CALIBRATION OF A PROJECTOR WITH A PLANAR GIRD

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ABSTRACT:

The projector is being frequently used in broad range of the photogrammetric measurements with the development of non-contact measurement in the close-range photogrammetry, because a projector can project any pattern onto the measured object for controlling and calculating. Before using a projector, the interior parameters of it have to be calibrated first. Hence, the projector calibration is a necessary and important preceding step. The paper proposes a flexible technique to make the calibration of a projector with a planar grid. This technique only requires an ordinary projector, a digital camera and a planar grid. The planar grid provides the main control ground and the camera takes photos as the image data. The camera demands to be calibrated or its intrinsic parameters are known. The algorithm with 2D direct linear transformation (2D-DLT) and collinear equations is used to calibrate the projector. The operation method in detail and the algorithm are addressed systematically and entirely. First, the image coordinates of the projector are designed carefully and the space coordinates of the projector are computed by the image data and the intrinsic and extrinsic parameters of the digital camera. Then, the decomposition of initial values of the projector intrinsic and extrinsic parameters using the correspondence of 2D-DLT and collinear equation is deduced. Finally, the projector calibration parameters are worked out by the whole adjustment. The feasibility and the exactness of the calibration technique of a projector put forward in this paper are verified by the results of real experimentations and data.

1. INTRODUCTION

1.1 Advantages of a Projector

The use of the ordinary projector becomes familiar and frequent, because a projector can project any pattern onto the measured object. It can provide the points of interest conveniently and simply. The pattern can be designed differently according to all kinds of requirements of the measured object. These points projected on the surface of the object are stable, high contrast and quality. They have no inhere target thickness. Their size can grow up and their intensity can decrease down as the projector gets further from the measured object. Hence the projected points can be adjusted to suit for a good measurement. At same time, the projector is set far enough away the surface of the measured object so that it is convenient and easy to be installed and is almost not affected by the special object such as high temperature iron block or quickly moving bus. Because of the advantages of the ordinary projector above, it is being used in broad range of the close-range photogrammetric measurements. In order to take full advantage of the projector in the procedures of diverse photogrammetric measurements, the intrinsic parameters of the projector have to be calibrated in advance. Namely, the projector needs to be calibrated at first. Hence, the projector calibration is a necessary and important preceding step.

1.2 Calibration of a Projector

In this paper a flexible technique is proposed to calibrate the ordinary projector easily. The technique only requires an ordinary projector, a digital camera and a planar grid. The projector illuminates a target grid slide onto the planar grid. The digital camera is used to take the images of the planar grid from a few (at least two) different orientations. The camera has already been calibrated or its intrinsic parameters are known first. The planar grid functions as a control ground to provide the coordinates of space points. Using the correspondence of collinear equations and 2D direct linear transformation, the intrinsic parameters of the projector can be worked out. By this time, the projector is calibrated entirely.

Compared with other techniques, which use the expensive equipment and the complex algorithm, the proposed technique is easy to realize and flexible. It is hardly affected by the space factor or time factor. It paves the path for the using in future of an ordinary projector, which is calibrated. The feasibility and the correctness of the projector calibration technique proposed in this paper are verified by the results of real data.

1.3 Using of a Calibrated Projector

After a projector has been calibrated correctly, it can be used to control on and project to the measured objects as the same function as a camera. Especially, it can supply the points of interest on the surface of the objects which are lack of the texture. Using these points, all kinds of values of the measured objects can be computed easily. When the intrinsic parameters of a projector are correct, the using of it and the calculation with its parameters are effective and exact.

2. METHODOLOGY AND ALGORITHM

2.1 Equipment and Method

An ordinary projector, a digital camera and a planar grid are the main equipments required in this technique. For the projector, a

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target grid slide is designed first so that the coordinates of its grid points can be acquired as the known data. The camera requires to have been calibrated already or its intrinsic parameters are known in advance. The planar grid is also designed first to be able to obtain the coordinates of the points of the grid.

First, the positions of the projector and the camera are adjusted to two appropriate places. The camera is focused to take clear photos of the planar grid and the projector is also focused to project clearly a target grid slide onto the plane. Second, the camera is used to take an image of the planar grid. Then the planar grid is covered by a white paper (or other things), which makes the planar grid seem to be a plane. The projector illuminates the target grid slide onto the plane. The camera is used again to take the images of the grid illuminated. In this process of taking the image of the illuminated grid, it is important that the positions of the camera and the planar grid are both never changed. Using the image processing method, the coordinates of the grid points of the image of planar grid are extracted. Because the space coordinates of the planar grid points are known as design, the extrinsic parameters of the camera can be calculated by the space resection method. So that the intrinsic and extrinsic parameters of the camera have been gotten at present. Using the image processing method again, the coordinates of the points of the image of the projected grid are extracted, too. By the collinear equations, the space coordinates of the projected points of the image are calculated in reverse, because the Z value of the space coordinates of these points is zero always. For the projector, the space coordinates of the grid points projected have been acquired by the process above, and the image coordinates of these points are known as design in advance. Using the correspondence of 2D-DLT and collinear equation, the decomposition of initial values of the projector intrinsic and extrinsic parameters is deduced. Third, the positions and orientations of the projector and the camera are changed to take another two images as the second step. At least all images need to be taken from two different positions and orientations. Then at least two sets of the initial values of the projector parameters can be computed out. Finally, the intrinsic parameters of the projector can be worked out by the whole adjustment based on these initial values above. So the ordinary projector has been calibrated entirely by this way.

2.2 Algorithm

The collinear equations are:

$$x - x_0 = -f\frac{a_1(X-X_s)+b_1(Y-Y_s)+c_1(Z-Z_s)}{a_3(X-X_s)+b_3(Y-Y_s)+c_3(Z-Z_s)}$$
$$y - y_0 = -f\frac{a_2(X-X_s)+b_2(Y-Y_s)+c_2(Z-Z_s)}{a_3(X-X_s)+b_3(Y-Y_s)+c_3(Z-Z_s)} \quad (1)$$

where f, x_0, y_0 = the intrinsic parameters of the projector
X_s, Y_s, Z_s = the coordinates of the projector centre
X, Y, Z = the space coordinates of points
x, y = the image coordinates of the relative points
R {a_i, b_i, c_i, i=1,2,3} = the rotated matrix made up of rotated angles ϕ, ω, κ
Z = 0

2D-DLT is expressed as:

$$\begin{cases} x = \dfrac{h_1X + h_2Y + h_3}{h_7X + h_8Y + 1} \\ y = \dfrac{h_4X + h_5Y + h_6}{h_7X + h_8Y + 1} \end{cases} \quad (2)$$

where H_i(i=1,2…8) = parameter
X, Y = the space coordinates of the points projected
x, y = the image coordinates of the relative points

When the number of points observed in an image is more than 4, the parameter H_i can be calculated out.

According to the formula (2), formula (1) can be transferred into formula (3):

$$x = \frac{\left(f\frac{a_1}{\lambda} - \frac{a_3}{\lambda}x_0\right)X + \left(f\frac{b_1}{\lambda} - \frac{b_3}{\lambda}x_0\right)Y + \left(x_0 - \frac{f}{\lambda}(a_1X_s + b_1Y_s + c_1Z_s)\right)}{-\frac{a_3}{\lambda}X - \frac{b_3}{\lambda}Y + 1}$$
$$y = \frac{\left(f\frac{a_2}{\lambda} - \frac{a_3}{\lambda}y_0\right)X + \left(f\frac{b_2}{\lambda} - \frac{b_3}{\lambda}y_0\right)Y + \left(y_0 - \frac{f}{\lambda}(a_2X_s + b_2Y_s + c_2Z_s)\right)}{-\frac{a_3}{\lambda}X - \frac{b_3}{\lambda}Y + 1} \quad (3)$$

Compared formula (2) and (3), then:

$$\begin{cases} h_1 = f\dfrac{a_1}{\lambda} - \dfrac{a_3}{\lambda}x_0 \\ h_2 = f\dfrac{b_1}{\lambda} - \dfrac{b_3}{\lambda}x_0 \end{cases} \quad (4)$$

$$\begin{cases} h_4 = f\dfrac{a_2}{\lambda} - \dfrac{a_3}{\lambda}y_0 \\ h_5 = f\dfrac{b_2}{\lambda} - \dfrac{b_3}{\lambda}y_0 \end{cases} \quad (5)$$

$$\begin{cases} h_3 = x_0 - \dfrac{f}{\lambda}(a_1X_s + b_1Y_s + c_1Z_s) \\ h_6 = y_0 - \dfrac{f}{\lambda}(a_2X_s + b_2Y_s + c_2Z_s) \end{cases} \quad (6)$$

$$\begin{cases} h_7 = -\dfrac{a_3}{\lambda} \\ h_8 = -\dfrac{b_3}{\lambda} \end{cases} \quad (7)$$

where $\lambda = (a_3 X_s + b_3 Y_s + c_3 Z_s)$

From formula (4), (5), (6), (7), then:

$$\begin{cases} \dfrac{(h_1 - h_7 x_0)}{f} = \dfrac{a_1}{\lambda} \\ \dfrac{(h_2 - h_8 x_0)}{f} = \dfrac{b_1}{\lambda} \end{cases} \tag{8}$$

$$\begin{cases} \dfrac{(h_4 - h_7 y_0)}{f} = \dfrac{a_2}{\lambda} \\ \dfrac{(h_5 - h_8 y_0)}{f} = \dfrac{b_2}{\lambda} \end{cases} \tag{9}$$

$$\begin{cases} -h_7 = \dfrac{a_3}{\lambda} \\ -h_8 = \dfrac{b_3}{\lambda} \end{cases} \tag{10}$$

From formula (8), (9), (10) to reduce the parameter λ, and $a_1 b_1 + a_2 b_2 + a_3 b_3 = 0$, then:

$$\frac{(h_1 - h_7 x_0)\cdot(h_2 - h_8 x_0)}{f^2} + \frac{(h_4 - h_7 y_0)\cdot(h_5 - h_8 y_0)}{f^2} + h_7 h_8 = 0 \tag{11}$$

$$f = \sqrt{\frac{-(h_1 - h_7 x_0)\cdot(h_2 - h_8 x_0) - (h_4 - h_7 y_0)\cdot(h_5 - h_8 y_0)}{h_7 h_8}} \tag{12}$$

Considering the primary point (x_0, y_0) close to the centre of the image, the initial value of focus f is able to be computed on the assumption that the primary point is in the centre of the image. At present the initial values of the intrinsic parameters of the projector f, x_0, y_0 have been gotten from the formulas above. Then the initial values of the extrinsic parameters of the projector $(\phi, \omega, \kappa, X_s, Y_s, Z_s)$ are deduced from the next formulas.

From formula (8) and (10), and from the formula (9) and (10), then:

$$\frac{a_1}{a_3} = -\frac{(h_1 - h_7 x_0)}{f h_7} \quad \frac{b_1}{b_3} = -\frac{(h_2 - h_8 x_0)}{f h_8} \tag{13}$$

$$\frac{a_2}{a_3} = -\frac{(h_4 - h_7 y_0)}{f h_7} \quad \frac{b_2}{b_3} = -\frac{(h_5 - h_8 y_0)}{f h_8} \tag{14}$$

Angle κ is able to be calculated from the formula (15).

$$tg\kappa = \frac{b_1}{b_2} = \frac{h_2 - h_8 x_0}{h_5 - h_8 y_0} \tag{15}$$

From the formula (13), (14), and $b_1^2 + b_2^2 + b_3^2 = 1$, then:

$$b_3^2 = \frac{1}{1 + \dfrac{(h_2 - h_8 x_0)^2}{f^2 h_8^2} + \dfrac{(h_5 - h_8 y_0)^2}{f^2 h_8^2}} \tag{16}$$

Angle ω is able to be calculated from the known angle κ and the formula (17).

$$\sin\omega = -b_3 \tag{17}$$

Angle ϕ is able to be calculated from the known angle κ, angle ω and the formula (18).

$$tg\phi = \frac{a_3}{a_1 b_2 - a_2 b_1} = \frac{1}{\dfrac{a_1}{a_3} b_2 - \dfrac{a_2}{a_3} b_1} \tag{18}$$

From the formula (8), (9), (10) and (6), then:

$$\begin{cases} h_3 = x_0 - \dfrac{f}{\lambda}(a_1 X_s + b_1 Y_s + c_1 Z_s) \\ h_6 = y_0 - \dfrac{f}{\lambda}(a_2 X_s + b_2 Y_s + c_2 Z_s) \\ \lambda = (a_3 X_s + b_3 Y_s + c_3 Z_s) \end{cases} \tag{19}$$

The initial values of X_s, Y_s, Z_s are able to be calculated from the formula (19).

So, the initial values of the intrinsic and extrinsic parameters of the projector can be worked out from the deduction above.

Because the distortion is sure to exist in the projector lens, the distortion parameters of it should be worked out in the process of the projector calibration.

The distortion parameters are added into the collinear equation, then:

$$x-x_0-\Delta x=-f_x\frac{a_1(X-X_s)+b_1(Y-Y_s)+c_1(Z-Z_s)}{a_3(X-X_s)+b_3(Y-Y_s)+c_3(Z-Z_s)}=-f_x\frac{\overline{X}}{\overline{Z}}$$

$$y-y_0-\Delta y=-f_y\frac{a_2(X-X_s)+b_2(Y-Y_s)+c_2(Z-Z_s)}{a_3(X-X_s)+b_3(Y-Y_s)+c_3(Z-Z_s)}=-f_y\frac{\overline{Y}}{\overline{Z}} \tag{20}$$

$$\Delta x=(x-x_0)(K_1\cdot r^2+K_2\cdot r^4)+ P_1(r^2+2\cdot(x-x_0)^2)+2\cdot P_2(x-x_0)\cdot(y-y_0)$$

$$\Delta y=(y-y_0)(K_1\cdot r^2+K_2\cdot r^4)+ P_2(r^2+2\cdot(y-y_0)^2)+2\cdot P_1(x-x_0)\cdot(y-y_0) \tag{21}$$

where $r^2=(x-x_0)^2+(y-y_0)^2$

K_1, K_2 = Radial Distortion

P_1, P_2 = Decentering Distortion

From the formula (21), then:

$$v_x=\frac{\partial x}{\partial X_s}\Delta X_s+\frac{\partial x}{\partial Y_s}\Delta Y_s+\frac{\partial x}{\partial Z_s}\Delta Z_s+\frac{\partial x}{\partial \phi}\Delta\phi+\frac{\partial x}{\partial \omega}\Delta\omega+\frac{\partial x}{\partial \kappa}\Delta\kappa +\frac{\partial x}{\partial X}\Delta X+\frac{\partial x}{\partial Y}\Delta Y+\frac{\partial x}{\partial Z}\Delta Z+\frac{\partial x}{\partial f_x}\Delta f_x+\frac{\partial x}{\partial f_y}\Delta f_y+\frac{\partial x}{\partial x_0}\Delta x_0+\frac{\partial x}{\partial y_0}\Delta y_0 +\frac{\partial x}{\partial K_1}\Delta K_1+\frac{\partial x}{\partial K_2}\Delta K_2+\frac{\partial x}{\partial P_1}\Delta P_1+\frac{\partial x}{\partial P_2}\Delta P_2-l_x$$

$$v_y=\frac{\partial y}{\partial X_s}\Delta X_s+\frac{\partial y}{\partial Y_s}\Delta Y_s+\frac{\partial y}{\partial Z_s}\Delta Z_s+\frac{\partial y}{\partial \phi}\Delta\phi+\frac{\partial y}{\partial \omega}\Delta\omega+\frac{\partial y}{\partial \kappa}\Delta\kappa +\frac{\partial y}{\partial X}\Delta X+\frac{\partial y}{\partial Y}\Delta Y+\frac{\partial y}{\partial Z}\Delta Z+\frac{\partial y}{\partial f_x}\Delta f_x+\frac{\partial y}{\partial f_y}\Delta f_y+\frac{\partial y}{\partial x_0}\Delta x_0+\frac{\partial y}{\partial y_0}\Delta y_0 +\frac{\partial y}{\partial K_1}\Delta K_1+\frac{\partial y}{\partial K_2}\Delta K_2+\frac{\partial y}{\partial P_1}\Delta P_1+\frac{\partial y}{\partial P_2}\Delta P_2-l_y \tag{22}$$

The formula (22) is the error equation of the whole adjustment finally.

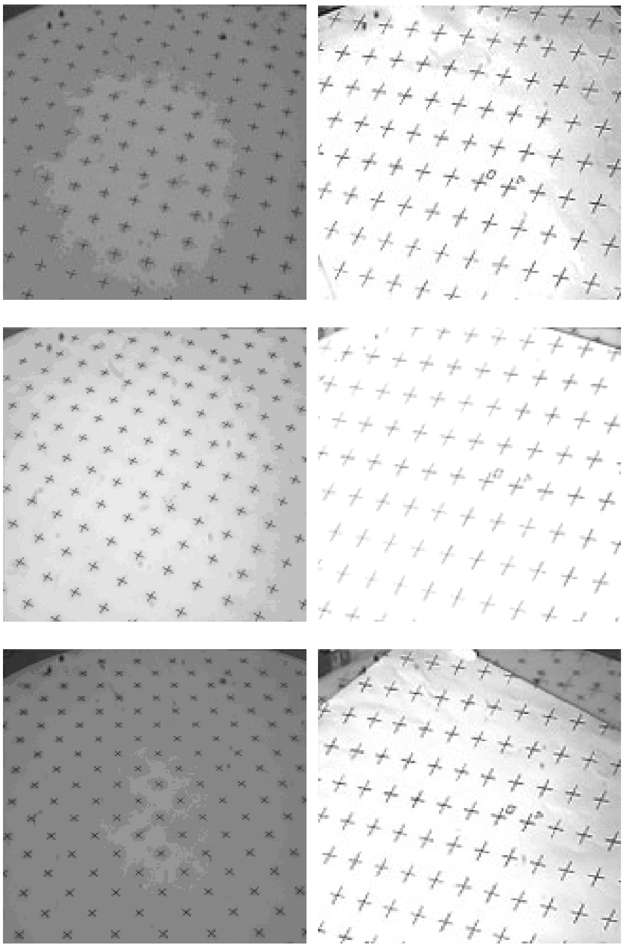
3. DATA AND EXPERIMANTAL RESULTS

3.1 Design Data

The size of the planar grid is 60cm×60cm and there are 18×18=324 points in the planar grid. The interval of these points is the same and is 30mm. Each point has its own serial number which is exclusive. The size of the target grid slide designed is 1024 pixels×768 pixels and there are also 18×18=324 points in the slide grid. The interval of these points is also the same and is 40 pixels. Every point has its own only serial number, too.

3.2 Image Sequences Data

When the positions of the camera and the projector are adjusted well and fixed relatively, both of them need to be focused respectively. Then the camera is used to take an image of the planar grid without the projector illuminating. Following the planar grid is covered by a white paper and projector illuminates the target grid onto the paper. The camera is used to take another image of the target grid. Then the planar grid is rotated and the camera and the projector are changed to a different orientation. Taking two images is done by the same step above. There are 4 sets of the two images as shown Figure 1. The size of each image is 1300pixels×1030pixels. The distance between the digital camera and the planar grid is about 0.5 meters. The distance between the projector and the planar grid is about 1.5 meters.



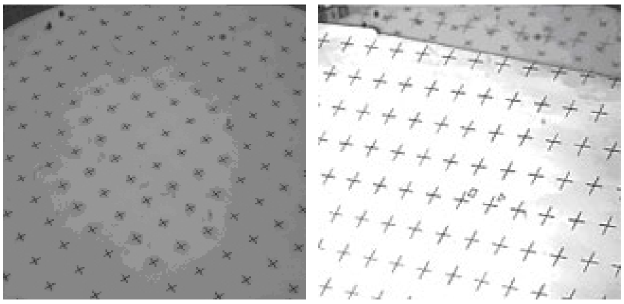


Figure 1. Image data

3.3 Experimental Results

There are respectively 113 points, 84 points, 115 points and 97 points extracted out from the four images with the projected grid ant there are 409 points in total. Using the algorithm to compute these observation values of the points, the parameters of the slide projector are computed by the calibration method mentioned above shown in Table 1.

f (pixel)	x_0 (pixel)	y_0 (pixel)
1676.835624	509.105746	381.938052

Table 1. Parameters of the slide projector

In order to verify the results of calibration, the space coordinates of the points projected in 4 images are calculated respectively using the parameters of the slide projector calibration and the parameters of the digital camera calibration. Comparing the two sets of the calculated coordinates, the root-mean-square errors of the space coordinate residuals are calculated. The number of points and the values of the root-mean-square error are shown in Table 2.

Number of points	RMSEx (mm)	RMSEy (mm)
409	0.594	0.892

Table 2. The root-mean-square error of the coordinates of the points

Because the control ground is not very stable and exquisite, there exists system error in the designed space coordinates integrally. At the same time, the distance between the projector and the planar grid is relatively far so that the distortion of the projector is relatively hard. Because of the two reasons above, the values of the root-mean-square error in this experiment are poor relatively. However, the results of the experiment are satisfied with the requirement of the close-range photogrammetry essentially.

4. CONCLUSIONS

The paper provides a flexible technique to calibrate the slide projector easily and practically. The technique only requires an ordinary slide projector, a digital camera, and a planar grid and it is hardly affected by the space factor or time factor. The paper also deduces the detail algorithm of this technique and supplies the real data slide projector calibration. These two aspects have proved that the technique this paper prospered is feasible and proper.

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