APPROXIMATION OF NON PROJECTIVE MAPPING AND THEIR EFFICIENT APPLICATION FOR A GEOMETRICALLY BASED 3D POINT DETERMINATION USING MULTIPLE VIEWS

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ABSTRACT:

In this paper an unusual taxonomy for optical mappings is introduced based on their geometric characteristics: 1. type of projection center (single viewpoint, non single viewpoint) and 2. type of transformation (projective, non projective). Under this background we survey the multi media geometry (refraction resulting from different optical media). Strict physical models of non projective mappings can be very complex in dependency on their geometric nature. Therefor different methods for reducing the complexity exist. This paper describes a method of ascertaining a virtual camera to approximate non projective mappings by a projective model and their application for a 3D point determination using multiple views with non projective multi media geometry. As will be seen, the approximation can be used without loosing the quality of the strict model significantly. For the matching process we introduce a new algorithm for multiple views based on geometric constrains alone which uses all images simultaneously.

1 INTRODUCTION

1.1 Motivation

The nature of an optical mapping process between a 3D object space and a 2D image space depends on the geometry of the imaging system, its physical laws and the scene structure.

In this context we use the term *imaging system* instead of camera system, because it should contains all parameters, which have an effect on the nature of the optical mapping. In particular the effect on the way of mapping light-rays, influenced by light refraction or reflection.

Based on the different nature of optical mappings and their characteristic, several kinds of classifications exist (e.g in (Hartley and Zisserman, 2003) whether they have a finite centre or a centre "at infinity", or whether they preserve straight lines or not).

An unusual feature for a classification of mappings is the kind of image distortion, resulting from light refraction or reflexion of the mapping rays. Therefore we introduce a classification of imaging systems, based on the following geometric features, which influence the characteristics of image distortions:

Geometrical characteristics of optical mappings:

1. **Single Viewpoint (SVP) or Non Single Viewpoint (NSVP)**
   mapping rays intersect in one single point or not
2. **Invariance or variance of straight lines**
   straight lines in the scene appear as straight lines in the 2D image space (*projective mapping*) or appear as curves

From the combination of these geometric features, we get a *classification of imaging systems* with three different classes, summarised in table 1. It is based on the classification of optical mappings in (Wolff and Förstner, 2001) and on the taxonomy of distortions published in (Swaminathan et al., 2003).

Table 1: Classification of imaging systems

Class	*Mapping*	*Viewpoint*	*Imaging System*	*Modeling distortion*
1	Projective	Single Viewpoint	Pinhole	-
2	Non Projective	Single Viewpoint	Wide-angle cameras, fish-eye cameras, central catadioptric cameras, Approximation of objective distortion	based on position in image space
3	Non Projective	Non-Single Viewpoints	Wide-angle cameras, fish-eye cameras , catadioptric cameras, camera clusters, moving cameras, multimedia geometry, objective distortion	based on position in object space

1. Class 1 is the *perspective mapping*, also named *pinhole camera*. It is the most specialized and simplest model, where the straight projecting rays intersect in a single viewpoint (the pinhole) and preserves straight lines. This results in no image distortions (not taking distortions into account, which result from the perspective mapping). All cameras modelling central projection are specialisations of the general *projective camera*, therefore we use the term *projective mapping* which could be presented by a projective model. Every deviation from this model causes image distortion.
2. Class 2 is created by mappings with single viewpoints, not preserving straight lines. However, this leads to image distortions, which depends on the image position. Its influence

or error can be calculated in dependency on their image coordinates (*image space based*). No information about the scene structure are needed. An example for such an projection is the general used model for the optical distortion. Strictly, it has not a single viewpoint, but the accuracy of this approximation is well enough.

3. Class 3 is formed by projections with *non single viewpoints which do not preserve straight lines*. The projection rays are no straight lines and they do not intersect in one point. But their envelope forms a locus of viewpoint in three dimensions which is called caustic surface or just caustic (Swaminathan et al., 2001). The resulting image distortions are called caustic distortions. Their exact determination bases on the position of the observed feature in object space (*object space based*). Therefor information about the scene structure are necessary to determine the influence of the distortions. Imagesystems like wide-angle, fish- eye and catadioptric cameras with a spherical and conical reflector based design (Nayar et al., 2000), camera clusters, strict model of objective distortion and multi media geometry (e.g. air and water) belong to this class. In section 2 we will present the caustic of a multi-media system.
4. The combination of a non single viewpoint with an invariance of straight lines is under the valid physical laws not possible.

The influence of image distortion using imaging systems with non single viewpoints is object space based, that means it cannot be determined or corrected without any information of the scene structure. If no information about the scene structure are given, it is necessary to make some assumption about the scene structure (e.g. (Swaminathan et al. 2003)). For the mapping process between the object and the image space, special algorithms are needed. For example the iterative algorithms for the multi media geometry in (Maas, 1995), which could be very complex.

Another method is to replace the non single viewpoint by a single viewpoint, so that the mapping process can be modeled without any information about the object space. Swaminathan et al. presented in (Swaminathan, 2001) a method to determine a single viewpoint by estimation the best location to approximate the caustic by a point for catadioptric cameras. This methods based on the determination of singularities of the caustic.

A method which is used here to define a single viewpoint is first mentioned in (Wolff and Förstner 2000) and was published in more detail in (Wolff and Förstner 2001): the explicit strict physical model with non single viewpoints is replaced and approximated by a less complex projective mapping with a single viewpoint. Therefor no pre-informations about the scene structure are needed. The estimation of the approximation is posed as the minimization of the back projection error in image space. The introduced approximation is applicable for all kinds of optical, non projective mappings. The degree of approximation can be augmented by partitioning the object space into small segments and calculating a local approximation for every part of the object space separately. For this partitioning we need the extension of the observing area approximately. The method was presented in (Wolff and Förstner 2001) used for a matching process based on the trifocal tensor.

1.2 Goal of this paper

In the context of non projective projections, the paper makes the following key contributions:

- Under the background of the taxonomy of imaging systems we survey the non projective multi media geometry (projecting rays passes different media e.g. air, perspex and water). It belongs to class 3 with a caustic as a non single viewpoint.
- We presend a new image point matching algorithm for a 3D reconstruction using multiple views, based on geometrically constraints alone. The method uses all images simultaneously. The test of hypotheses is placed on object space.
- The approximation for a non projective mapping by a virtual, projective camera is used for the image point matching process for multiple views with multi media geometry. As we will see , this is implemented without loosing the quality of the strict model significantly.
- Different quality tests for the approximation and the point matching algorithm are realized.

1.3 Projective Geometry

We use multiple-view geometry as it has been developed in recent years and is documented in (Hartley and Zisserman 2003).

Assuming straight lines preserving mappings, the projection of object points $\mathbf{X}$ to image points $\mathbf{x}'$ can be modeled with the direct linear transformation (DLT):

$$\mathbf{x}' \quad = \quad \mathsf{P}\mathbf{X} = \begin{pmatrix} \mathbf{1}^{\mathsf{T}} \\ \mathbf{2}^{\mathsf{T}} \\ \mathbf{3}^{\mathsf{T}} \end{pmatrix} \mathbf{X} = \mathsf{K}R(I| - \boldsymbol{Z})\mathbf{X}$$

for object points $\mathbf{X}$ represented in Plücker coordinates. P is the projection matrix, K the calibration matrix, R the rotation matrix and $\boldsymbol{Z}$ the projection center of the camera.

2 GEOMETRY OF IMAGING SYSTEMS WITH NON SINGLE VIEWPOINTS

2.1 Caustics as Loci of Viewpoints

For the modeling of point projection we need two relations:

1. A projection relation predicting the image point $\mathbf{x}'$ of a given object point $\mathbf{X}$.
2. An inverse projection relation, giving the mapping ray $\mathbf{L}$ in the object space. In case of projective mapping a light ray is build by the projection center and the image point. In case of non projective mappings only that part of the broken ray is important, which intersects the object point.

For Class 1 and 2 of our classification the realization of these two relations is geometrically trivial. The mapping ray is built by the object point or rather the image point and the projection center. In the case of image distortion a correction of the image points can be calculated image space based.

For class 3 relation 2 is also trivial. The projecting light rays change their direction because of refraction and reflection (see Fig. 1). These changes can be directly determined using the Snell's refraction law and reflection law. Relation 1 is not as trivial like the others, because the direction of the ray coming from the object point is not directly determinable if the object point and the physical pupil of the lense is given alone. But, as seen in Fig. 1, the envelope of the rays, which do not intersect in one point, forms a locus of viewpoints in three dimensions, the so called caustic. The light rays in object space are the tangent on this surface. Each point on the caustic surface represents the three-dimensional position of a viewpoint and its viewing direction. Thus, the caustics completely describes the geometry of the catadioptric camera (Swaminathan et al., 2001).

Swaminathan et al. uses this caustic in (Swaminathan et al., 2001) for an analyzation of an catadioptric camera for its characteristics like field of view, resolution and geometric singularities. They also present a calibration technique to estimate the caustic surface and camera parameters for a conic catadioptric system using known camera motion.

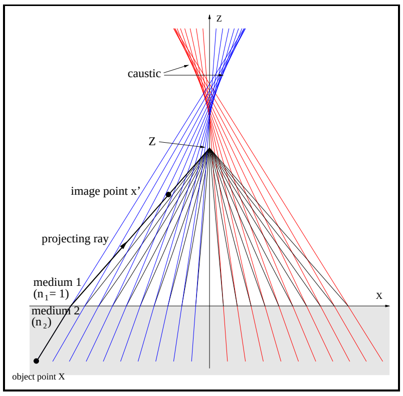

Figure 1: Geometry of a multi media system consisting of air and water, showing the projecting light rays and the caustic of the system.

2.2 Multi Media Geometry

Multi media geometry results from the observation of an object through several transparent physical media with different refraction indices. The light rays are refracted at the interface surfaces between two media. The classical application is to aquire images of objects through the media air, perspex and water, e.g. the 3D tracking of particles for modelling in fluid dynamics (Maas, 1991) or the observation of fluvial transportation processes under water (Wolff and Förstner, 2000).

The parameters of an imaging system with multi media geometry consists of the parameters of the used camera and the parameters of the refraction (exterior orientation of the refracting interfaces and the refraction indices). As seen in Fig. 1, refracted light rays coming from object points do not intersect in one point, but their envelope forms a caustic (Wolff and Förstner 2001). The imaging systems involve the projection center of the camera and the non single viewpoint. Therefor the mapping in general is not projective, it does not preserve straight lines. The resulting image distortions could not be modeled image space based and the complexity of the calculation of the mapping between object and image space increase.

For the strict realisation without approximation of the two relations mentioned in section 2.1, we use the multi media geometric models described in (Maas, 1995). He used a strict multi media geometric model based on Snell's Law for the effect of a ray being twice broken. For relation 1. a radial shift of each object point relative to the nadir point of the camera can be calculated and used as a correction term in the collinearity condition. This projection actually inverts a ray tracing process. As the algebraic expression cannot be directly inverted, the calculation is an iterative procedure. For relation 2 the Snell's Law is used to inverse the projection directly by ray tracing.

3 APPROXIMATION OF NON PROJECTIVE MAPPINGS BY MEANS OF VIRTUAL PROJECTIVE CAMERAS

For the approximation we assume the expected volume V in object space to be approximately known. We assume the non projective mapping $\mathbf{x}' = f(\mathbf{X})$ to be known, i. e. the orientation and calibration to be available. The task is now to find a projective mapping $\overline{\mathbf{x}}' = \mathsf{P}\mathbf{X}$ such that the systematic errors of the image coordinates $\overline{\mathbf{x}}' - \mathbf{x}'$ are minimum. This leads to the well known problem: determine P such

$$\Omega = \int_{\mathbf{X}\in V} (\mathbf{x}' - \mathsf{P}\mathbf{X})^{\mathsf{T}}(\mathbf{x}' - \mathsf{P}\mathbf{X})d\mathbf{X} \rightarrow min,$$

where the integral is to be taken over the expected volume V of interest. The camera with this projection matrix P is a *virtual projective camera*, which we use for the approximation. Its quality impair with the enlargement of the volume V. To get the acquired quality of the approximation, we may partition the object space and in corresponding way the image space such that for every part V_i, $\sum_i V_i = V$ of the volume a local DLT with the local projection matrix ${}^a\mathsf{P}$ is solved. To define the partition of the volume V a priori quality analysis of the projective model have to be carried out (Wolff and Förstner, 2001).The determination of the Direct Linear Transformation contains the following steps:

Determination of the Direct Linear Transformation:

1. determination of the parameters of the imaging system orientation using the strict model
2. define of a regular $o \times p \times q$ grid of object points $\mathbf{X}_i$, $i = 1..(o \cdot p \cdot q)$ lying in the expected object volume.
3. a priori quality analysis of the projective model
4. subdividing of the object volume V into parts V_i accordingly to the a priori quality analysis
5. calculation of the corresponding image points $\mathbf{x}'_i$, $i = 1..(o \cdot p \cdot q)$ using the strict non projective model
6. estimation of the projection matrices ${}^a\mathsf{P}$ for every V_i by minimizing the back-projection error using the object points $\mathbf{X}_i$ and the image points $\mathbf{x}'_i$.

If enough well distributed control points and corresponding non projective image points can be measured, the DLT could be also determined by using these real data.

4 MATCHING AND 3D DETERMINATION OF POINTS FOR MULTIPLE VIEWS USING GEOMETRICALLY CONDITIONS

4.1 The Algorithm

The algorithm for finding matching candidates in multiple image views assumes the extracted points of $n > 2$ images to be given with their projection matrix P. The radiometric information of the images were only used to extract the image points and are not required for the matching algorithm. It has the following characteristics:

1. only geometric conditions: all projection rays of corresponding image points intersect in one object point.
2. all images are used simultaneously.
3. test of hypotheses placed in object space.
4. if necessary, correspondency tests using radiometric information can be easily implemented.

Methods for multiple image matching based on geometrically constraints use often only three or four images simultaneous, like the matching algorithm using the trifocal tensor (Wolff and Förstner, 2000), the quatrifocal tensor or using the intersection of two epipolarlines (Maas 1997). To take all images into account, the algorithms are used for different combinations of three or four images.

The number of ambiguities using geometrically conditions alone, was examined by Maas in (Maas, 1992) for different numbers of images. The complexity of the matching strategy arise with the number of images, but the high amount of ambiguities to be expected requires more images to be reduced. Therefore we use $n > 2$ images and the constrain for an object point, that its image points are seen in at least three images, to eliminate wrong hypotheses.

The presented algorithm uses all images simultaneously, therefor the test of hypotheses is realised in the object space. The geometric condition is that all projection rays of corresponding image points intersect in one object point. Therefor we first find matching hypotheses by using the epipolar constraints defining one image as the starting image. The epipolarlines between the starting image and all other images are calculated and every image point close to the epipolarline is a hypotheses for a corresponding point to the point in the starting image. The epipolarline can be shorten by considering the height extension in object space. To get also the points, which are not seen in the first image, but maybe in at least three other images, this step should be calculated also for other images as starting images. The number of starting images depends on the constellation of the image system. Then we determine the object points belonging to these two point hypotheses. The result is a 3D point cloud, where a group of at least m close points define one object point. The number of points m depends on the number of starting images. To test the hypotheses of correspondences, a clustering of the point cloud is calculated using the k-means algorithm. The resulting clusters containing at least m points belong to one object point. The mean value of the points in one cluster is a first approximation of the 3D point determination. If a higher quality is required, all points can be finally determined with by estimating a bundle adjustment. Therefor we use the image point correspondences resulting from the points belonging to one cluster. We summarize the algorithm into the following steps:

The main steps of the algorithm for 3D prediction of points are:

1. Extraction of points $\mathbf{x}_i^{(j)}$ in all n images, where j is the number of the image and i the number of the point.
2. define one image as the starting image a.
3. for all points $\mathbf{x}_i^a$ in image a determine hypotheses of point correspondences using epipolar lines in all other images.
4. define another image as second starting image b.
5. for all points $\mathbf{x}_i^b$ in image b determine hypotheses of point correspondences using epipolar lines in all other images.
6. if necessary repeat point 4 and 5 for as much different images as it is convenient.
7. clustering of the 3D point cloud P resulting from point 2 to 6 $\rightarrow$ approximated 3D object points $\tilde{\mathbf{X}}_i, i = 1..m$.
8. final bundle adjustment of all matched points, using $\tilde{\mathbf{X}}_i, i = 1..m$ as approximated values $\rightarrow$ final object points $\mathbf{X}_i$.

If the imaging system and the resulting images are not projective, then there exist two different possibilities:

1. The specialized strict physical model of the mapping process will be implemented in the algorithm, which is sometimes not possible, or the strict physical model can be very complex and the computational time can rise in dependency on the algorithm.
2. An approximation for the non projective mapping is used for the matching process. For the a priori quality control of the percentage reduction of computation complexity for the replacement of the multi media geometry by a normalized projective model see (Wolff and Förstner, 2001).

4.2 Application for Non Projective Views

The application of the approximation by a virtual projective camera presented in section 3 contains the following steps:

Implementation of the virtual camera for an effective 3D point determination using non projective mappings

1. determine virtual projective mappings ${}^a\mathsf{P}$ for the observation space
2. matching the image points using ${}^a\mathsf{P}$
3. final bundeladjustment using the strict model

5 EXAMPLE AND QUALITY CONTROLS

5.1 Data: a surface of a fluvial sediment

Our work on using multi media geometry is motivated by investigations on the generation of fluvial sediments (Wolff and Förstner, 2000). The aim is to derive a physical model of the underlying process of the dynamical sediment transport. The surface of the water is smoothed by a perspex pane. We get the standard case of multi media geometry: air, perspex and water with plane interfaces. The observed sediment surface is shown together with the extracted points of one image in Fig. 2 (for the extraction of interest points see (Förstner, 1994). The surface of the sediment was formed by a jet of water hitting the sediment. We used four Sony XC-77 CE cameras (748×564 pixel) for the acquisition of the images.

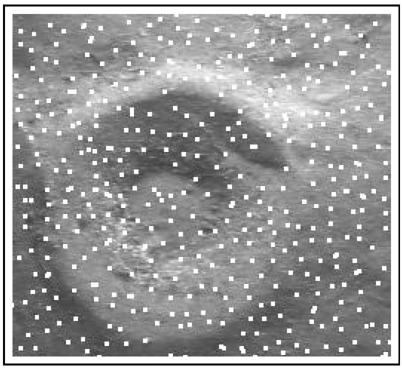

Figure 2: Image of the sediment surface with extracted points.

5.2 Determine the reference data using the strict model

To get reference data for the quality analysis of the matched image points and determined object points we carried out the presented algorithm using the strict multi media model. We use the same software and values for its parameters to calculate reference data and the approximated data.

5.3 Quality analysis

For the quality analysis of the 3D determination of points using the approximation, we want to examine the following points:

Table 2: Quality analysis 1: estimation of the virtual cameras

observing area: left lower corner observing area: right upper corner	$(3, 3, -17)$ [cm] $(21, 23, -11)$ [cm]
number of points used for estimation distance between points	845 2 [cm]
$\hat{\sigma}_{approxVC}(cameraC1)$	0.04 [pel]

Table 3: Coordinates of the projection centre of camera C1 for the strict model and the approximation VC

Projection center	X_1 [cm]	X_2 [cm]	X_3 [cm]
Strict model	3.08	4.53	63.46
Approximation VC	3.07	4.51	85.98

Quality analysis:

1. A priori quality DLT:
   residuals as backprojection errors in image space
2. Quality DLT:
   residuals in object space for new points
3. Quality point matching algorithm:
   comparison of the reconstructed points (before final estimation) using the strict and the approximated model
4. Quality point matching algorithm:
   comparison of the reconstructed points (after final estimation) using the strict and the approximated model

5.4 Prediction of 3D points using the virtual camera

5.4.1 Estimation of the virtual cameras To define the segmentation of the object space a priori quality test have to be calculated (see (Wolff and Förstner, 2001)). These a priori tests show, that the determination of only one virtual camera (VC) for the whole object space is enough. For the position of the four cameras see Fig.4.

To investigate the quality of the determined virtual cameras (Quality analysis 1), we project the object points which were used for the estimation of P into the image space and get the image points $\overline{\mathbf{x}'}$. The estimated DLT (11 independent parameters) yields residuals $\overline{\mathbf{x}'} - \mathbf{x}'$ being systematic errors. To get an a priori quality of the projective model we give the r. m. s. error

$$\widehat{\sigma}\text{approx} = \sqrt{\frac{\sum_i (\overline{\mathbf{x}}'_i - \mathbf{x}'_i)^2}{2n - 11}}$$

where n is the number of points used.

Tab. 2 gives the entities and results of estimating the virtual cameras of camera C1. The number of points used for the estimation need not to be as high as in this case. Tab. 3 gives the coordinates of the camera projection center for the three different orientations. The multi media geometry influence mostly the hight of an object point, which is here the X_3 coordinate of the projection center. Therefor the projection center of the two orientations differ mostly in the hight.

5.4.2 Results of the point matching using the approximation As mentioned above, the algorithm should be calculated for different starting images, to guarantee that also the points, which are not extracted in the starting image, can be found. Here we use four cameras, every camera could see the whole object scene. Together with the constrain, that at least three corresponding image points of an object point are needed, it is enough to have two different starting images. Therefor and because of the constraints, that the image points of an object point should be seen in at least three image points, we got the constraint for our clustering algorithm: a group of at least three define an object point.

First, we want to examine if the constraint for a object point, that at least three close points in a group define an object point, is sufficient. Fig. 3 shows the hypotheses of two matched image points by there corresponding object points (seen from the side). The distribution of the 3D points shows a very dense part, where the sediment surface is supposed to be. All the other points might be wrong hypotheses and should be deleted by the clustering algorithm. Fig. 4 shows the results after the clustering. All points which differ significantly from the surface are eliminated (Fig. 4 a). Fig. 4 b) shows the distribution of the object points on the surface, which are evenly distributed.

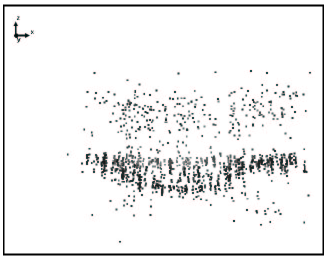

Figure 3: Hypothesis of 3D point matchings before clustering. A group of at least three points define an object point.

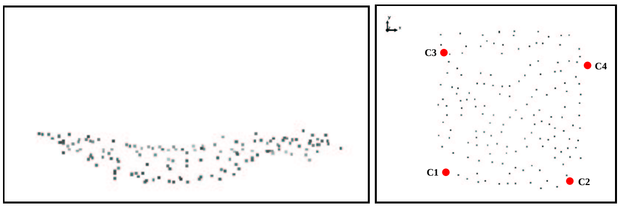

Figure 4: Results after clustering the point hypotheses. The right figure shows the point cloud from the side, the left figure shows is from above together with the positions of the cameras.

Using the strict model gave 156 reconstructed 3D points, the use of the virtual camera VC found 161 points. For the quality analysis 3. we have to compare the two sets of points. Therefor a threshold ϵ is defined, so that a point $\tilde{\mathbf{X}}_k$ is defined as equal to a referent point $\mathbf{X}_j$ if $|\tilde{\mathbf{X}}_k - \mathbf{X}_j| < \epsilon$. The number of points found as equal in dependency of the threshold is shown in Fig. 5.

The main influence of the approximation refers to the hight of the object points. The r. m. s. error of the X_3 coordinate of the reconstructed object points $\mathbf{X} = (X_1, X_2, X_3)$ is

$$\widehat{\sigma}_{X_3} = \sqrt{\frac{\sum_i (\overline{X}_{3_i} - X_{3_i})^2}{n - 1}},$$

where n is the number of points used. The error of the approximation is given in table 5 in comparison to the referent data before calculating the final bundle adjustment.

5.5 Final 3D determination of the predicted points using the strict model

After the matching process, including an approximated determination of the object point, we calculate a final bundle adjustment for the strict model and for the approximation VC. The clusters resulting from the clustering algorithm contain that points, which were found as corresponding points. To compare this clusters

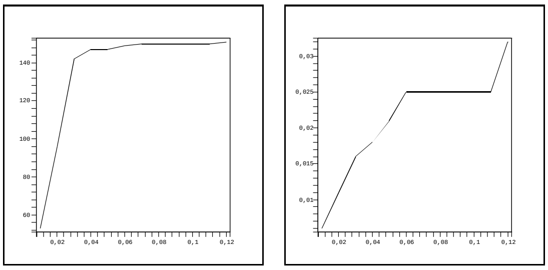

Figure 5: Quality analysis 3: comparison of the point prediction between the strict and the approximated mapping. Left figure: number of points found as equal, depending on the used threshold ϵ. Right figure: histogram of σ_{X_3}, depending on threshold ϵ.

Table 4: Quality analysis 4: Comparison of the final estimation using the strict and the approximated model

	Test
number of points ($\epsilon \leq 0.001$)	149
$x_{max}(\epsilon \leq 0.001)$	$1.0 \cdot 10^{-5}$ [cm]
$y_{max}(\epsilon \leq 0.001)$	$3.0 \cdot 10^{-5}$[cm]
$z_{max}(\epsilon \leq 0.001)$	$2.0 \cdot 10^{-4}$ [cm]
number of points ($\epsilon \leq 0.1$)	1
number of points ($\epsilon \leq 0.15$)	2
number of points ($\epsilon > 0.15$) or not found	4

(quality analysis 4), the corresponding object points were determined by fixed parameters of the orientation of the imaging system. A cluster is defined as equal, if the difference between the reconstructed points is smaller than 0.001 cm. The results are given in Tab. 4. 149 clusters are identical, 1 object point has a difference which is smaller than 0.1 cm, 2 points smaller than 0.15 cm and 4 points have a bigger difference than 0.15 cm or were not found by using the virtual projective camera as an approximation.

For quality analysis 2 we compare the final estimated 3D points using the strict model of the multi media mapping and the approximation. The error is given in Fig.fig:histogram. The differences are normal distributed.

Fig. 7 shows the digital terrain model of the sediment surface resulting from the estimated object points.

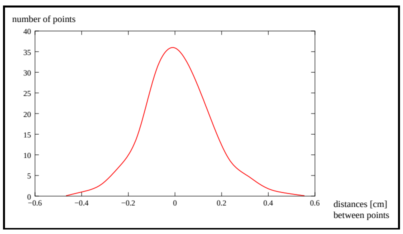

Figure 6: Quality analysis 2: comparison of the final estimated 3D points using the strict model and the approximation.

6 SUMMARY

In this paper we introduced a classification of optical mappings based on the geometry of the imaging system having a single viewpoint or a non single viewpoint. From this classification we got different kinds of image distortions: image space based and object space based. The models for optical mappings belonging to the second kind of mappings need information about

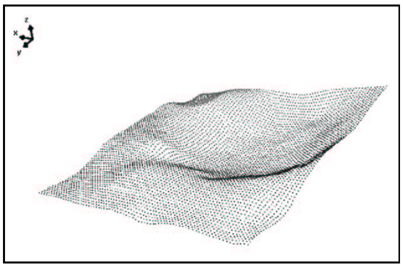

Figure 7: Reconstruction of the sediment surface resulting from the matched points using the virtual camera.

the scene structure and special complex algorithms for the projection between object and image space. Under this background we surveyed the multi media geometry. We presented a method to calculate a virtual projective camera which approximate the strict non projective mapping. The approximation was used for a point matching process using multiple views of a sediment surface with multi media geometry. We introduced a new matching process for multiple views based on geometric constraints alone, which is usable for projective mappings and the approximation of non projective mappings. Different quality tests show, that the approximation is sufficient for the reconstruction of a sediment surface.

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