FRACTAL AND SURFACE MODELING

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ABSTRACT:

Digital surface representation from a set of three-dimensional samples is an important issue of computer graphics that has applications in different areas of study such as engineering, geology, geography, meteorology, medicine, etc. The digital model allows important information to be stored and analyzed without the necessity of working directly with the real surface. In addition, we can integrate products from digital terrain model (DTM) and other data in a geospatial information system (GIS) environment.

The objective of this work is to model surfaces from a set of scattered three dimensional samples. The basic structure used to represent the surface is the triangulated irregular network (TIN). Another goal of the paper is evaluation of the quality of digital terrain models for representing spatial variation. This work presents stochastic methods for triangular surface fitting.

One of the most popular stochastic models to represent curves and surfaces are based on fractal concept**.** A fractal is a geometrical or physical structure having an irregular or fragmented shape at all scales of measurement**.** In addition, a fractal is based on self-similarity concept indicating that each part of its structure is similar to the whole**.**

Brownian motion is the most popular model used to perform fractal interpolations from a set of samples.

The Fractional Brownian motion (FBM)**,** derived from Brownian motion, can be used to simulate topographic surfaces. FBM provides a method of generating irregular, self-similar surfaces that resemble topography and that have a known fractional dimension.

Fractal concept has been used for optimum sampling in generating a digital terrain model. Results of the research have shown that the method can be successfully used in DTM generation. In addition fractals allow us to create realistic surfaces in shorter time than with exact calculations. Another advantage of the fractal concept is the possibility of computing surfaces to arbitrary levels of detail without increasing size of the database.

1. INTRODUCTION

So far we have assumed that sharp boundaries or smooth shapes exist for real entities. This assumption reflects a map model or geometric bias rather than an appropriate model to represent nature.

Smoothed curves and surfaces are subjects of Euclidean geometry and are adequate to represent artificial shapes like parts of mechanical and aeronautical projects, furniture, toys, etc. Natural objects like clouds, coastlines and mountains have irregular or fragmented features.

These are better represented by the Fractal geometry that was first formalized by Mandelbrot (1982).

Fractal geometry has enough capability to represent more adequately than Euclidean geometry real world entities that are not smoothly formed, as in the case with most natural objects.

The word fractal implies properties as in fraction or fragmented. In essence fractal geometry has ideas of fragmentation and self-similarity (Laurini and Thompson 2002).

Even though objects may be rough or irregular, there is fragmented, they may at the same time have some similar semblance of shape or pattern when viewed from different distance. Self-similarity is symmetry across different scales; there are patterns within patterns. Or, as Mandelbort says, fractals are geometric shapes that are equally complex in their details as in their overall form (Mandelbrot 1982).

To better understand this concept we describe steps for producing the geometrical shape of natural objects like snowflake. Suppose that we have a triangle. By dividing each side to three parts and replacing the middle part with two equal parts a polygon with twelve sides will be generated. By repeating the above stage for each side, at step two there are forthy-eight sides. At each step, the number of sides are multiplied by four. So for an initiator perimeter of length l, the perimeter becomes $l\left(\frac{4}{3}\right)^N$ which number evidently tends to infinity, although the area tends to a finite limit. The self-similarity ratio is $\frac{1}{3}$.

Therefore, fractal geometry has promise for some of the requirements of spatial information systems. Two-dimensional stochastic interpolations are useful for terrain modeling (Felgueiras and Goodchild 1995; Goodchild and Mark 1987). One dimensional application use of fractal concept is for coastlines (Cheng et al 2001) or boundaries of entire continents. Fractals may also be used for image error analysis (Kolibal and Monde 1998), assigning color palette (Cheng and Qingmou

2002), and land cover classification of forest (Blanco and Garcia 1997).

In this paper we use fractal for optimum sampling in generating a digital terrain model (DTM).

2. CONCEPT OF FRACTAL DIMENSION

Recent advances in the area of fractal geometry have allowed us to model natural objects dimensionality. For example, the length of a coastline can vary depending on scale, ranging from an apparently infinitely high length to a very short distance if we highly generalize the shape. It is interesting that fractal geometry can give us measure of the dimensionality of objects that are different from Euclidian geometry. The fractal dimension tells us how densely a phenomenon occupies the space in which it is located. It is independent from the measurement units used or alteration of the space by stretching or condensing.

The fractal dimension of many entities can be obtained by the Equation 1 or 2 (Laurini and Thompson 2002):

$$d = \frac{\log n}{(\log(1/r))} \tag{1}$$

Or

$$d = \frac{\log n}{(\log(s))} \tag{2}$$

Where   n =number of pieces in the repetitor
r =self-similarity ratio
d=fractal dimension.

Alternatively, s, the scaling factor, the inverse of the self-similarity ratio, can be thought of as the number of pieces that an entity is split into. In the case of the snowflake example already mentioned, n=4, r=1/3, giving d=1.2619.

We can imagine a continuum where a value of d close to 0 would mean an entity is close to a point, a value of 1 means a line, and if it is near 2, it is an area.

Similarly, a smooth line will have a dimensionality of 1, but an irregular line has a higher value, certainly greater than 1. For coastlines the mean fractal dimension is d=1.2, wherease for terrain, d is about 2.3

Brownian motion is the most popular model used to perform fractal interpolations from a set of samples. Brownian motion, first observed by Robert Brown in 1827, is the motion of small particles caused by continual bombardment by other neighboring particles. Brown found that the distribution of the particle position is always *Gaussian* with a variance dependent only on the length of the time of the movement observation (Laurini and Thompson 2002).

The Fractional Brownian motion (FBm), derived from Brownian motion, can be used to simulate topographic surfaces. FBm provides a method of generating irregular, self-similar surfaces that resemble topography and that have a known fractional dimension.

The FBm functions can be characterized by variograms (graphic that plots the phenomenon variation against the spatial distance between two points) of the form (Felgueiras and Goodchild 1995):

$$E[(z_i - z_j)]^2 = K * (d_{ij})^{2H} \tag{3}$$

where   E=statistical expectation

z_i, z_j =heights of the surface at the points i and j

d_{ij} =spatial distance between these points

K=constant of proportionality
H=parameter in the range 0 to 1

K is also related to a vertical scale factor S that controls the roughness of the surface. H describes the relative smoothness at different scales and has a relation with the fractal dimension D as formulated in Equation 4 (Felgueiras and Goodchild 1995):

$$D = 3 - H \tag{4}$$

When H is 0.5 we get the pure Brownian motion. The smaller H, the larger D and the more irregular is the surface. On the contrary, the larger H, the smaller D and the smoother the surface.

Fournier et al (1982) presented recursive procedures to render curves and surfaces based on stochastic models. They described two methods to construct two-dimensional fractal surface primitives. The first one is based on a *subdivision of polygons* to create fractal polygons while the second approach is based on the *definition of stochastic parametric surfaces.*

The *subdivision of polygons* is based on the *fractal poly line subdivision* method. A fractal poly line subdivision is a recursive procedure that interpolates intermediate points of a poly line. The algorithm recursively subdivides the closest extreme intervals and generates a scalar value at the midpoint which is proportional to the current standard deviation σc times the scale or roughness factor **S.** So, the z_m value of the middle point between two consecutive points, i and j, of a poly line is determined by the Equation 5 (Felgueiras and Goodchild 1995):

$$z_m = (z_i + z_j)/2 + S * \sigma_c * N(0,1) \tag{5}$$

Where σ_c varies according to Equation 5 and $N(0,1)$ is a Gaussian random variable with zero mean and unit variance**.**

The subdivision of polygons method is suitable to create stochastic surfaces based on TIN digital models. Each triangle of the TIN model can be subdivided into four smaller triangles by connecting the midpoints of the triangles. The z value of the midpoints is calculated by the fractal poly line subdivision

method presented above. The subdivision can be continued until the area or a side of the current triangle reaches a predefined limit. So the original triangle is transformed into a fractal triangle whose irregular surface consists of many small triangular facets.

As pointed out by Fournier and Goodchild (1995) , the presented methods for rendering curves and surfaces are satisfactory approximations of fractional Brownian motion. They allow us to create realistic surfaces in faster time than with exact calculations. Another advantage of these approaches is the possibility of computing surfaces to arbitrary levels of detail without increasing size of the database.

Figure 1 illustrates the behavior of fractal curves created using fractional Brownian motion, different values of **H,** and a constant vertical scale factor. The curves were rendered using the fractal poly line subdivision method**.**
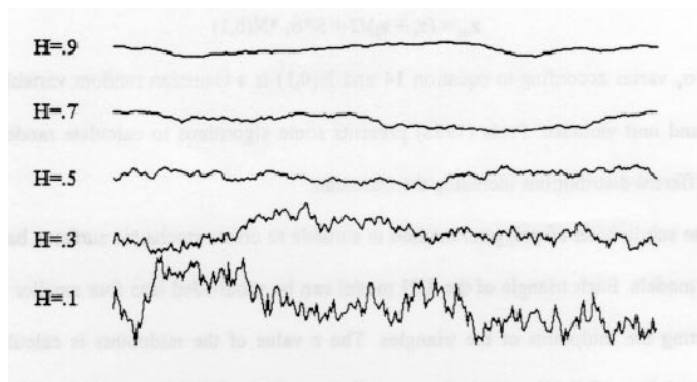
Figure 2. TIN model of one rough land in Hamedan, Iran. Units are in meter.

Figure 1. Stochastic curves rendered for different values of the parameter H (Fournier and Goodchild 1995).
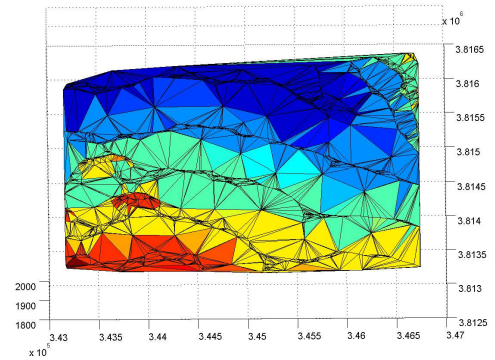
Figure 3. TIN model of one smooth land in Hamedan, Iran. Units are in meter.

3. METODOLGY

This section describes the methodology used to analyze surface fitting on TIN models.

The first step for modeling surfaces is the definition of the input sample set that will be used to construct the surface. This sample set must be representative of the phenomenon to be modeled.

The next step involves the use of the sample set to construct the basic structure of the DTM model. Here the input samples were transformed on the vertices of the triangles of a TIN model.

Our case study was in two parts of Hamedan city in Iran (rough, smooth) that sample set has been extracted from a map at a scale of 1:25000 and then TIN model was constructed (Figure 2, 3).
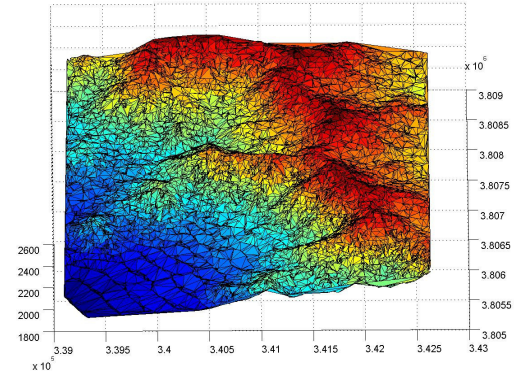
After this stage we use fractal concept for interpolation. This method used to estimate the z values of the rectangular grid, which was based on the polygon subdivision approach presented in Felgueiras and Goodchiled (1995). The method begins finding the current triangle T_c, of the original TIN model, that contains the grid point $P_i(x_i, y_i, z_i)$. Then the triangle T_c is subdivided recursively in four smaller triangles by connecting the midpoints of its sides. The z values of these midpoints are defined by a fractal poly line subdivision approach described in section 2. A new triangle T_c, that contains the point P_i, is chosen among the four smaller triangles. The subdivisions continue until the point P_i is within a defined proximity criterion of one of the vertices of the triangle T_c. When the proximity is reached, one can define z_i equal to the z of this vertex. In this paper, sample points of the map were condensed with fractals up to contour interval of 5 meters (Figure 4, 5)
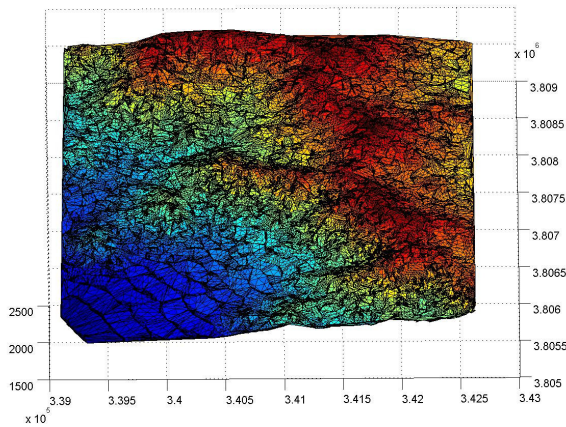
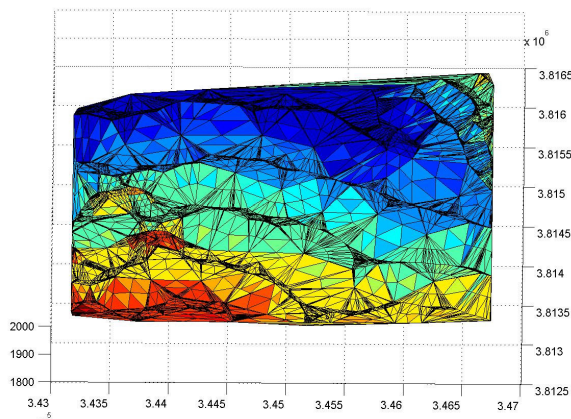Figure 4. TIN model of Figure 2 that is condensed with fractals H=0.8, units are in meter.

Figure 5. TIN model of Figure 3 that is condensed with fractals H=0.3, units are in meter.

In order to perform a quantitative analysis of the surfaces rendered by the fractal interpolator approaches, we compared the results with the real surfaces. So 15-20 check points was selected for each region. Then for each point *error function* that defined as the difference between the *real elevation* and the *estimated elevation* was computed. Table 1 shows the residuals on the check points for the rough and smoothed region.

In addition for H=0.1-0.9 with increment 0.1, standard deviation of the error function was computed that results have shown that the best H for irregular and regular surfaces are 0.3 and 0.8 respectively (Table 2) .It means that irregular surface has fractal dimension 2.7 while regular surface has fractal dimension 2.2.

Table 1, Residuals on check points for smooth and rough region (m)

Point number	Residuals on check points for smooth region (m)	Residuals on check points for rough region (m)
1	-0.090	0.340
2	0.100	0.280
3	0.060	0.210
4	-0.332	0.726
5	-0.381	-1.196
6	1.159	0.141
7	0.066	0.267
8	-0.016	6.262
9	0.037	-1.364
10	0.558	6.763
11	0.164	1.726
12	0.121	-0.056
13	-0.423	0.570
14	-0.045	-2.015
15	-0.890	-3.635
16	0.041	-0.088
17	-0.357	
18	0.010	
19	0.427	
20	0.022	

Table 2, standard deviation of the error function for smooth and rough region (m)

H	standard deviation of the error function for smooth region	standard deviation of the error function for rough region
0.1	2.675	0.419
0.2	2.680	0.425
0.3	2.682	0.404
0.4	2.713	0.435
0.5	2.676	0.465
0.6	2.666	0.550
0.7	2.789	0.598
0.8	2.618	0.626
0.9	2.888	1.283

4. CONCLUSION

Fractal methods can be successfully used when the real surface represents a natural phenomenon like elevation. The major problem seems to be the definition of the appropriate parameters H and S (H is relative smoothness at different scales and S is scale or roughness factor) to best represent the variations of the real surface.

Fractal interpolation is recommended for modeling natural terrain surfaces when interest lies in visualization, and when the parameters of the fractal interpolation can be adjusted to create a realistic-looking representation.

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