

GENERALIZED POINT PHOTOGRAMMETRY AND ITS APPLICATION

Zuxun Zhang*, Jianqing Zhang

School of Remote Sensing and Information Engineering, Wuhan University, Wuhan, China, 430079
(zxzhang, jqzhang)@supresoft.com.cn

Commission V, WG V/1

KEY WORDS: Photogrammetry, Digital, Theory, Industry, Calibration, Orientation, Reconstruction, Inspection

ABSTRACT:

Point based co-linearity is an essential conception in Photogrammetry. But point in photogrammetry means only physical or visible points, such as dots, cross points and corner point etc. Especially in the stage of either analogue or analytical, even digital photogrammetry, the main primitive feature, which can be measured by human operator, is physical point. It can be referred to as Point Photogrammetry contrary to Line photogrammetry, which is proposed and used only in the past about 20 years.

For a straight line a coplanar equation is used, where the line on the image lies on the plane, passing through corresponding spatial straight line and the photographing centre in line photogrammetry. All straight lines and curves are normally referred to linear feature rather than point feature in line photogrammetry. The co-planarity can be normally explain as $Sa \cdot (SP \times SQ) = 0$, which means the image point a must lie on the plane composed by perspective centre S and two end point P and Q of the 3D line.

However, in mathematics all lines or curves are consists of points, which is expressed as generalized point in this paper. Collinear equation could be used to linear features similar as the visible point, and all kinds of features existing in photogrammetry can be concentrated on "point" – called as Generalized Point Photogrammetry. According to this philosophy the adjustment form would be consistent, simpler and more convenient comparing with line photogrammetry.

In the Generalized point photogrammetry the only difference between physical points and feature line point is that two collinear equations of both x and y are used for the physical point and only one collinear equation of x or y depending on the local direction of the line segment is used for feature line point. The collinear equation, used for feature line point, includes the parameters of feature line. For example, the coordinates of point in a straight line or circle can be expressed by their parameters, and be substituted to the collinear equation, and the parameters can be computed in the same time.

Besides physical point and feature line point, the invisible point – infinity is also included in generalized point scope, and collinear equations are used for them too.

After the theory of the generalized point photogrammetry is introduced, its applications are presented, including extraction of vanish points, determination of interior parameters of the image, and sheet-metal part inspection and measurement. Corresponding experiments and results are demonstrated with the conclusion.

1. INTRODUCTION

The co-linear equation is the core of photogrammetry (Wang, 1990), because it is source from forward and backwards intersection for point measurement in the surveying. The co-linearity of points is the basic concept of the photogrammetry. The points concern the dot, the intersection, the corner and so on. Whether analogue or analyse even digital photogrammetry, the basic feature point, which can be measured by manual way, is physical or visual. Then the photogrammetry can be called as point photogrammetry.

However, There are a lot of straight lines in extraction of buildings, architecture photogrammetry and industry part measurement. Line based photogrammetry (based on co-planar equation) was studied and applied (F.A.Van.den Heuvel, 1999). In line photogrammetry, co-planar equation can be used for straight line, where the line on the image passed through the plane determined by photographic centre and corresponding spatial line. In line photogrammetry all lines and curves are considered as line features instead of point feature. The coplanar equation expressed as $Sa \cdot (SP \times SQ) = 0$, where image

point a is within the plane determined by photographic centre S and two points P and Q of a spatial line.

In the real world, there are large number of curves, such as roads, rivers and lakes on the ground, circles, arcs and curves in architecture photogrammetry and industry measurement. In addition, the corresponding points between old map and new image are selected as control points in the map update, which is a quite difficult job. Usually, there are few distinct points, such as dot point and corner point, and their matching is not precise. Therefore, if a mount of lines in the map and image could be applied as control information for the matching, it would be significative in both theory and practise.

In mathematics, all line and curve consist of points. Same as visual point, co-linear equation can be used to the points in line feature, and all feature points can come down to the point in order to fit the co-linear equation----that is so called as generalized point photogrammetry. According to the principle, the form of the adjustment is consistent, and simpler and more convenient than line photogrammetry. The only different between feature point and physical point is that the latter is fitting two co-linear equations relative to x and y respectively,

* Corresponding author.

and the former is only fitting one co-linear equation relative to x or y . The co-linear equation applied by feature point includes the parameters of feature line. For example, the coordinates of points in the line and circle can be expressed by their parameters, and substituted to the co-linear equation, and the parameters can be solved at one time. Besides physical and feature point, un-visual point---infinite point is also included in the category of generalized point. For example, vanish point---the intersection of projects from a group of parallel line in the space is the project of the infinite point, and it is fitting the co-linear equation. Consequently, it is easy to reduce the point, line, circle, curve and infinite point in to one mathematical model: co-linear equation, and perform uniform adjustment.

2. GENERALIZED POINT PHOTOGRAMMETRY

In traditional photogrammetry, all physical points are fitting the co-linear equations:

$$x = x_0 - f \frac{a_1(X - X_s) + b_1(Y - Y_s) + c_1(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} \quad (1)$$

$$y = y_0 - f \frac{a_2(X - X_s) + b_2(Y - Y_s) + c_2(Z - Z_s)}{a_3(X - X_s) + b_3(Y - Y_s) + c_3(Z - Z_s)} \quad (2)$$

where x and y are the observations with corrections v_x, v_y, X, Y and Z are ground coordinates with corrections $\Delta X, \Delta Y, \Delta Z, X_s, Y_s, Z_s, \varphi, \omega, \kappa, f, x_0, y_0$ are parameters to be solved, which can be expressed by approximations and corresponding corrections. If p_1, \dots, p_n are additional parameters, the linearized error equations are:

$$v_x = a_{11}\Delta X_s + a_{12}\Delta Y_s + a_{13}\Delta Z_s + a_{14}\Delta\varphi + a_{15}\Delta\omega + a_{16}\Delta\kappa + a_{17}\Delta f + a_{18}\Delta x_0 + a_{19}\Delta y_0 + b_{11}\Delta X + b_{12}\Delta Y + b_{13}\Delta Z + c_{11}\Delta p_1 + \dots + c_{1n}\Delta p_n - l_x \quad (3)$$

$$v_y = a_{21}\Delta X_s + a_{22}\Delta Y_s + a_{23}\Delta Z_s + a_{24}\Delta\varphi + a_{25}\Delta\omega + a_{26}\Delta\kappa + a_{27}\Delta f + a_{28}\Delta x_0 + a_{29}\Delta y_0 + b_{21}\Delta X + b_{22}\Delta Y + b_{23}\Delta Z + c_{21}\Delta p_1 + \dots + c_{2n}\Delta p_n - l_y \quad (4)$$

where

$$\begin{aligned} a_{11} &= \frac{\partial x}{\partial X_s}, & a_{12} &= \frac{\partial x}{\partial Y_s}, & a_{13} &= \frac{\partial x}{\partial Z_s}, \\ a_{14} &= \frac{\partial x}{\partial \varphi}, & a_{15} &= \frac{\partial x}{\partial \omega}, & a_{16} &= \frac{\partial x}{\partial \kappa}, \\ a_{21} &= \frac{\partial y}{\partial X_s}, & a_{22} &= \frac{\partial y}{\partial Y_s}, & a_{23} &= \frac{\partial y}{\partial Z_s}, \\ a_{24} &= \frac{\partial y}{\partial \varphi}, & a_{25} &= \frac{\partial y}{\partial \omega}, & a_{26} &= \frac{\partial y}{\partial \kappa}, \\ b_{11} &= -a_{11}, & b_{12} &= -a_{12}, & b_{13} &= -a_{13}, \\ b_{21} &= -a_{21}, & b_{22} &= -a_{22}, & b_{23} &= -a_{23}, \\ c_{11} &= \frac{\partial x}{\partial p_1}, & \dots & & c_{1n} &= \frac{\partial x}{\partial p_n}, \\ c_{21} &= \frac{\partial y}{\partial p_1}, & \dots & & c_{2n} &= \frac{\partial y}{\partial p_n} \end{aligned}$$

Constants $l_x = x - (x)$, $l_y = y - (y)$, where (x) and (y) are computed by the approximations of parameters substituted into equation (1) and (2).

2.1 Vanish Point

Let $(x_{X\infty}, y_{X\infty})$, $(x_{Y\infty}, y_{Y\infty})$ and $(x_{Z\infty}, y_{Z\infty})$ be the coordinates of intersections (vanish points) $p_{X\infty}$, $p_{Y\infty}$ and $p_{Z\infty}$ of projects from three bundles of straight lines, which are parallel to X, Y and Z axes respectively. The numerator and denominator of equation (1) and (2) are divided by X , and let X tend towards to limitless, the co-linear equations of vanish point $p_{X\infty}$ relative to the lines parallel to X axis can be acquired:

$$x_{X\infty} = x_0 - f \frac{a_1}{a_3}, \quad y_{X\infty} = y_0 - f \frac{a_2}{a_3}$$

The co-linear equations of vanish points $p_{Y\infty}$ and $p_{Z\infty}$ relative to the lines parallel to Y and Z axis respectively can be acquired in the same way:

$$x_{Y\infty} = x_0 - f \frac{b_1}{b_3}, \quad y_{Y\infty} = y_0 - f \frac{b_2}{b_3}$$

$$x_{Z\infty} = x_0 - f \frac{c_1}{c_3}, \quad y_{Z\infty} = y_0 - f \frac{c_2}{c_3}$$

Above six equations show that the three interior parameters x_0, y_0, f and three exterior direction parameters φ, ω, κ can be computed using vanish points relative to three limitless points in X, Y and Z axes respectively

2.2 Point in Feature Line

Each point in the straight line and curve can be directly used to only one of two co-linear equations.

2.2.1 Straight Line: Each point (x, y) in the line parallel to y axis on image is fitting equation (1) and (3), wether the vertical coordinate v is what value. In the same reason, each point (u, y) in the line parallel to x -axis on image is fitting equation (2) and (4), wether the horizontal coordinate u is what value. The equation of a line l with arbitrary other orientation θ is

$$a x + b y + c = 0, a \neq 0, b \neq 0, \theta = \arctg(a/b), \theta \neq 0^\circ, \theta \neq 90^\circ$$

When $-45^\circ < \theta \leq 45^\circ$ or $135^\circ < \theta \leq 225^\circ$, every point $p(x, y)$ of line l is fitting equation (1) and (3), otherwise p is fitting equation (2) and (4).

If the equation of spatial line L in plane $Z = Z_0$, which project is l , is

$$X = X_0 + t \cdot \cos \theta \quad Y = Y_0 + t \cdot \sin \theta \quad (8)$$

Substitute equation (8) into co-linear equations (1) and (2), then

$$\begin{aligned} x &= x_0 - f \frac{a_1(X_0 + t \cos \theta - X_s) + b_1(Y_0 + t \sin \theta - Y_s) + c_1(Z - Z_s)}{a_3(X_0 + t \cos \theta - X_s) + b_3(Y_0 + t \sin \theta - Y_s) + c_3(Z - Z_s)} \\ y &= y_0 - f \frac{a_2(X_0 + t \cos \theta - X_s) + b_2(Y_0 + t \sin \theta - Y_s) + c_2(Z - Z_s)}{a_3(X_0 + t \cos \theta - X_s) + b_3(Y_0 + t \sin \theta - Y_s) + c_3(Z - Z_s)} \end{aligned}$$

Because $p_1 = X_0, p_2 = Y_0, p_3 = \theta$, so that among equation (3) and (4):

$$\begin{aligned} b_{11} &= b_{12} = b_{21} = b_{22} = 0, \\ c_{11} &= a_{11}, \quad c_{12} = a_{12}, \quad c_{13} = -t a_{11} \sin \theta, \\ c_{21} &= a_{21}, \quad c_{22} = a_{22}, \quad c_{23} = t a_{21} \cos \theta \end{aligned}$$

2.2.2 Circle: If the equation of spatial circle with centre (X_0, Y_0) and radius R in plane $Z = Z_0$ is:

$$X = X_0 + R \cdot \cos t, \quad Y = Y_0 + R \cdot \sin t \quad (9)$$

Substitute equation (9) into co-linear equations (1) and (2), then

$$x = x_0 - f \frac{a_1(X_0 + R \cdot \cos t - X_s) + b_1(Y_0 + R \cdot \sin t - Y_s) + c_1(Z - Z_s)}{a_3(X_0 + R \cdot \cos t - X_s) + b_3(Y_0 + R \cdot \sin t - Y_s) + c_3(Z - Z_s)}$$

$$y = y_0 - f \frac{a_2(X_0 + R \cdot \cos t - X_s) + b_2(Y_0 + R \cdot \sin t - Y_s) + c_2(Z - Z_s)}{a_3(X_0 + R \cdot \cos t - X_s) + b_3(Y_0 + R \cdot \sin t - Y_s) + c_3(Z - Z_s)}$$

Because $p_1 = X_0$, $p_2 = Y_0$, $p_3 = R$, so that among equation (3) and (4):

$$b_{11} = b_{12} = b_{21} = b_{22} = 0,$$

$$c_{11} = a_{11}, \quad c_{12} = a_{12}, \quad c_{13} = a_{11} \sin t,$$

$$c_{21} = a_{21}, \quad c_{22} = a_{22}, \quad c_{23} = a_{21} \cos t$$

2.2.3 Curve: Ground: If the tangent direction of a point $p(x, y)$ in a curve is θ , similar as the case of straight line, when $-45^\circ < \theta \leq 45^\circ$ or $135^\circ < \theta \leq 225^\circ$, point $p(x, y)$ is fitting equation (1) and (3), otherwise p is fitting equation (2) and (4).

The tangent equation of the curve in point (X_i, Y_i) of plane $Z = Z_0$ is:

$$X = X_i + t \cdot \cos \theta_i \quad Y = Y_i + t \cdot \sin \theta_i \quad (10)$$

Similar as the case of straight line, here $p_i = \theta_i$ among equation (3) and (4):

$$c_{1i} = -t a_{11} \sin \theta_i, \quad c_{2i} = -t a_{21} \sin \theta_i$$

By the principle mentioned above, it is easy to conclude the visual point, infinite point, straight line, circle, arc and curve in to an uniform mathematical model: co-linear equation, and perform unitive adjustment.

3. APPLICATION OF GENERALIZED POINT PHOTOGRAMMETRY

3.1 Computation of Image Parameter from Vanish Point and Modelling by Single Image

Three interior parameters x_0, y_0, f and three exterior parameters φ, ω, κ , can be calculated by equations (5), (6) and (7). The principle point (x_0, y_0) of image is the intersection of three perpendiculars in the triangle consisted of three vanishes. The focus can be calculated by formula:

$$f^2 = -(x_{x_\infty} - x_0)(x_{y_\infty} - x_0) - (y_{x_\infty} - y_0)(y_{y_\infty} - y_0) \quad (11)$$

The angle elements are

$$\tan \varphi = \sqrt{f^2 + x_{z_\infty}^2 + y_{z_\infty}^2} / \sqrt{f^2 + x_{x_\infty}^2 + y_{x_\infty}^2} \quad (12)$$

$$\tan \omega = f / \sqrt{x_{y_\infty}^2 + y_{y_\infty}^2} \quad (13)$$

$$\tan \kappa = x_{y_\infty} / y_{y_\infty} \quad (14)$$

The computation of the coordinates of project center and building modeling is performed synchronously. Taking simplest building as an example (Figure 1), let point 2 as origin, $X_2 = Y_2 = Z_2 = 0$, $\vec{23}$ is x-axis, and let its length be L , $X_3=L$, $Y_3 = Z_3 = 0$, then:

$$0 - X_S = (Z_2 - Z_S) \frac{a_1(x_2 - x_0) + a_2(y_2 - y_0) - a_3 f}{c_1(x_2 - x_0) + c_2(y_2 - y_0) - c_3 f} = (0 - Z_S) \frac{u_2}{w_2}$$

$$0 - Y_S = (Z_2 - Z_S) \frac{b_1(x_2 - x_0) + b_2(y_2 - y_0) - b_3 f}{c_1(x_2 - x_0) + c_2(y_2 - y_0) - c_3 f} = (0 - Z_S) \frac{v_2}{w_2}$$

$$L - X_S = (Z_2 - Z_S) \frac{a_1(x_3 - x_0) + a_2(y_3 - y_0) - a_3 f}{c_1(x_3 - x_0) + c_2(y_3 - y_0) - c_3 f} = (0 - Z_S) \frac{u_3}{w_3}$$

$$0 - Y_S = (Z_2 - Z_S) \frac{b_1(x_3 - x_0) + b_2(y_3 - y_0) - b_3 f}{c_1(x_3 - x_0) + c_2(y_3 - y_0) - c_3 f} = (0 - Z_S) \frac{v_3}{w_3}$$

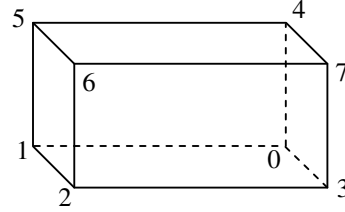


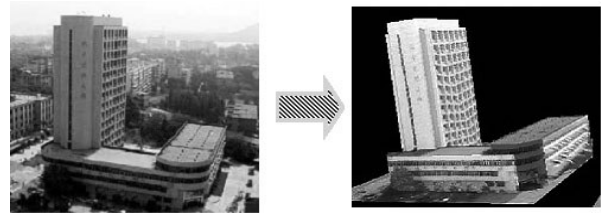
Figure 1 Stereo Building

The coordinates of project center can be calculated:

$$Z_S = -\frac{w_2 w_3}{u_3 w_2 - u_2 w_3} L, \quad X_S = Z_S \frac{u_2}{w_2}, \quad Y_S = Z_S \frac{v_2}{w_2}$$

The other two parameters width W and height H of the building can be computed by the coordinates of project center and co-linear equation with right angle condition.

Based on the computed initial values, suppose 1(0, W , 0), 2(0, 0, 0), 3(L , 0, 0), 4(L , W , H), 5(L , W , 0), 6(0, 0, H), 7(L , 0, H), 11 parameters can be solved. Figure 2 is an example of computation of image parameters from vanish point and modelling by single image.

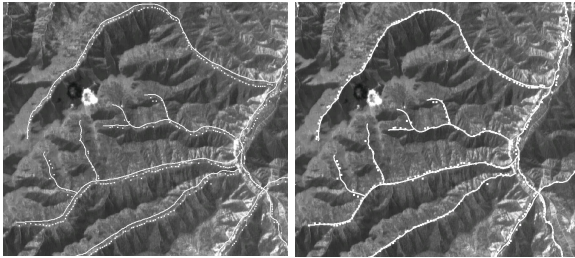


a Original Image b Stereo Model
Figure 2 Computation of Image Parameter from Vanish Point and Modelling by Single Image

3.2 Determination of Image Parameter by Matching between Vector and Image

The problem of data update is how to use exist data, be more exact, how to use exist information. AS well-known, if the new image would be applied to update the exist map, their superposition is the most important step. After determining the image parameter, rectifying the image to ortho-image consistent with the map, the image could be superposed on the map. Therefore, in data update, except selecting control point by traditional manual way, whether the large mount of line elements of objects on the map could be applied as control is very important for the realization of the automation for data

update and the improvement of work efficiency. Using the theory of generalized photogrammetry, it can be realized let linear objects be the control. Figure 3a shows the projects of linear objects on the image and corresponding generalized points nearby, which can't be superposed. Figure 3a shows the superposed result of the linear objects and image, after orientation and rectification by the control of linear objects, where the vector is better superposed.



a. Linear Objects & Image b. Superposition

Figure 3. Superposition of Linear Objects & Image

3.3 Inspection of Sheet-metal Part

How to inspecting the parts in industrial production fast and exactly is one of the keys for improving the industrial product quality. The generalized point photogrammetry theory is applied successfully into the inspection of the metal-sheet parts. A rotated disk and CCD camera consist of the inspection platform (Figure 4). The part is put on the disk, which rotation is controlled by computer. The images are taken when the disk is rotating. Usually 25 images are taken after the disk rotates one circuit. By the bundle adjustment based on generalized point photogrammetry theory, the precise and measurable geometric model of the metal-sheet part is created (Figure 5).

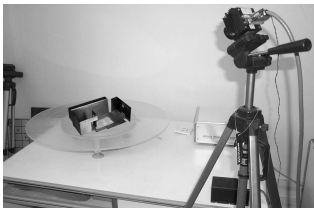


Figure 4. Inspection Platform

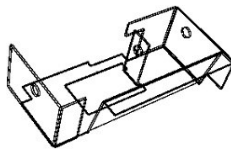


Figure 5. Measurable Geometric Model

Based on generalized point photogrammetry theory, not only straight line but also the circle and the rectangle hole with arcs in the corners can be measured accurately (Figure 6). Because each pixel in the edge is participating in least squares matching, the result with high precise can be insured, even if there is much rust (Figure 6).

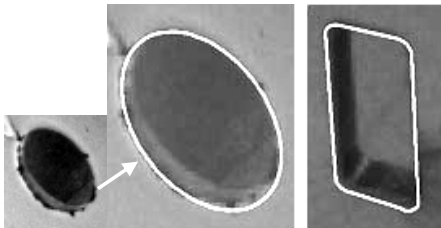


Figure 6. Measurement of circle and arc

3.4 Determination of Plane Pose by Contour Line Matching

Any mark point and line in a flying plane is hard to recognize, and only its contour line can be extracted. The position of the plane can be traced using the laser, and the image of the plane can be acquired by photograph simultaneously. However, how can the pose of the plane be determined?

Determining shape of an object by contour line is an important research content (Li Li, Mao Shongde, 1996), and the pose of a moving object could be measured by contour line also. According to the coordinates traced by the laser and initial value of the pose, a virtual plane could be projected to the image (Figure 7). Then, based on generalized point photogrammetry theory, the pose of flying object could be determined using the co-linear equation (Table 1).

Table 1. Measurement of Flying Plane by Contour Line

	$\varphi (^{\circ})$	$\omega (^{\circ})$	$\kappa (^{\circ})$
True Value	15.99	-15.59	-31.19
Measured Value	15.92	-15.33	-31.17
Error	-0.07	0.26	0.02

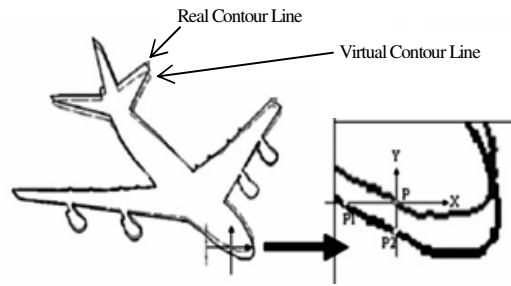


Figure 7. Measurement of Flying Plane by Contour Line

4. CONCLUSION

The generalized point photogrammetry theory introduced in this paper reduces the point, straight line, circle, arc, curve and limitless point to one mathematical model: co-linear equation and uniform adjustment. The theory discards the localization of traditional and few feature points, the information can be utilized adequately. The generalized point photogrammetry theory has been successfully applied in 3D modelling of the city, data update, measurement of industrial part, circle and arc, and determination of the pose of flying plane. Consequently the correctness and validity of the generalized point photogrammetry theory are proved.

Acknowledgements

Thanks for the supporting from Natural Science Fund of P. R. China (No. 40337055).

References

- [1] Wang Zhizhuo, Principles of Photogrammetry (with Remote Sensing), Press of Wuhan, Technical University of Surveying and Mapping, Publishing House of Surveying and Mapping, 1990

- [2] Zhang Zuxun, Zhang Jianqing, 3D Reconstruction of House with Single Image, Proc. of Symposium on Automation of Engineering Surveying, 2001 (in Chinese).
- [3] Cipolla R. Robertson D. and Boyer E. 1999, *Photobuilder—3D models of architectural scenes from uncalibrated images*. Proc. IEEE International Conference on Multimedia Computing and systems, Firenze, volume I, pp.25-31, June, 1999.
- [4] F. A. Van.den Heuvel, 1999, *A line-photogrammetric mathematical model for the reconstruction of polyhedral objects*, in Videometrics VI, Sabry F. El-Hakim (ed.), Proceedings of SPIE Vol. 3641, ISBN 0-8194-3112-5, pp. 60-71.
- [5] Li Li, Ma Shongde, Reconstructing the 3D Shape of 2-order curve Surface from 2D Contour Line, Computer Journal, Vol. 19, No. 6, p. 401, 1996