obviously, if $\partial A \cap \partial B = \neg\phi$, then $dist(\partial B, \partial A) = 0$. Accordingly, both nearness parameters of IC_b and EC_b are equal to 0. The interior nearness parameter, IC_b, is applicable to the topological relation of $dist\,(\partial B, \partial A) \neq 0$ and $A^0 \subseteq B^0$, that is,

$$\partial A \cap \partial B = \phi, \quad A^0 \cap B^- = \phi \tag{15}$$

In another word, it is only suitable for the case of (*h*) in Figure 4. The exterior nearness parameter, EC_b, can only be applicable to the following situation

$$\partial A \cap \partial B = \phi, \quad A^0 \cap B^- = \neg\phi \tag{16}$$

Similarly, it is only suitable for cases of (*c*) and (*f*) in Figure 4.

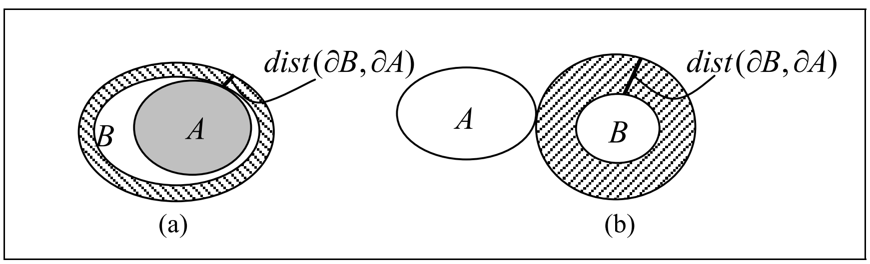


Fig. 5. Nearness parameters of the point sets, **(a)** an inner nearness metric, **(b)** an outer nearness metric

4 Describing Spatial Relations Under Topology and Metrics

From section 3, we know that spatial relations between two area objects can be described by the relevant metric parameters of point sets. For a certain topological relation, the metric parameters response the related values. The types of spatial relations, metric parameters and their values are three basic elements in describing spatial relations between two area objects. If the type of a spatial relation, N, is given and the value of metric parameter, m , is v , they constitute an ordered unit as $R = (N, m, v)$, which is regarded as a basic unit in describing the spatial relations. The value v is dependent on N and m , noted as $v = m(N)$. Accordingly, the unit can be written as

$$R = (N, m, m(N)) \tag{17}$$

In addition, all the basic units will form a generalized model $\Omega(R)$. Apparently, the model puts the qualitative and quantitative description into a uniform ordered 3-unit form. It can describe the spatial relations with high resolution.

5 Examples and Discussions

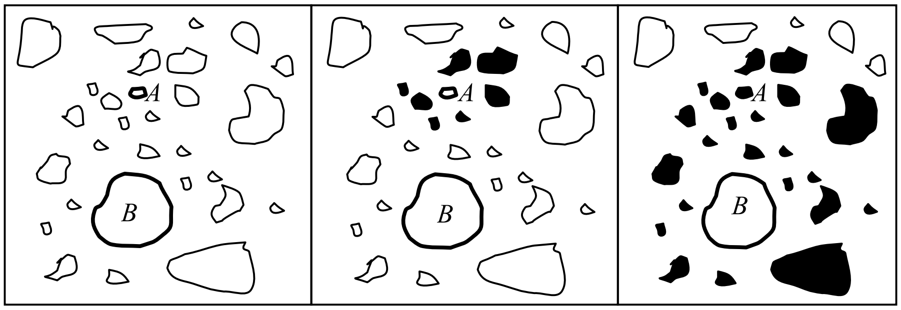
The approach proposed in this paper is the combination of the 9-intersection model and metric parameters. The new method is rather helpful for spatial analyses. The following examples are provided to show how to apply the new model in practice.

(1) Based on a coarse classification of spatial relations using topology of point sets, we can effectively analyze 'disjoint' and 'overlap' relations between area objects by adding geometrical metric of point sets.

In Figure 1, the relations between *A* and *B* are both 'disjoint' under the 9-intersection model. However, the values of outer nearness parameters are different, i.e., $EC_{ba} < EC_{bb}$, which means that the degree of 'disjoint' relations between *A* and *B* in Figure 1(*a*) is greater than that of in Figure 1 (*b*). The correlation between *A* and *B* in Figure 1(*a*) is smaller than that in Figure 1(*b*). For example, if the entity *A* is a strip mine, and B- a village, the impact of the strip mine on the village in Figure 1 (*a*) is much greater than that of in Figure 1 (*b*).

In Figure 3, topological relations between *A* and *B* at the four moments of t_1 , t_2 , t_3 and t_4 are same, called 'overlap', under the 9-intersection model. However, their interior segmentation ratios, II_b , of the point sets are different, i.e., $II_{b1} < II_{b2} < II_{b3} < II_{b4}$. For instance, if the entity *A* is an arable region, and B- a desert, the submergence of the desert to the arable region is increasingly serious and has an outspread trend.

(2) For the analysis of a topological neighborhood between two area objects, the adding information of geometrical metric of the point sets can help us explain the irreversibility of 'nearness' relation in cognitive geographical space.



(a) (b) (c)

Fig. 6. Graphic illustration of irreversibility about spatial neighborhood, **(a)** shape and location of A and B, **(b)** the neighborhood of A, **(c)** the neighborhood of B

The distances between *A* and *B* in Figure 6 (*a*), (*b*), and (*c*) are the same. However their neighborhood areas differ from one another. Here, *A* is one of the neighborhood areas of *B*, but the reverse proposition is not true. Meanwhile, if the entity *A* is a county and *B*- a province, it is clear that the neighborhoods of *A* are some surrounding counties and those of *B* are surrounding provinces during analyzing their neighborhoods.

(3) The new model can help us identify the dimension of the intersection between two area objects. For example, the dimension of intersection may be 0 or 1, while topological relations between two area objects under 9-intersection model are (*b*), (*d*) or (*e*), as shown in Figure 4. The boundary segmentation parameter BB_b of point sets can confirm the maximal dimension of the intersection. Therefore, if we have $BB_b = 0$, then the intersection between two area objects can only be points or else. If we have $BB_b \neq 0$, however, it at least contains a line.

6 Conclusion

In this paper, we reviewed the 9-intersection model, and proposed a new method of combining topology with metric of point sets to represent spatial relations in GIS. Since the 9-intersention model is a coarse classification method, it cannot help us perform spatial analysis at a higher-level. Because the metric relations have strong restriction on the location of spatial objects, the spatial relations can be represented more accurately by combining topology with metric parameters of point sets. The metric parameters defined in this paper can be easily calculated. The processes of describing spatial relations in GIS are generalized as three steps as outlined in the abstract.

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References

Chen J, Li CM, Li ZL, Gold CM (2001) A Voronoi-based 9-intersection model for spatial relations. International Journal of Geographical Information Systems 15 (3):201-220

Clementini E, Paolino DF, Peter VO (1993) A small set of formal topological relationships suitable for end-user interaction. In: Advances in Spatial Databases, Lecture Notes in Computer Science, no. 692. Springer-Verlag pp.277-295

Clementini E, Sharma J, Egenhofer M (1994) Modeling topological spatial relations: strategies for query processing. Computers and Graphics 18(6):815-822

Egenhofer M, Herring J (1991) Categorizing binary topological relationships between regions, lines and points in geographic databases. In: Egenhofer M, Herring J et.al (eds) A framework for the definition of topological relationships and an approach to spatial reasoning within this framework. Santa Barbara, CA pp1-28

Egenhofer M and Franzosa R (1991) Point-set topological spatial relations. International Journal of Geographical Information Systems 5(2):161-174

Egenhofer M and Franzosa R (1995) On the equivalence of topological relations. International Journal of Geographical Information Systems 9(2):133-152

Egenhofer M (1997) Query processing in spatial-query-by-sketch. Journal of Visual Languages and Computing 8(4):403-424

Shariff ABM, Egenhofer M, Mark D (1998) Natural-language spatial relations between linear and area objects: the topology and metric of English-language terms. International Journal of Geographical Information Systems 12 (3):215- 245

Winter S (2001) Uncertain topological relations between imprecision regions. International Journal of Geographical Information Sciences 14(5):411- 430