A UNIFIED APPROACH TO STATIC AND DYNAMIC MODELLING IN PHOTOGRAMMETRY AND REMOTE SENSING

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ABSTRACT:

Modern photogrammetry and, more generally, the current technology for Earth observation are dependent on various forms of data processing. After the sensing or acquisition step, the data are available in digital format and all what has to be done is to calibrate, to orient and to extract georeferenced information. In this context, data processing for trajectory determination, sensor calibration and sensor orientation follows various patterns, all of them particular cases of the general time dependent parameter estimation problem defined by the equation $f(t, \ell(t) + v(t), x(t), \dot{x}(t)) = 0$, where f is the mathematical functional model, t is the time, $\ell(t)$ is the time dependent observation vector, $v(t)$ is a white-noise generalized process vector, $x(t)$ is the parameter vector and $\dot{x}(t)$ the time derivative of $x(t)$. A number of different approaches to estimate parameters $x(t)$ from data $\ell(t)$ has been developed according to the particular form of the above model equation. $\ell + v = f(x)$, $f(\ell + v, x) = 0$, $f(t, \ell(t) + v(t), x(t)) = 0$ and $\dot{x}(t) = f(t, \ell(t) + v(t), x(t))$ are examples of model equations leading to network and Kalman filter/smoother solution strategies. Although these two procedures have proven to be well suited to their respective model equation structure, the paper discusses some of their limitations and alternatives, particularly for time dependent problems. The proposed family of methods uses numerical techniques that integrate the rigorous least-squares method and the finite difference methods for the solution of the Boundary-Value problem of Ordinary Differential Equations. Although we do not claim that this has to substitute existing, proven techniques, the paper indicates how hybrid static and dynamic data processing can be easily integrated with this new approach.

1 INTRODUCTION

Nowadays, trajectory determination [1] for navigation, geodetic positioning and remote sensing orientation is mainly based on two parameter estimation methodologies: least-squares network adjustment —the network approach (NA)— and Kalman filtering and smoothing —the state-space approach (SSA). It is known that Kalman filtering is a general form of sequential least-squares. However, in practice, there is no much connection between the two approaches other than some output estimated parameters following the network approach being used as input observations for a second estimation step following the state-space approach. And vice versa. It must be mentioned that the GPS research related community has since long been faced to the problem of making a decision between classical least-squares, Kalman filtering and smoothing and some intermediate approaches (Xu, 2003). The dilemma holds for both the processing of moving object trajectories and for the processing of stationary or quasi-stationary objects. For the family of problems just mentioned (static, quasi-static and kinematic) there are examples of successful application of both the state space approach and of the network approach. To illustrate the statement, we cite two "classics" that have had and still have a significant impact in geomatics in the past decade. The GLOBK system (Herring, 2003) uses Kalman filtering and has been successfully applied to time-dependent precise networks for deformation monitoring originating from VLBI and GPS. At the opposite end, the GPS aircraft trajectories for Earth observation applications like aerial triangulation or LIDAR aerial surveys were determined under the network approach (Frieß, 1990).

The goal of the ongoing research behind this paper is not to devise a "unified" algorithm that package both classical least-squares and state-space estimation in "one." The approach is rather pragmatic —numerical, algorithmic and software oriented— as the theories of least-squares estimation (Koch, 1995) and state-space estimation (Maybeck, 1979a, Maybeck, 1979b) are well established. The actual goal is to interpret stochastic dynamic models —i.e., differential or difference equations— and their time dependent unknown parameters —i.e., stochastic processes— in a way that, for the time dependent parameter estimation problem, both the network approach and the state-space approach are applicable. We do not claim that both approaches be fully interchangeable. We do claim that in some circumstances, it might be advantageous to apply the network approach to the estimation of time dependent parameters. As well, we claim that time dependent problems in geomatics do not necessarily require a SSA treatment.

In addition to the numerical, algorithmic, software data modelling and software use potential advantages of a unified approach, there are a number of estimation problems that might benefit from it. They include the modelling

[1] In this paper trajectory determination is understood as the determination of a time series of positions, velocities and attitudes.

of trajectories for airborne and spaceborne imaging linear arrays, the calibration of inertial instruments (angular rate sensors and accelerometers) with "cross-over" type of observation equations and the modelling/estimation of geodetic networks for monitoring and prediction purposes. It has to be mentioned that a parallel research effort is being conducted by A. Térmens for inertial strapdown kinematic airborne gravimetry (Térmens and Colomina, 2003, Térmens and Colomina, 2004) for an optimal calibration of accelerometers.

The key idea behind this investigation is that a stochastic dynamic model (a stochastic differential equation) and its stochastic processes can be transformed through discretization into a family of stochastic difference equations and discrete time processes. Those, in turn, can be seen as a family of observation equations and parameters that can be processed under the network approach.

The paper begins by reviewing some definitions and concepts from the theory of stochastic processes and stochastic differential equations. We take this approach because of the available sound theory that includes continuity theorems and numerical solution methods consistent with the stochastic nature of the problem. Then, the state-space and the network approaches are defined and compared. Once this is done, in section 6 we define time dependent networks in a way that generalize the traditional least-squares based networks. Here, the scope of the concept of a dynamic or time dependent network is precisely defined. The algorithmic and software implementation implications of section 6, should be clear at that point. However, we underline them in section 7 for readers not familiar with the development of network adjustment systems.

2 STOCHASTIC PROCESSES

A stochastic process is a parametrized collection of random variables defined on a probability space $(\Omega, \mathcal{F}, P)$ (Law ler, 1995). The parameter space T is usually the time or a time interval. In other words, a stochastic process x is a set of random variables indexed by time

$$x := \{x(t) \,|\, t \in T, \, T \subset R\}$$

where R is the set of real numbers. In this paper, and in most applications, the parametrizing, indexing or tagging subset T is either N, the set of natural numbers, or R. If $T = N$, x is called a discrete time process and in the other case, $T = R$ or $T = [a, b] \subset R$, it is called a continuous time process. The set where the random variables take values, typically R^n, is called the state space.

From the definition, it is clear that for each $t \in T$, we have a random variable $\omega \longrightarrow x(t)(\omega) := x(t, \omega)$ for $\omega \in \Omega$. But the function $x(t, \omega)$, for a given fixed ω, can be seen as a function of t, $t \longrightarrow x(t, \omega)$ for $t \in T$. This function is a *path*. We introduce the concept of a path because it is close to our intuition in INS and GPS trajectories, satellite orbits, etc. When we look at a trajectory, ω can be seen as a point or one of our repetitive experiments and thus $x(t, \omega)$ would represent the position of the point at time t or the result of the particular experiment.

A fundamental stochastic process is the *Brownian motion* (or *Wiener process* or *continuous random walk*) named after a 19th century botanist who observed that pollen grains on a liquid described an irregular trajectory. Its formal derivative is called white noise. White noise is formally considered a stochastic process to facilitate the visualization and interpretation of the continuous idealization of discrete time processes whose random variables are independent, normally distributed ones. (Sometimes, in the engineering literature, it is said that the white noise process is a helpful concept that does not exist in the world of mathematics. In fact, this statement is wrong. White noise exists as a generalized stochastic process (Øksendal, 1993), a slightly more complex concept than a stochastic process.)

The stochastic analogs of *ordinary differential equations* (ODE) are the *stochastic differential equations* (SDE). The theory for SDE can be found in (Øksendal, 1993). SDE arise naturally from real-life ODE whose coefficients are only approximately known because they are measured by instruments or deduced from other data subject to random errors. The initial or boundary conditions may be also known just randomly. In these situations, we would expect that the solution p of the problem be a stochastic process. We will call $p = p(t, \omega)$ a *prediction*. Under certain [non-restrictive] hypotheses p has a number of properties including that it is t-continuous (Øksendal, 1993, pp. 48-49).

Assume now that we have managed to predict the stochastic process p —the *system*— over a time interval $[t_0, t_f]$. In our application, determining p reduces to determine an estimate of the path $E(p(t))$ and estimates of the process auto-covariance functions

$$C(t_1, t_2) := E\left((p(t_1) - E(p(t_1)))(p(t_2) - E(p(t_2)))^T\right).$$

Assume further that we are able to relate p through some linear model —the observation equations— to another process z —the *observations*— so we have additional information of p. A natural question arises: can we improve our estimates of p with the additional information z?. The answer, in general, is yes, and the tool is the well known filtering and smoothing. Filtering at time s refers to finding a best estimate for the system $\hat{p}(s)$, $t_0 < s < t_f$ given the observations z in the interval $[t_0, s]$. Smoothing, refers to finding the best estimate for $\hat{p}(s)$ at any time by using the information of z all over $[t_0, t_f]$. Saying that $\hat{p}(s)$ is best means that $E\left(\|p - \hat{p}\|^2\right)$ is minimal over all solutions of the system SDE that verify the observation equations (see (Øksendal, 1993, pp. 58-59) for a detailed description of the probability function associated to the SDE and to the observations white noise processes).

3 THE STATE-SPACE APPROACH

We will call *state-space approach* (SSA), the methodology and principles of solving the above problem of prediction, filtering and smoothing for time discrete processes (section 2).

The SSA is the well known Kalman filtering and smoothing published by R.E.Kalman in 1960 (Kalman, 1960) and discussed in numerous textbooks from different points of view (Maybeck, 1979a, Øksendal, 1993). Equivalent later formulations in terms of sequential least-squares can be found in (Teunissen, 2001). The SSA has been successfully applied to precise navigation for surveying applications (Scherzinger, 1997).

We borrow the state-space name from the state-space representation of a dynamical system. A state vector is a minimal set of variables whose values are able to describe a system. The optimal solution to the prediction-filtering-smoothing (section 2) is obtained through one of the recursive algorithms of the Kalman filter type.

In the prediction-filter cycle, the most important entity is the state vector. All the rest are subordinated parameters. In a way, the state vector dominates the scene which, in some situations, may represent a problem. One example is the difficulty in the feedback of the results of adaptive Kalman filter steps to a correct scaling of the inertial observations (angular rates and linear accelerations) in the inertial navigation equations. (In the *network approach* (NA), this reduces to a classical estimation of variance components). Another example of the weaknesses of the SSA is the estimation of gravity error states in the inertial navigation equations. We may estimate the gravity error of our gravity model in better or worse ways, depending on a number of instrumental, modelling and mission related factors. But we cannot impose that the gravity error estimated at time t_1 at point x_1 is the same as the gravity error estimated at a later time t_2 at point x_2 if $x_2 = x_1$ —the so-called cross-over points— as discussed in (Térmens and Colomina, 2003, Térmens and Colomina, 2004).

4 THE NETWORK APPROACH

In geomatics, a network is a set of instruments, observations and parameters that are inter-related through mathematical models. The mathematical models are the observation equations. To *solve the network* is to perform an optimal estimation of its parameters in the sense of least-squares; i.e., the expectation of the parameters and their covariance is known. Moreover, their covariance is minimal (Koch, 1995). The network approach exhibits superior performance when the connectivity that observations create between the unknown parameters is high.

In the network approach, our network will be solved in a grand, single adjustment step where all parameters, time dependent and independent, will be simultaneously estimated. This is giving us some hint on how to implement the network approach for time dependent networks in a computer programme. We discuss this in sections 6 and 7.

An [unknown] random variable —a time independent parameter— is to the classical network approach what an [unknown] stochastic process —a time dependent parameter— is to the state-space approach. In the following, the names “time dependent parameter” and “stochastic process” will be used indistinctly.

Note that the state-space approach can be used, as well, for the estimation of time independent parameters as they can be modeled as stochastic constant processes. A stochastic constant takes the same value c over time. c may or may not be known before the estimation process; but once it is estimated it will not change over the time period where the stochastic process is defined. An example of a random constant is a GPS ambiguity —integer or real— in a phase observation equation.

Note, as well, that a stochastic dynamic model (stochastic differential equation) can be transformed into a set of stochastic difference equations. Then, the family of stochastic difference equations can be seen as a set of observation equations and the network approach can be used. To discretize a stochastic dynamic model, we propose the difference methods (it is the “natural” way to do it). We are aware of limitations and/or inferior performances of the numerical difference methods for the solution of ODEs. However, the comparative analysis between difference methods and other more sophisticated numerical methods (variational methods, multiple shooting, ...) is usually done in the context of deterministic ODE (Stoer and Bulirsh, 1992). But, while the extension or generalization of the difference methods for deterministic ODE to the SDE is straightforward, the extension of the other mentioned methods is less obvious. In future investigations we will explore these numerical issues. Further, we refer the reader to the specific literature on the numerical solution of SDE (Kloeden and Platen, 1999).

5 COMPARATIVE ANALYSIS

In the previous sections we have looked at the SSA and the NA as different approaches to, essentially, solve the same problem. Before we introduce and discuss time dependent networks we summarize their main advantages and disadvantages from a geomatic perspective.

NETWORK APPROACH

- Advantages:
    1. Support for connectivity of parameters regardless of time.
    2. Support for both traditional networks and for SDE.
    3. Possibility to compute the covariance of a limited number of selected parameters.
    4. Variance component estimation.
- Disadvantages:
    1. Large system of linear equations.[2]

[2] The matrices are essentially of the band-bordered type and we can apply sparse matrix techniques, fill-in reduction techniques and memory paging to solve the system of linear equations.

2. Real-Time parameter estimation not feasible in general.

STATE-SPACE APPROACH

- Advantages:
    1. Real-Time parameter estimation capability.
    2. The state vector dominates the scene.[3] That is, there is a clear definition of what the system is.
- Disadvantages:
    1. Connectivity of parameters through static observation equations is not supported.
    2. Filter divergence.
    3. Computation of covariance matrices for all the state vectors cannot be avoided.

The above list is by no means comprehensive but, in our opinion, the only situation where the SSA is clearly superior is real-time parameter estimation. This statement should not be taken as a recommendation. In real life problems, other factors may be taken into account. For instance, in INS/GPS trajectory determination, a SSA based software engine can be applied to both real-time and post-processing computation modes. This aspect may be fundamental before making implementation decisions.

6 TIME DEPENDENT NETWORKS

A *time dependent network* is a network such that some of its parameters are time dependent; i.e., that some of its parameters are stochastic processes. Analogously, we define that to *solve a time dependent network* is to perform an optimal estimation of its parameters which include some stochastic processes. (However, this is easier said than understood and done. In this section we clarify the meaning of the above statement and in section 7 we suggest some implementation mechanisms.) We recall that optimality in estimating a stochastic process means to estimate the best expectation function path $\hat{x}(t)$ in the sense of having minimal $E\left(\|x-\hat{x}\|^2\right)$ as mentioned in section 2.

Note that we are asked to solve for more information in time dependent networks that in time independent ones. Accordingly, as it was to be expected, we will be given more information before the estimation process. This new information is the dynamic observation model for the random process. If we now rename our traditional observation equations as the static observation model(s), then the global picture of time dependent networks becomes clear and clean.

An static observation model is an equation of the type

$$f(t, \ell + v, x(t)) = 0 \qquad (1)$$

[3]For some models this advantage could be a disadvantage. See section 3 for a related discussion.

where v is a normally distributed variable of null expectation. A dynamic observation model —or a stochastic dynamic model— is an equation of the type

$$f(t, \ell(t) + v(t), x(t), \dot{x}(t)) = 0 \qquad (2)$$

where $v(t)$ is a white noise process. In more global terms, we will refer to the family of static observation equations as the network static model. And to the family of dynamic observation equations[4] as the network dynamic model. Typically, a particular dynamic model (2) will be given for $t \in S'$ where $S' \subset S$. Note that a dynamic observation equation may include time independent parameters and that a static observation equation may include time dependent parameters but not its derivatives. Note, as well, that the static model may be of the form (1). This is not only consistent with the concept of an static observation equation but necessary when it contains a time dependent parameter.

The dynamic model is a key component of a time dependent network. Indeed, all what we know about $x(t)$ before solving the network is that $x(t)$ is a stochastic process. Indeed, the static model contributes to the determination of $x(t)$. However, without the dynamic model there is no "dynamics" in the process; i.e., we cannot guarantee that the set $\{\hat{x}(t) | t \in S'\}$ is a continuous path. In principle, strictly speaking, mathematical continuity does not tell us much about the roughness or smoothness of the solution path but practical experience proves its effectiveness. (The lack of dynamic modelling results, in practice, in somewhat rough solutions for $\hat{x}(t)$. A typical example of this is found in the determination of GPS trajectories under the network approach when compared with the same trajectory determined under the state-space approach which are, usually, smoother.)

Note, last, that in practice, we do not have to compute the auto-covariance function; we just have to provide a mechanism to compute it if requested.

We illustrate the above simple definition with two examples: a geodetic monitoring network and an airborne imaging network (block) with INS/GPS aerial control. These two examples are time dependent networks as they include dynamic observation models and time dependent parameters. Note, for instance, that the orientation parameters of a block can be seen as a set of time independent, unrelated parameters $\{p_i | i = 1, \ldots, n\}$ or as a time dependent parameter $\{p(t) | t \in [a, b], a, b \in R\}$.

The airborne network (block) with INS/GPS aerial control is a time dependent network because its unknown orientation parameters position, velocity and attitude depend on the time. The "flight" is a stochastic process. This one is a stochastic process over $[t_0, t_f]$, where t_0 and t_f are the initial and the final time of the flight respectively. The stochastic process is just defined over a finite time period and we cannot predict the system beyond t_f because

[4]In this paper no distinction is made between "equations" and "models" (both terms including the stochastic and functional components). We will use both terms as appropriate to highlight the parallelism between the dynamic and static aspects of the problem.

INS/GPS observations are required for the dynamic observation equations. The general network model is made up of the dynamic observation model —INS observation equations— and the static observation model —GPS observation equations, ground control points and the photogrammetric collinearity equations.

The geodetic monitoring network is a time dependent network in that it is a network of observed and measured points at given epochs and we want to know the situation of the network points within the time observation epochs and in future time epochs. We have the measured points at epochs $[t_0,\, t_1,\, \cdots,\, t_f]$ and we want to determine the position of the network points at epoch $t_f\, +\, \Delta t$. This is, in principle, a stochastic process over $[t_0,\, +\infty)$. This model is made up of the static observation model —GPS static observation equations, known control point equations, known constant 3D coordinate differences for points in a same tectonic plates, etc.— and the dynamic observation model — known variable coordinate differences according to some geophysical deformation model.

7 A UNIFIED APPROACH

The implications of the definition of time dependent networks of the preceding section are obvious. However, for the sake of clarity we underline them under the theoretical, algorithmic, software and production viewpoints.

7.1 A unified theoretical approach

The classical network is a set of instruments, observations and parameters. They are related through static observation models. The network approach is a procedure to estimate the parameters. The inputs are the values of observations and, if needed, the initial approximations of the parameters. The outputs are the estimated values of the parameters. On demand, the network approach can generate the covariance of the parameters and/or the auto-covariance function.

The time dependent network concept that we propose in this paper is a set of instruments, observations and time dependent and independent parameters. They are related through static and dynamic observation models. A time dependent parameter generates a set of equations, one equation for every time epoch. Now, the network approach is a procedure to estimate both time dependent and time independent parameters. The inputs are the values of the observations and, if needed, initial approximations of the parameters (note that, in this case, initial approximations are for time dependent and independent parameters). The outputs are the estimated values of the parameters including the stochastic processes. On demand, the network approach can generate the covariance of the parameters and/or the auto-covariance function. We insist on the parallelism of the time dependent and time independent network concepts.

We claim that the time dependent network concept proposed provides a unified theoretical framework that covers the estimation of time dependent and time independent parameters. The time dependent network is based on static and dynamic observation models. The time independent network is (solely) based on static observation models. Thus, the classical network can be seen as a particular case of the new time dependent networks.

This unified approach is the basis for the reasonable development of time dependent network determination software, which is at the same time rigorous and simple. We discuss this aspect in the next section.

7.2 A unified algorithmic and software approach

A modern well designed software system of the class we are discussing here is based in the object-oriented paradigm. Combining object-oriented design and the previous theory, a simple and powerful time dependent network determination software can be generated. This software system shall include these fundamental entity classes: observation, instrument, parameter and model. See (Colomina et al., 1992) for a related discussion and modelling in time independent networks.

The observations may have an associated time (time epoch of the observation). We call them time-tagged observations. However, we emphasize that our observations, although time dependent, are stochastically independent as they are only subject to a white noise process. In principle, it should not come as a surprise that for a time dependent networks, all what we have to do is to generalize time dependent parameters and dynamic observation models from time independent parameters and static observation models, respectively.

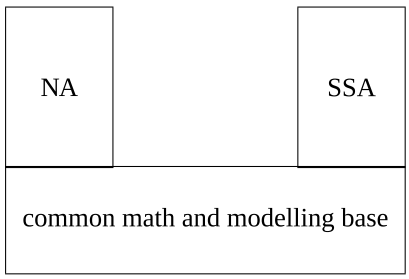

Figure 1: Unified SW approach

Interestingly enough, in our unified software approach, the mathematical foundation libraries are not much different from the classical approach. This applies both to internal software aspects and to interface aspects. Moreover, with minor changes, most of the organizational parts and discrete mathematical components of existing [well designed] network adjustment packages can be kept. Even more interesting is the fact that the NA and SSA computational engines can share the same model libraries, as the estimation engines work with the same models, their software implementation and their external interfaces. In other words, the parallel development and maintenance of an NA and an SSA engine within the frame of a general system is possible.

7.3 A unified exploitation approach

Unified theoretic frameworks lead to simple and efficient algorithms and software. Unified software approaches lead to simple and efficient exploitation procedures. In particular, an eventual software implementation of the concepts presented, would lead to common shareable input/output formats for a number of estimation engines.

A benefit of a unified approach is that we can follow different strategies and that we can combine them. In some situations, one approach should be preferred. In other situations we can combine them. For a family of problems, one approach may be preferred for calibration tasks whereas the other may be preferred for orientation tasks.

Note, as mentioned in section 1, that the output estimated parameters of a static network may be used as input observations for a time dependent network. Similarly, an SSA engine can be used to generate initial approximations for a NA engine. In all the cases, it is clear that interoperability is easier to achieve with a unified approach.

8 CONCLUSION, ONGOING WORK AND FURTHER RESEARCH

In this paper we have defined in a precise way the concept of time dependent networks. The proposed concept extends the classical unified (from geodesy, photogrammetry and remote sensing) geomatic concept of network. In short, a time dependent network is a classical network that incorporates stochastic processes —that we call time dependent parameters— and dynamic models —that we call dynamic observation models. We have related time dependent networks and their solution approaches to the existing Kalman filtering/smoothing and network methodologies —what we call the SSA and the NA solution approaches— and have discussed their advantages and disadvantages. Last, we have given some hints on how this unified approach can be exploited at the software development and data processing levels.

We are currently developing an experimental software prototype that implements the concepts presented in this paper. Further research will be related to the numerical solution of SDEs for geomatic applications and to their optimization in terms of speed and memory/disk storage requirements.

REFERENCES

Colomina, I., Navarro, J. and Térmens, A., 1992. GeoTeX: a general point determination system. In: International Archives of Photogrammetry and Remote Sensing, Vol. 29-B3, International Society of Photogrammetry and Remote Sensing, pp. 656–664.

Frieß, P., 1990. Kinematische Positionsbestimmung für die Aerotriangulation mit dem NAVSTAR Global Positioning System (Ph.D. Thesis). C, Vol. 359, Deutsche Geodätische Kommission, München, DE.

Herring, T., 2003. GLOBK: Global Kalman filter VLBI and GPS analysis program, version 10.1. Technical report, MIT, Cambridge, MS, US.

Kalman, R., 1960. A new approach to linear filtering and prediction problems. Transactions of the ASME, Journal of Basic Engineering 82(1), pp. 34–45.

Kloeden, P. and Platen, E., 1999. Numerical solution of Stochastic Differential Equations. Springer Verlag, New York, US.

Koch, K., 1995. Parameter estimation and hypothesis testing in linear models. Springer Verlag, Berlin, DE.

Law ler, G., 1995. Introducttion to Stochastic Processes. Chapman & Hall/CRC, Boca Ratón, FL, US.

Maybeck, P., 1979a. Stochastic models, estimation and control. Mathematics in science and engineering, Vol. 141-1, Academic Press Inc., New York, NY, US.

Maybeck, P., 1979b. Stochastic models, estimation and control. Mathematics in science and engineering, Vol. 141-2, Academic Press Inc., New York, NY, US.

Øksendal, B., 1993. Stochastic differential equations: an introduction with applications. Universitext, third edn, Springer-Verlag.

Scherzinger, B., 1997. A position and orientation postprocessing software package for inertial/GPS integration (POSPROC). In: Proceedings of the KISS'97 Symposium, Calgary, AB, CA, pp. 197–204.

Stoer, J. and Bulirsh, R., 1992. Introduction to Numerical Analysis. second edn, Springer-Verlag, New York, US.

Térmens, A. and Colomina, I., 2003. Sobre la corrección de errores sistemáticos en gravimetría aerotransportada, (in Spanish). In: Proceedings of the 5. Geomatic Week, Barcelona, ES.

Térmens, A. and Colomina, I., 2004. The Network Approach versus the State-Space Approach for strapdown inertial kinematic gravimetry. Abstract submitted to the IAG International Symposium Gravity, Geoid and Space Missions - GGSM2004, Porto, PT.

Teunissen, P., 2001. Dynamic data processing. Delft University Press, Delft, NL.

Xu, G., 2003. GPS theory, algorithms and applications. Springer-Verlag, Berlin, DE.

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