# SIMULATOR OF INTERFEROGRAM FOR SPACEBORNE SAR SYSTEM

K. Ren<sup>a, b,</sup> \*, G. Wu<sup>b,</sup> \*, X.Q.Shi<sup>a</sup>, and V. Prinet<sup>b</sup>

<sup>a</sup> Dept. of Electronic Engineering, Nanjing University of Sciences and Technology, Nanjing, China-kren@nlpr.ia.ac.cn <sup>b</sup> National Laboratory of Pattern Recognition, Institute of Automation, CAS -gwu@nlpr.ia.ac.cn

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### **ABSTRACT:**

In interferometric synthetic aperture radar (InSAR) processing, simulation of interferogram is a common practice. It is used as synthetic data to test and validate the whole chain of InSAR processing from the interferogram creation to the DEM reconstruction. The objective of this paper is to develop a simulator for validation of geocoding processing and phase-to-height conversion processing algorithm.

The simulator includes two parts: digital elevation model (DEM) of terrian simulation and interferogram simulation with DEM and satellite orbit parameters. DEM is realized by the fractal Browian motion (fBm) model with the midpoint displacement method and the terrain roughness and average slope are determined by two describing parameters of the model. Simulators for interferometry are generally developed in a simplified imagery geometry model. Here the geolocation method is adopted and the geographic coordinates (latitude, longitude) of the synthetic DEM in the imagery region are assigned according to the real orbit. Simulation experiments on simulated and ERS-1/2 tandem real orbit data demonstrate the efficiency of the proposed simulator. The developed simulator can also be used to test the accuracy of basline estimation.

## 1. INTRODUCTION

Since increasing spaceborne synthetic aperture radar (SAR) images over the most part of world is nowadays widely available, SAR Interferometry (InSAR) technology, as a new application of SAR, has been an important observation measurement of Earth in remote sensing community in recent ten years.

In application research of SAR or InSAR, simulators are developed for selecting an optimum imagery mode of SAR, understanding the effects of illumination angle and terrain relief on SAR images, testing and optimizing interferometric SAR (InSAR) processing algorithms, or going insight into radar received signal of given the terrain [Wary L. S., etc, 2000]. These simulators can be generally divided into two groups. One is to simulate SAR raw data based on the backscatter model of land surface and SAR pulse transformation function [Franceschetti G., 1998]; and the other one is to simulate interferogram based on a digital elevation model (DEM)[Xu W.and Cumming B., 1997]. Here we pay attention on the interferogram simulator. With a high quality DEM as the simulator input, the topographic phase contribution can be isolated completely in a real interferogram from other phase variables caused by object deformation or temporal decorrelation. When input DEM is coarse, the synthetic interferogram is useful for flat phase removal processing and improving the phase unwrapping processing.

In this paper a simulator of interferogram for spaceborne SAR system, combined with terrain simulation, is developed to validate phase-to-height conversion and geocoding processing algorithm. DEM is realized by the fractal Browian motion (fBm) model with the midpoint displacement method and the terrain

roughness and average slope are determined by two describing parameters of the model. For the interferogram simulation, an improved method is developed, which works with two real radar sensor parameters and their warp relationship of coregistration. The simulator implementation is described in detail. Experiment results on real senor and orbit parameters are given to demonstrate the efficiency of the presented method.

This paper is organized as follows: Sections 2 and 3 describe DEM simulation and interferogram simulation respectively. Section 4 shows the simulation results and Section 5 is conclusion.

## 2. DEM SIMULATION

## 2.1 Principle

In this paper, the terrain data are simulated by fractal Browian motion (fBm). A fractal is the shape made of parts similar to the whole in some ways; it can describe many complex objects too irregular to be dealt with in traditional geometrical language. It has been shown that terrain surface possesses some fractal characteristics under a wide range of scale, and fBm is regarded as a proper model for terrain representation [Peitgen Heinz-Otto etc., 1988], [Jin Y.W. and Lu S.J., 1998]. FBm is a random process, whose increment is stationary, and satisfies Gaussian distribution as follows:

$$\frac{f(X + \Delta X) - f(X)}{\left\|\Delta X\right\|^{H}} \sim N(0, \sigma^{2})$$
(1)

or

<sup>\*</sup> Corresponding authors.

$$E(|f(X + \Delta X) - f(X)|^2) = \sigma^2 ||\Delta X||^{2H} \quad (0 < H < 1) \quad (2)$$

Here H and  $\sigma$  are two important parameters to control the model, and the meanings of them can be illustrated in 1D case [Zhu Q., 1995].

Let

$$Y = f(0) - f(-X)$$
  
 $Z = f(X) - f(0)$ ,

be the past and future increment of f, respectively, thus the correlation coefficient of Y and Z is described as

$$\rho(Y,Z) = \frac{E(YZ)}{\left(E(Y^2)E(Z^2)\right)^{\frac{1}{2}}} = 2^{2H-1} - 1$$
(3)

It indicates that H is a measure of correlation between increments of f. The smaller H is, the rougher f shows in shape.

In addition, let  $\Delta X = 1$ , then (2) yields

$$E(|f(X+1) - f(X)|^{2}) = \sigma^{2}$$
(4)

(4) indicates that  $\sigma$  is an index of the average difference of f at unit distance, or is regarded as average slope.

Generalizing this to 2D case, the terrain surface can be modeled as a 2D fBm. Then H reflects the roughness of the surface, and  $\sigma$  reflects the average slope of the surface. In general, the flat region is with big H and small  $\sigma$ . The contrary case is with the high mountain region. And the middle mountain or hill region is between the former two cases.

Therefore, the different types of the terrain can be generated by adjusting parameters H and  $\sigma$ , and used to validate the interferogram simulation algorithm.

#### 2.2 Algorithm

Here we use the midpoint displacement algorithm [Zhu Q., 1995] to simulate terrain data. The main idea of this algorithm is to form the final regular elevation grid by recursive subdivision. In each subdivision, the elevation of a point *h* is obtained by interpolating its four neighbors  $h_k(k = 1, 2, 3, 4)$  and a random stationary displacement  $\Delta_i$ , where the displacement is determined by *H*,  $\sigma$  and the recursive number *i*.

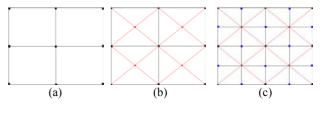


Figure 1. (a) The *i*th subdivision  $d_i$ (b) The (i+1)th subdivision  $d_{i+1} = \frac{1}{\sqrt{2}}d_i$ (c) The (i+2)th subdivision,  $d_{i+2} = \frac{1}{2}d_i$ 

In Figure 1, the *i*th subdivision is illustrated in (a) with the grid interval  $d_i$ ; in (b), the (i+1)th subdivision is completed by interpolating each four neighbors to get the red points and then form the red grid that rotate 45 degrees against the *i*th one; (c) is the (i+2)th subdivision, with the blue points interpolated, the grid becomes denser with the interval of  $d_{i+2}$  as half as  $d_i$ . During the subdivision, the interpolation is formulated as follows,

$$h = \frac{1}{4} \sum_{k=1}^{4} h_k + \Delta_i$$
 (5)

$$\Delta_i \sim N(0, v_i^2) \tag{6}$$

$$v_i = d_i^{\ H} \sigma \sqrt{1 - 2^{2H-2}} \tag{7}$$

$$d_{i} = \frac{1}{\sqrt{2}} d_{i-1}$$
(8)

So repeating this process can reach a small enough grid interval  $d_i$ .

### 3. INTERFEROGRAM SIMULATION

#### 3.1 Principle

It is well-known that the distance between the satellite and the target on the ground can be determined accurately by the phase information of the SAR image. Based on the observation, the phase difference of two SAR images is defined as interferogram  $\phi$  given by

$$\phi = \psi_1 - \psi_2 = -\frac{4\pi}{\lambda}(r_1 - r_2)$$
(9)

where  $r_1, r_2$  = the distances between a target point *P* and satellite  $S_I, S_2$ , respectively.

 $\lambda$  = wavelength of radar.

The simulation of interferogram relies therefore on the accurate determination of the range difference between two imagery distances.

### 3.2 Target point position in Cartesian coordinates system

The DEM data (latitude  $\phi$ , longitude  $\psi$ , height *h*) are generally defined in geodetic coordinates. For the simulation, the data should be firstly converted to the corresponding  $(P_x, P_y, P_z)$  in the Earth Center Cartesian coordinates system. The

transformation from geodetic coordinates to Cartesian coordinates is given as

$$\begin{cases} P_x = (R+h)\cos(\phi)\cos(\psi) \\ P_y = (R+h)\cos(\phi)\sin(\psi) \\ P_z = [R(1-e^2)+h]\sin(\phi) \end{cases}$$
(10)

where  $P_x$ ,  $P_y$ ,  $P_z$  = coordinates of target point P in Earth center Cartesian coordinates system

a = the semi-major axe of the Earth

b = the semi-minor axe of the Earth

 $e = \sqrt{(a^2 + b^2)/a^2}$ , the earth numeric eccentricity  $R = a/\sqrt{1 - a^2 \sin^2(\phi)}$ , local Earth's radius of curvature

## 3.3 Calculating satellite positions

For two satellite imaging the target point P, their positions need to be calculated. According to the geolocation principle of SAR image, the image coordinates of a target point (row and column numbers) can be calculated from its Cartesian coordinates  $(P_x, P_y, P_z)$  by solving a system of the range equation and the Doppler equation.

Assume that the target point  $P = (P_x, P_y, P_z)$  is in a SAR image and has coordinates (*row*, *col*). We know that the satellite position is a function of azimuth time, namely rows number, and the satellite state vectors can be expressed by image coordinates (*row*, *col*) of the target P. So the range equation and the Doppler equation can be converted to an equation system with two independent variables: *row & col*.

Let the time of the first state vector be the reference time origin. For the single look complex (SLC) data, the absolute time of the image rows is known by means of the *PRF* (Pulse Repeat Frequency, given in SLC header file)

$$t(row) = \frac{row}{PRF} + dt \tag{11}$$

where dt = the difference of time of the first image row with respect to the time of the first state vector

*PRF* = Pulse Repeat Frequency

row = index of SAR image in azimuth direction

The satellite position and its velocity can be expressed as function of the image rows:

$$S(row) = [Sx(t(row)), Sy(t(row)), Sz(t(row))]$$
(12)

and

$$Vs(row) = [Vx(t(row)), Vy(t(row)), Vz(t(row))]$$
(13)

where  $(S_x(t), S_y(t), S_z(t))$  and  $(V_x(t), V_y(t), V_z(t))$  are satellite position and velocity respectively interpolated from the state vectors given in SLC header file with a cubic spline interpolation at the time *t*.

The slant range of target point P can be expressed by target column index *col*:

$$r(col) = r_0 + col \cdot spc \tag{14}$$

where  $r_0$  = slant range of first column,

*spc* = range resolution

In order to achieve image coordinates (*row*, *col*) of the target point P, the following equations system must be solved:

$$\begin{aligned} \left| S(row) - P \right| &= r(col) \quad range \ equation \\ \frac{2}{\lambda} \frac{Vs(row) \cdot (S(row) - P)}{|S(row) - P|} &= f_d \quad Doppler \ equation \end{aligned}$$
(15)

where the symbol  $\cdot$  stands by the inner product of two vectors.

By assuming the Doppler frequency is equal to zero, the image coordinates (*row*, *col*) of the point are retrieved using a minimization method with the pair of image center coordinates (*rowc*, *colc*) as an initial guess.

So satellite position  $(S_x, S_y, S_z)$  in the Earth center Cartesian coordinates system can be obtained by the imagery time which is decided by row numbers of the target point.

Here we calculate the slave statellite position with respect to the targe point by the coregistration warp function of master and slave images. This approach not only can decrease the geolocation errors in synthetic interferogram but also can eliminate the phase errors caused by coregistration when synthetic interferogram is used in D-InSAR processing. For the simulation example (see the next section), the geolocation errors are about in a resolution cell of ERS-1/2 image, namely 3.9m in azimuth direction and 7.9m in range direction. Because the accuracy of co-registration is sub-pixel level, the position accuracy of the target point in slave SAR image is better than that of geolocation.

The index of the target point P in slave SAR image equals to

$$\begin{bmatrix} row_{S} \\ col_{S} \end{bmatrix} = \begin{bmatrix} a_{0} & a_{1} & a_{2} \\ b_{0} & b_{1} & b_{2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ row_{M} \\ col_{M} \end{bmatrix}$$
(16)

where  $row_M$ ,  $col_M$  = index of target point P in master image

 $row_s, col_s = index of target point P in slave image$ 

$$\begin{bmatrix} a_0 & a_1 & a_2 \\ b_0 & b_1 & b_2 \end{bmatrix}$$
 = warp matrix of master to slave image

From the known positions of a target and the satellites in the Cartesian coordinates system, the distance of the target to each of the two satellites can be derived. Thus, the interferogram can be simulated.

In radar image coordinates, row and col index of the target points form an irregular grid. In order to get a phase image in regular grid, a regridding interpolation processing is necessary. [Eineder M., 2003]

## 4. **RESULTS**

Three groups of terrain data are randomly generated with fBm model, and each comprises three types of terrain, the flat region ( $\sigma = 3, H = 0.9$ ), the hill region ( $\sigma = 10, H = 0.6$ ) and mountain region ( $\sigma = 30, H = 0.3$ ). In the visual sense, these simulated terrain models are obviously different in terms of roughness and shape. The simulation results are shown in Figure 2.

In the study case, the orbit parameters of a pair of ERS-1/2 tandem data are assumed and a location for synthetic DEM in common area of two SAR images is assigned. Here the geographic location of left-low corner point of synthetic DEM is (30.2500°, 111.625°) and the space of pixels is 1.5 arcsecond (about 45 meter in ground). The difference of the elevation can change arbitrarily. Other parameters for the simulation are given in Table 1.

Item	Parameters
DEM pixel spacing	45 m×45 m
DEM data size	256×256
DEM location	LAT: 30.2500° ~30.3567° LON: 111. 625° ~111.7317°
Orbit of ERS-1/2	ERS-1 orbit = 25070 ERS-2 orbit = 5397 FRAME = 2997
Imagery time of ERS-1/2	1996-5-1/2
PRF	1679. 902 Hz
Wavelength	0. 05667 m
azimuth×range pixel spacing	3. 985 m ×7. 904 m
Nearest slant range	829. 213 km
Parallel and normal components of the baseline	$B_p = 52m$ $B_n = 102m$
Master to Slave Warp Matrix	-6507.176         1.00001         -0.00009           5.9979         -0.00001         1.00027

Table 1. The simulation parameters

We choose the synthetic hill region ( $\sigma = 10, H = 0.6$ ) to test the interferogram simulation algorithm with different elevation differences. Assume the look angle is 23 degree for ERS data. It is found that the height ambiguity approximates 90.1m when the normal baseline equals to102m. Figure 3(a) shows the synthetic DEM with the elevation changes from 16m to 536m. Figure 3(b) gives the corresponding interferogram after removing the flat phase. It is seen that the synthetic interferogram can well reveal the variety of DEM. Figures 3(c) and (d) give another example with the elevation changes from 263m to 367m.

### 5. CONCULSION

An interferogram simulator combining DEM simulation and interferometric phase simulation has been developed and tested with real orbit parameters and geometry model. It is a good assistant tool not only for validaition of phase-to-height converion and geocoding processing, but also for D-InSAR processing.

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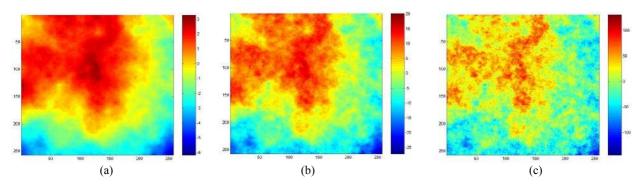


Figure 2 (a) The synthetic flat region with  $\sigma = 3, H = 0.9$ (b) The synthetic hill region with  $\sigma = 10, H = 0.6$ (c) The synthetic mountain region with  $\sigma = 30, H = 0.3$ 

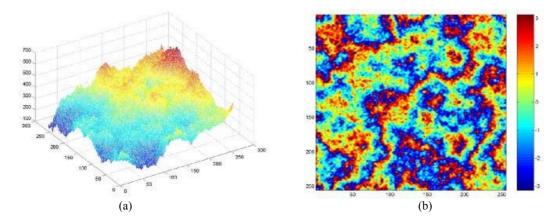


Figure 3 (a) The synthetic DEM with  $\sigma = 10, H = 0.6$ , elevation changes from 15m to 536m (b) The synthetic interferogram corresponding to DEM shown in (a)

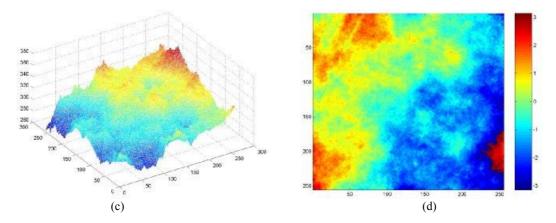


Figure 3 (c) The synthetic DEM with  $\sigma = 10, H = 0.6$ , elevation changes from 263m to 367m (d) The synthetic interferogram corresponding to DEM shown in (c)