DEVELOPMENT OF A SIMULATOR FOR RELIABLE AND ACCURATE 3D RECONSTRUCTION FROM A SINGLE VIEW

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Commission V, WG 2

KEY WORDS: *Photogrammetry, Architecture, Simulation, Reconstruction*

ABSTRACT

Reconstruction from a single view of architectural scenes is possible thanks to the automatic identification of structural perspective elements in the view. Reliable and accurate reconstruction depends strongly on a robust estimation of vanishing points. In this paper we compare different approaches for estimation of vanishing points, and we justify our choice of Danish estimator. The intersection of pairs of pencils of perspective lines through vanishing points generates maps of quadrilaterals in the image which provide automatic grouping criteria for interpreting the scene. Quadrilaterals are not enough for an accurate reconstruction. Triplets of pencils of perspective lines provide a wire-framed structure of traditional architectonic scenes used for visualization tasks by constructing trapezoidal and cuboid maps superimposed to views with a low computational cost. By identifying multiple junctions in images we obtain the relative orientation between adjacent planes containing bundles of perspective lines. The introduction of multivector representation simplifies the visualization and management of 3D information arising from the lifting of adjacent quadrilaterals to 3D cuboids for visualization tasks.

1. INTRODUCTION

Increasing needs for the integration of Architectural Photogrammetry, Motion Planning, Simulation and Augmented Reality Visualization require flexible and friendly interfaces for 3D reconstruction, which can be easily updated by minimizing the human intervention. A robust estimation of vanishing points is crucial for an accurate and reliable 3D reconstruction of architectural scenes. Vanishing points arise from the intersection of a bundle of parallel lines in the scene, which become perspective lines in its representation. Thus, vanishing points are traditionally estimated from intersections of putative perspective lines or by minimizing the distance to projections of parallel lines. Both procedures are not robust ones, and errors are unacceptable for photogrammetric applications. Then, we apply a robust methodology support by the area's triangle minimization and robust estimators in order to obtain more accurate results.

Different interesting approaches are developed in [Wil02], [Heu98] and [Lut94], which are based on the simultaneous estimation of the best projection matrix for a parallelepiped, parallel and colinearity constraints application and hierarchical Hough transform respectively.

A robust and accurate identification of vanishing points allows to superimpose in an automatic way a perspective model on the view by taking in account the intersections of bundles of coplanar perspective lines. The line through two vanishing points is a vanishing line. In the same way, a vanishing plane contains 3 non-aligned vanishing points. We shall label as perspective pencils, perspective nets and perspective webs to the one, two and three-parameter families of bundles of perspective lines. Perspective lines (resp. planes) pass through a vanishing point (resp. lines). A perspective line (resp. plane) is not in general a vanishing line (resp. plane). An automatic realistic rendering of perspective planes must be take in account their mutual intersections, the information about visibility in the scene, and to delete volumetric regions which can occlude visible regions of background w.r.t. the chosen cuboids in the foreground. Thus, to avoid the manual intervention it is necessary to maintain simultaneous information about perspective lines and perspective planes, and their incidence relations. The incidence information is performed in terms of doubly connected lists linked to "oriented projective flags". Oriented projective flags are given as pairs of an incidence variety $\{(l_i, \pi_i) \mid l_i \subset \pi_i\}$ and contain the information relative to oriented projective lines l_i (pencils of perspective lines) and projective planes π_i (nets of perspective lines). The relative orientation of the wired structure superimposed to the image arises from identifying the type of double (L and T-type) and triple (Y and ↑-type) junctions linked to the intersections between perspective planes, and contents retrieval assigned to each junction type. Junctions can be estimated from a Deriche's filter or a Harris corner's detector. Advances in automatic visualization of 3D information and interactive visual navigation require the identification and management of cuboids in 3D structures linked to the intersection of pairs of pencils of perspective planes through vanishing lines. Both of them require to identify structural constraints as collinearity and coplanarity, and incidence constraints for automatic grouping of perspective lines. Often in photogrammetry, perspective lines in architectural scenes are manually drawn to avoid sensitivity errors. In our case, vanishing points are estimated by a robust methodology. From vanishing points we retrace perspective lines through vanishing points in an automatic way, by taking in account clouds of mini-segments which are grouped following some variant of non-linear regression or RANSAC methods [Tor03]. In this way, we obtain bundles of perspective lines, by avoiding the lack of robustness of least squares methods linked to an automatic grouping of mini-segments, which is a specially cumbersome problem in highly textured images. There are essentially two approaches to recover and visualize a 3D structure: features and primitive-based approaches. In the current work, we have followed a primitive-based approach for perspective models because it supports better vectorized information. Robust and accurate estimation of vanishing points is obtained integrating different methodologies belonging to Photometry and Computer Vision with robust procedures. Parameters linked to transformations between views are expressed in terms of affine maps between quadrilaterals determined by intersections of perspective lines. Planar collineations between pairs of views are lifted to collineations in the ambient 3D space by constructing cuboids from the third vanishing point. We have developed a symbolic management of information based on propagation models from an initial cuboid located around the focal point and generated from pencils of lines through vanishing points. Perspective lines provide the main directions for propagation phenomena in our model. Thus, the robust identification of vanishing points allows to avoid "tedious" local compatibility and global coherence conditions for homologue cuboids in different constructions, and simplify the tracking in presence of self-motion [Fin02]. We have restricted ourselves to simple indoor and outdoor scenes with a static camera, by avoiding occlusion problems and possible cumbersome alternance between concave/convex regions for evaluating the goodness of our results.

2. METHODOLOGY

With the goal of investigating and testing the behavior of our robust methodology, we have developed a program that allows us to test and analyze under different geometric and stochastic conditions several perspectives models and oblique images. The next scheme shows the steps of the methodology developed:

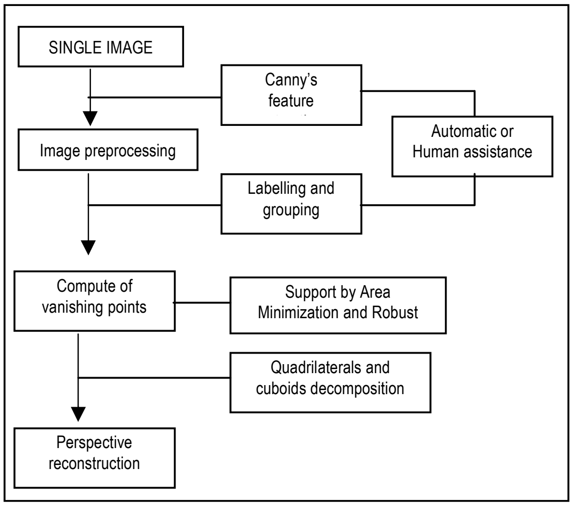


Fig.1: Methodology developed.

2.1 Automatic extraction of basic elements

Automatic grouping in Computational Geometry is usually performed by using auxiliary constructions such as decompositions in triangles, trapezoids or, more generally, convex sets associated to a multivectorized treatment of data [Ber97] Minisegments are obtained from Canny's detector. Minisegments are grouped along lines to determine ordinary multiple junctions. Large segments are grouped depending on the slope and collinearity restrictions. In this way, we attenuate the striped effect which appears from boundaries and minisegments obtained from the application of Canny's detector. Furthermore, we eliminate short segments with length under a selected threshold. In indoor scenes, we apply an active discrimination for vertical lines, due to the characteristics of scene and bad illumination (reflectances, irradiance of the ground and floor, etc). Large segments make part of putative candidates to become perspective lines. To validate them as perspective lines, we must have a robust an accurate estimation of the vanishing points, and verify incidence conditions of large segments through vanishing points.

2.2 Elements of perspective

Vanishing points are computed from bundles of perspective lines in the framework of Projective Geometry [Har00]. The relative location of vanishing points w.r.t. the image determines the perspective model (frontal view, angular perspective, and oblique perspective, respectively) which is useful for visualization tasks (linked to one, two or three vanishing points to finite distance, respectively). Selection of constraints in manually driven reconstruction is typical of photogrammetry, and it depends on the perspective model chosen. In frontal and angular perspective, rigidity constraints relative to motions and objects are translated to parallelism and perpendicularity constraints for typical representations of architectural scenes. The projective extension of Euclidian approach is clearly very useful for oblique perspective. It is based on maps of cuboids appearing as the image of parallelepipeds by a projective transformation. This kind of volumetric representation has been extensively developed and applied for reconstruction from a single view and their applications [Wil02].

2.3 Robust estimators for vanishing points

In scenes with bad illumination or annoying architectural elements, sometimes it is difficult to identify perspective elements to visualize 3D scenes. In this case, we have developed a real-time implementation based on the intersection of putative perspective lines obtained by regression and Hough transform [Fin02]. A weighted average around a vanishing line allows to identify a coarse approach to a putative vanishing point V_p and retracing perspective lines through V_p. The high rate of errors (around a 5%) can be corrected in mobile navigation [Fin02]. However, this error is not allowed in more accurate 3D Reconstruction. Thus, we have concentrated our attention in the implementation of algorithms for robust estimation of vanishing points. In this section we compare different several estimators for vanishing points: Danish method, Minimal Sum, Huber, German-McClure. The main goal of all of them is the automatic elimination of lines which are not adequate for the "right" determination of vanishing points. The rightness is measured in terms of the area minimization. In typical architectural scenes we have a high redundancy degree, more precisely, we have enough elements arising from an automatic vectorization to provide bundle of errors in perspective lines.

Robust estimators provide an adjustment method, and detect wrong observations appearing as outliers in an efficient way. Robustness corresponds to their independence w.r.t. the errors distribution. Robust estimators are based on applying variable weight functions $P(w)$, and they can be written as $P(w_i^{k-1})$ where i is a residual number and k the iteration number. The function P allows to modify original weights in order to reject observations having blunders errors in the adjustment. We have considered the following robust estimators:

Danish estimator	**GermanMcClure estimator**
$W(v) = Exp(-\lvert V(i) \rvert^2)$	$W(v) = \frac{\psi(v)}{v} = \frac{1}{(1+v^2)^2}$
Minimum Sum estimator	**Huber estimator**
$W(i) = \frac{1}{\lvert V_i \rvert}$	$W(v) = \begin{cases} 1 & para \quad \lvert v \rvert / \sigma \leq a \\ \frac{a}{\lvert v \rvert / \sigma} & para \quad \lvert v \rvert / \sigma > a \end{cases}$

Fig. 2: *Robust estimators*

2.3.1 Strategy of resolution

The typical algebraic resolution of the problem relies on the condition that the area of the triangle formed by each edge points $P_1(x_1, y_1)_i$, $P_2(x_2, y_2)_i$ of the extracted segments and the candidate vanishing point $V_P(x,y)$ equals zero (collinearity condition). This leads to the minimization of the residuals (V_i) of the observation equation:

Thus, the most critical step of the resolution is to invert the normal matrix which gives the solution of the Least Squares Adjustment equivalent to minimizate areas of triangles.

The solution corresponds to the minimum eigenvalue of the diagonal matrix D corresponding to a SVD decomposition $A=UDV^T$ of A, being U, V are orthogonal matrices. With more details, the SVD consists of two steps: 1) Obtain a bi-diagonal matrix by operating with Householder's reflectors, and 2) Develop an iterative process by using Given's rotations. As it is well known, the exact algebraic resolution is a typical example of an ill-conditioned problem due to two issues: a) The apparition of a great number of null entries in the matrix A which give serious round errors in normal matrix inversion. b) Redundant information can generate blunders errors that appear irregularly distributed among observations, by making difficult their identification. Several strategies based on using some a priori knowledge have been developed including more advanced variants of RANSAC methods [Tor03]. In this work, we have selected a more down-to-earth solution based on the robust Danish estimator, which improve results of more traditional adjustment based on LSM. To justify our choice, we shall compare different estimators.

2.4 Grouping tools: maps of quadrilaterals and cuboids

The automatic management of convex quadrilaterals related to the intersection of pairs of pencils from perspective lines in each view is symbolically performed with quadtrees according to the quadrangular segmentation. To avoid an excessive fragmentation of intersections of pencils of perspective lines, and to lower the high complexity of corresponding quadtrees, we use typical tricks of ray-tracing allowing us to identify partial occlusions following a typical multilayered model. Similarly the management of closed regions in the orthogonal 3D model of the scene is performed in terms of octrees obtained from triplets of pencils of perspective planes through three vanishing lines. Exterior or cross-product and contraction in Geometric Algebra allow to transfer information between simplified 2D and 3D models. A convex quadrilateral is the image of a rectangle by a projective transformation in the plane. A quadrilateral map of convex quadrilaterals is automatically generated from the intersection of pairs of pencils (bundles of perspective lines) through two vanishing points V_1, V_2. A pair of pencils is called a net of lines. To simplify the management of 2D projective primitives linked to nets of lines, we introduce a symbolic representation given by quadtrees linked to templates of convex quadrilaterals. By using a third vanishing point away from the vanishing line, we lift quadrilateral 2D nets to cuboid 3D families of lines (called webs) which provide an easily adjustable 3D template. The computer management of nets is performed by quadtrees supported on planar templates given by perspective quadrilaterals. In the same way, the computer management of webs of lines is performed by octrees supported on volumetric templates given by cuboids. A cuboid is the image of a rectangle parallelepiped by an affine transformation in 3D space. The quadrilateral map linked to changing quadrilaterals is easily updated in an incremental way by inserting/deleting quadrilaterals associated to elementary events given by segments. The splitting/grouping process of quadrilaterals arising from such updating can be described by an algorithm with a linear complexity in the number of elementary events. Multiple junctions are extracted by means of a variant of the Deriche's filter. Types of junctions inform to us about typical occlusions, or about convex or concave features. The allowed types of multiple junctions are double, triple and quadruple (only allowed at vanishing points in our case). Typical junctions at architectural scenes correspond to a) two incident walls (without additional information about ceiling or floor) are L-type double junctions, b) corners of inserted elements (doors, windows, etc) on a wall are T-type double junctions, c) corners associated to the perspective representation of trihedrals are Y or ↑-type triple junctions, d) vertices of wireframed 3D representations or vanishing points linked to 4-tuples of lines are quadruple junctions. The automatic identification of collections of junctions along a closed polygonal allow us to generate facets, by including information about partially occluded regions, due to the relative localization of the camera. The comparison between regions in different views is reduced to find isomorphism between maximal ordered collections of junctions along candidate to be homologue polygonals.

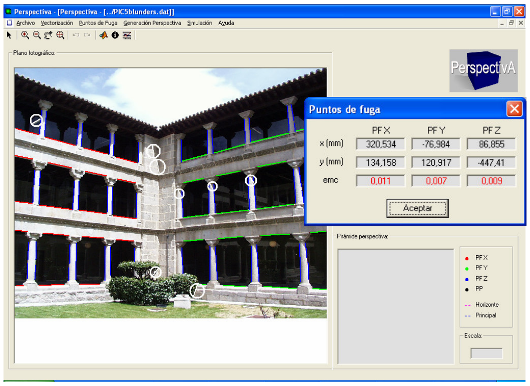
An automatic interpretation of saliencies in the 3D scene is performed with corners labelled. To begin with, let us suppose that the sweep out has given us segmentation by quadrilaterals supported on perspective lines. Each salient or entrant corner is characterized by three walls confluent at a typical Y-triple corner, i.e., a corner where discontinuities arising from extending visible segments are alternant with visible segments incident at such junction. So, in a typical architectural scene each triple junction Y is the common vertex for three quadrilaterals. The union of such three quadrilaterals give an hexagon H with.8 the triple point inside connected to three vertexes of H which are also triple junctions. The hexagon H inheritates the natural orientation induced by compatible positively oriented quadrilaterals that are incident at the triple junction. If central triple junction is a right Y, then the central junction is salient. Otherwise, i.e. if central point is an inverted Y, then it corresponds to a corner of a concave region w.r.t. the observer. Typical hall indoor scenes or piecewise linear approach to baroque facades exhibit an alternant behaviour between right and inverse Y junctions. The continuous or alternant character between triple junctions provides tools to connect local and global aspects. So, we obtain easily verifiable criteria for an automatic interpretation of the scene w.r.t. the observer's viewpoint. Intersections of perspective lines determine a structure given by convex quadrilaterals, which are automatically superimposed to each image. For each pair of vanishing points V_i and V_j we generate a map of quadrilaterals; this map is specialized to a rectangular grid for orthogonal perspective and a trapezoidal grid for frontal view with a vanishing point in the view. The segments lying onto each quadrilateral of such a map make part of the pencil *i* and *j* of perspective lines through the vanishing points V_i and V_j . A coarse management of almost flat facades in outdoor scenes or flat walls in indoor scenes is performed with map of quadrilaterals. However, the sudden jumps in relative depth or the alternance between concave and convex regions (typical in Baroque style) reduce the performance of quadrilaterals maps. The solution of this problem involves the labelling of vertices onto the map of quadrilaterals as multiple junctions.

2.4.1 Automatic generation of superimposed perspective models

The intersection of two pencils *i* and *j* of perspective lines through two vanishing points V_i and V_j determine a map of quadrilaterals. In the same way as for trapezoids in frontal perspective in indoor scenes [Fin02], we represent each quadrilateral by means of a bivector given as one half of the external product of the diagonal of the quadrilateral multiplied by the difference of vectors arising from a selected vertex of the diagonal. Thus, quadrilaterals inheritate a natural orientation depending on the sweep out process and labelling processes. Each map of quadrilaterals generated by two planar pencils from perspective lines provides a support for the automatic grouping and for their associated bilinear maps. In fact, optimization processes can be performed directly onto the space of bilinear maps, with the additional advantage linked to similarity relations between each map of quadrilaterals. In fact, propagation of similarities simplifies the matching process between candidates to homologue quadrilaterals. Above process can be performed for each pair of vanishing points. In this way, we obtain three maps of quadrilaterals that are matched together between them thanks to the existence of cuboids linked to three non-aligned vanishing points. An oriented triple product algebraically represents every cuboid. The set of lines through a vanishing point can be interpreted as the support of a radial vector field centered at the vanishing point. In the same way, each map of quadrilaterals (resp. cuboids) can be interpreted as the support of 2D (respectively, 3D) distribution *D* of vector fields whose singular points are located at vanishing points. The transformation from cuboids to quadrilaterals it is automatically performed by applying the contraction of distribution *D* along the vector field supported onto the set of lines through a vanishing point. The relative position of vanishing points in each view modify the vector field according to the semidirect product of translations by the special unitary group *SU(2)* (the universal double covering of the most common special orthogonal group *SO(3)* linked to rigid transformations in ordinary space. Maps of quadrilaterals provide the support for reading the information relative to the infinitesimal transformations of the Lie algebra *su(2)* of the group *SU(2)*.

3. EXPERIMENTAL RESULTS

The program developed by ourselves is helpful to experiment and visualize varied estimators behaviours within different geometrical and statistical conditions under a flexible and friendly context. We have applied some robust estimators to several images obtained of Santa Ana's Cloister.



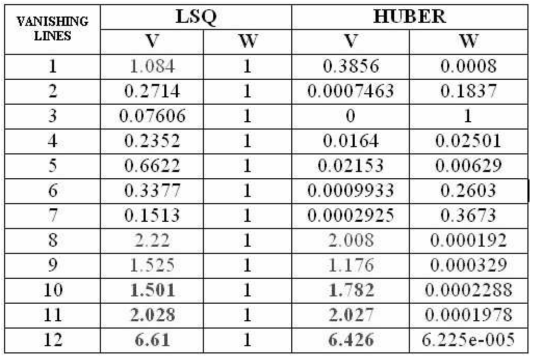
Puntos de fuga

	PF X	PF Y	PF Z
x (mm)	320,534	-76,984	86,855
y (mm)	134,158	120,917	-447,41
emc	0,011	0,007	0,009

Aceptar

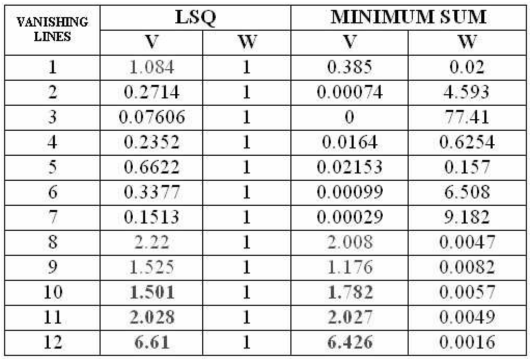
Fig. 3: *Interface of the program*

The results of perspective lines and vanishing points are expressed as a residuals (V) and weights (W) in the following tables, in which we have 3 blunders errors: 10, 11, 12.



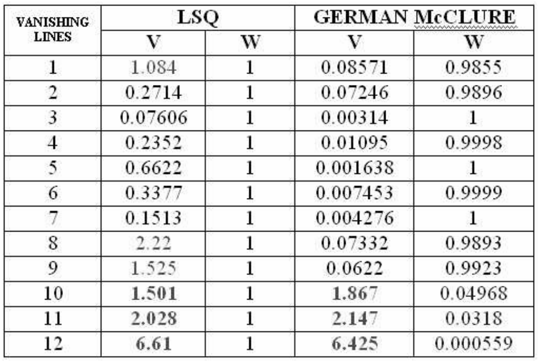
VANISHING LINES	LSQ		HUBER	
	V	W	V	W
1	1.084	1	0.3856	0.0008
2	0.2714	1	0.0007463	0.1837
3	0.07606	1	0	1
4	0.2352	1	0.0164	0.02501
5	0.6622	1	0.02153	0.00629
6	0.3377	1	0.0009933	0.2603
7	0.1513	1	0.0002925	0.3673
8	2.22	1	2.008	0.000192
9	1.525	1	1.176	0.000329
10	1.501	1	1.782	0.0002288
11	2.028	1	2.027	0.0001978
12	6.61	1	6.426	6.225e-005

Fig. 4: LSQ vs. Huber estimator in vanishing points detection



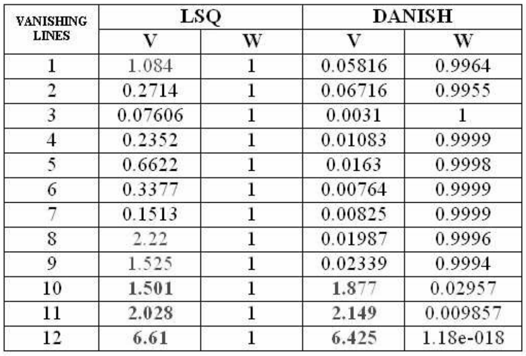
VANISHING LINES	LSQ		MINIMUM SUM	
	V	W	V	W
1	1.084	1	0.385	0.02
2	0.2714	1	0.00074	4.593
3	0.07606	1	0	77.41
4	0.2352	1	0.0164	0.6254
5	0.6622	1	0.02153	0.157
6	0.3377	1	0.00099	6.508
7	0.1513	1	0.00029	9.182
8	2.22	1	2.008	0.0047
9	1.525	1	1.176	0.0082
10	1.501	1	1.782	0.0057
11	2.028	1	2.027	0.0049
12	6.61	1	6.426	0.0016

Fig. 5: LSQ vs. Minimum Sum estimator in vanishing points detection



VANISHING LINES	LSQ		GERMAN McCLURE	
	V	W	V	W
1	1.084	1	0.08571	0.9855
2	0.2714	1	0.07246	0.9896
3	0.07606	1	0.00314	1
4	0.2352	1	0.01095	0.9998
5	0.6622	1	0.001638	1
6	0.3377	1	0.007453	0.9999
7	0.1513	1	0.004276	1
8	2.22	1	0.07332	0.9893
9	1.525	1	0.0622	0.9923
10	1.501	1	1.867	0.04968
11	2.028	1	2.147	0.0318
12	6.61	1	6.425	0.000559

Fig.6: LSQ vs. German estimator in vanishing points detection



VANISHING LINES	LSQ		DANISH	
	V	W	V	W
1	1.084	1	0.05816	0.9964
2	0.2714	1	0.06716	0.9955
3	0.07606	1	0.0031	1
4	0.2352	1	0.01083	0.9999
5	0.6622	1	0.0163	0.9998
6	0.3377	1	0.00764	0.9999
7	0.1513	1	0.00825	0.9999
8	2.22	1	0.01987	0.9996
9	1.525	1	0.02339	0.9994
10	1.501	1	1.877	0.02957
11	2.028	1	2.149	0.009857
12	6.61	1	6.425	1.18e-018

Fig. 7: LSQ vs. Danish estimator in vanishing points detection

In summary, Minimum Cost estimators (Minimum Sum and Huber) distribute blunder errors among the other observations, and make more difficult the right estimation of vanishing points. Contrarily, Danish and German & McClure estimators detect perfectly the computation by residuals of vanishing lines, discard the erroneous artificially introduced vanishing line, reject blunders errors out of adjustment by virtue of low weight and keep very small the other residuals.

4. CONCLUSIONS AND FUTURE WORK

We have developed several methods for a robust estimation of vanishing points inside the views corresponding to projections of indoor and outdoor scenes, and we have compared on a specially designed program. From the comparison between them, we have obtained robust and accurate results for Danish and German operators.

But, apart from the results obtained by applying Danish and German estimators are more accurate than the others in performed experiments, in general, the robust methodology developed overcomes in power and effectiveness to the classical method (LSQ), in blunders detection and vanishing point compute. The main reason about the discrepancy between estimators is that the oblique images and their structural elements present a great weakness, what increase the complexity of our algorithms when we face real situations, demanding us, to experiment and adapt our robust estimator to our particular problem. On the other hand, we are aware that to get that the user doesn't take part in the whole process will be a mete difficult to overcome. In this way, an interactive behaviour is still needed for the treatment of alternant behaviour in buildings. An automatic approach to be developed in the next future requires to improve our graphical model, by developing a low-cost implementation of algorithms allowing us to add variable visibility constraints linked to an automatic robust estimation of oriented facets in terms of inserting-deleting voxels.

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