BAYESIAN-BASED DESPECKLING IN WAVELET DOMAIN USING "*A TROUS*" ALGORITHM

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ABSTRACT:

In this paper an improved speckle noise reduction method is presented based on wavelet transform. A 2D Gaussian function is found to be the best model fitted to the speckle noise pattern cross-section in the logarithmically transformed noisy image. Therefore, a Gaussian low pass filter using *a trous* algorithm has been used to decompose the logarithmically transformed image. A Bayesian estimator is then applied to the decomposed data to estimate the best value for the noise-free wavelet coefficients. This estimation is based on alpha-stable and Gaussian distribution hypotheses for wavelet coefficients of the signal and noise, respectively. Quantitative and qualitative comparisons of the results obtained by the new method with the results achieved from the other speckle noise reduction techniques demonstrate its higher performance for speckle reduction in SAR images.

1. INTRODUCTION

Imaging techniques using coherent illumination, such as laser imaging, acoustic imagery and synthetic aperture radar (SAR), which generate coherent images [4], are subject to the phenomenon of speckle noise. Speckle noise is generated due to constructive and destructive interference of multiple echoes returned from each pixel. As a result, a granular pattern is produced in the radar image which corrupts significantly the appearance of the image objects. Speckle noise can be modeled as multiplicative random noise in spatial domain [16].

Many attempts were made to reduce the speckle noise. An appropriate method for speckle reduction is one which increases the signal to noise ratio while preserving the edges and lines in the image. Generally, there are two main approaches for speckle noise removal. The first is applied before image generation which is called multi-look processing [16]. In this method the synthetic aperture is divided into some pieces. Each of these apertures is processed separately to obtain a pixel with a special along-track dimension. The N images are summed to form an N-look SAR image. The N-look processing reduces the standard deviation of the speckle. The second approach is filtering the image using different filters [14,9]. Two types of filters are used for speckle reduction. Low pass filters such as mean or median generally smooth the image. The second type is adaptive filtering [13,10]. These filters *adapt* themselves to the local texture information within a box surrounding a central pixel in order to calculate a new pixel value. Adaptive filters demonstrated their superiority compared to lowpass filters, since they take into account the local statistical properties of the image. Adaptive filters perform much better than low-pass smoothing filters, in preservation of the image sharpness and details while suppressing the speckle noise. Both multi-look processing and spatial filtering reduce speckle at the expense of resolution and they both essentially smooth the image. Therefore, the amount of speckle reduction desired must be balanced with the particular application and the amount of details required.

Generally, a successful speckle reduction method has to accomplish these requirements: *i)* variance reduction in homogeneous areas, *ii)* texture, edge and line preservation, *iii)* point scatterer exclusion, and i*v)* artifact avoidance.

In all speckle noise reduction techniques, the statistical distribution of SAR data plays an important role. These statistical properties can be used to develop specialized filters for speckle noise reduction. However, in the above mentioned methods some information in the image such as edges and lines will be lost. Therefore, methods based on spatial filtering are not appropriate in applications in which preserving of spatial details is important.

Recently, few attempts have been made to reduce the speckle noise using wavelet transform as a multi-resolution image processing tool [6]. Speckle noise is a high-frequency component of the image and appears in wavelet coefficients. One common method used for speckle reduction is wavelet shrinkage [15]. According to this method, large wavelet coefficients of the image correspond to the signal and the smaller ones represent the noise. The threshold is computed based on statistical properties of the noisy data using different shrinkage rules. A shrinkage function such as Garrote-thresholding, hard-thresholding or soft-thresholding uses this threshold to modify the wavelet coefficients [5]. The main difficulty with this method is to optimally determine the threshold value.

Achim *et al.* [2] presented a Bayesian-based method for speckle noise reduction in medical ultrasonic images. They used a least square method for estimation of the wavelet coefficient distributions corresponding to the signal and noise. Then a Bayesian estimator has been used for estimating the noise free wavelet coefficients.

This paper presents an efficient algorithm for Bayesian-based speckle noise reduction using optimally implemented *a trous* algorithm for image decomposition [1, 11, 12]. It is shown that the Lapalacian of Gaussian (LOG) is the best wavelet function to be used for image decomposition in speckle reduction problem. Since complete reconstruction of the image using this wavelet function is not possible, another wavelet function called *Coiflet* which is similar to LOG function is used. Further improvement is achieved by using *a trous* algorithm for image decomposition applying a lowpass Gaussian filter. This algorithm uses an undecimated wavelet transform to avoid the artifacts produced by subsampling.

In the next section, the improved Bayesian-based algorithm for speckle noise reduction is presented. Section 3, is devoted to quantitative and qualitative evaluation of the results. Finally, concluding remarks are given in section 4.

2. SPECKLE NOISE REDUCTION MODEL

A simple model for speckle noisy image has a multiplicative form [16],

$$Y(x,y) = S(x,y).N(x,y) \tag{1}$$

where *Y, S* and *N* represent the noisy data, signal and speckle noise, respectively. In order to change the multiplicative nature of the noise to additive one, we apply a logarithmic transformation to the image data. Taking logarithm of the both sides of Eq. (1), we will have:

$$f(x,y) = s(x,y) + e(x,y) \tag{2}$$

where *f, s* and *e* represent logarithms of the noisy data, signal and noise, respectively. The next step is the computation of wavelet transform of $f(x,y)$. One of the important issues to be considered in wavelet transform is the choice of the best wavelet function as well as the transformation algorithm. Since we are interested in isolating the speckle noise in the image, the most appropriate wavelet function is one, which its shape looks like the speckle pattern. For this purpose, we computed the average of *x* and *y* cross sections of several speckle samples in the logarithmically transformed data. According to this study, the 2D Gaussian function has been found to be the best model fitted to the speckle pattern cross-section. Figure (1) illustrates the shapes of the x and y cross sections of the averaged speckle noise and Gaussian curves fitted to them.

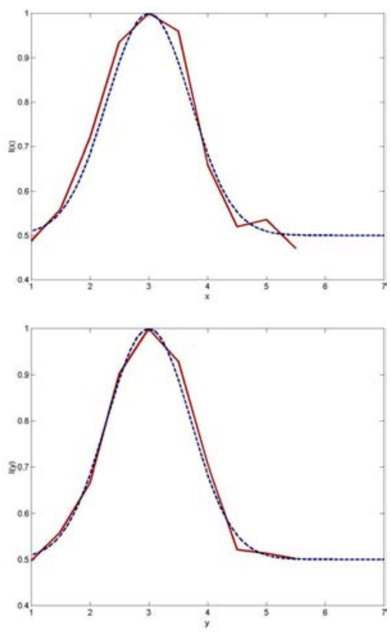

Figure1. The average of *x* and *y* cross sections (solid line) and the Gaussian curve (dotted line) fitted to the speckle pattern.

The Laplacian of Gaussian (LOG) function is therefore considered as the best wavelet among other filters for wavelet decomposition. Unfortunately, complete reconstruction of the image using LOG, is not possible. Hence another wavelet basis called *Coiflet* (with the filter length of 6) whose shape is similar to LOG may be used. Using this wavelet and Mallat's algorithm for wavelet decomposition, the complete reconstruction of the image is possible. Further improvements may be achieved by using Gaussian low pass filter and *a trous* algorithm for decomposition. This algorithm is well-known for using non-decimated wavelet transform which minimizes the artifact in the denoised data [5]. Shift invariancy is one of the important properties of *a trous* algorithm. In speckle noise reduction this property can improve the performance of the algorithm.

Wavelet coefficients of the logarithmically transformed image are best modeled by alpha-stable distribution, $S\alpha S$,which is the family of heavy-tailed densities [2]. The alpha-stable distribution does not have a direct expression but it can be defined by its characteristic function as follows:

$$\varphi(\omega) = \exp(j\delta\omega - \gamma\,|\,\omega\,|^{\alpha}) \tag{3}$$

where $\alpha\ (0 < \alpha \le 2)$ is the characteristic exponent. Small values of this parameter reflect the non-Gaussianity of the distribution function. $\delta\ (-\infty < \delta < \infty)$ is the location parameter and $\gamma\ (\gamma > 0)$ is the dispersion similar to variance used in the Gaussian distribution.

The noise component, *e*, can be modeled as a zero mean Gaussian random variable [1]. The characteristic function of the Gaussian distribution is:

$$\varphi_e(\omega) = \exp(J\delta\omega - \frac{\sigma^2}{2}\,|\,\omega\,|^2) \tag{4}$$

where δ , is the median value of the noise and σ is the variance or noise level. In the proposed method a Bayesian estimator is used for estimating the noise free signal. This estimator uses the wavelet coefficients distribution as *a priori* information. The goal is finding the estimator $\hat{s}$, which minimizes the conditional risk, $R(\hat{s}\mid d)$:

$$R(\hat{s}\mid d) = E[L(\hat{s}\mid d)] = \sum_{j=1}^{k}\ [L(\hat{s}\mid s_j)P(s\mid d)] \tag{5}$$

In this equation, $L[.]$, $\hat{s}$ and s , represent the loss function, estimated noise-free signal and signal, respectively. The estimated signal $\hat{s}(d)$ is the loss averaged over the conditional distribution of *s*, given a set of wavelet coefficients, *d* [7, 8]. The above Bayes risk estimator under a quadratic cost function minimizes the mean-square error and is given by the conditional mean of *s* given *d*:

$$\hat{s}(d) = \sum s p_{s|d}(s\mid d) \tag{6}$$

The mean-square error is defined for random variables that have finite second order moments. Since alpha-stable distribution does not have finite second-order statistics, we use absolute error as the loss function. Using the Bayes' theorem, the estimator is then given by [11]:

$$\hat{s}(d) = \frac{\sum P_e(e)P_s(s).s}{\sum P_e(e)P_s(s)} \tag{7}$$

where $P_e(e)$ and $P_s(s)$ are the PDFs of the noise and signal, respectively. In order to use Eq. (7), we have to estimate the

PDFs parameters which are α_s, γ_s and σ_e. For this purpose, it is preferable to use the characteristic functions rather than the PDFs. Since the wavelet coefficient of the noisy data *d* is equal to the sum of the wavelet coefficients of the signal *s* and noise *e*, we have [3]:

$$P_d(d) = P_s(s) * P_e(e) \tag{8}$$

where $*$ represents the convolution operation. The corresponding characteristic function $\phi_d(\omega)$ will be the product of the signal and noise characteristic functions:

$$\phi_d(\omega) = \phi_s(\omega).\phi_e(\omega) \tag{9}$$

For estimating the unknown parameters, α_s and γ_s, $\phi_d(\omega)$ is fitted to the Fourier transform of the wavelet coefficients histogram. For this purpose, the least square (LS) method is used which gives a robust solution. Moreover, the noise level, σ_e can be estimated using

$$\hat{\sigma}_e = 1.3 * MAD(d_J) \tag{10}$$

where the operator *MAD* represents the mean absolute deviation and d_J is the wavelet coefficients of the highest level. After estimating the unknown parameters, $\hat{s}(d)$ in each level of the wavelet transform can be computed using Eq. (7).

At the final stage, by applying the inverse wavelet and exponential transformations, the denoised image is reconstructed. In the next section, experimental results of the proposed technique are presented and compared to other noise reduction methods.

3. EXPERIMENTAL RESULTS

In order to compare different noise reduction techniques, speckle noise with the variance of 0.005 was added to an aerial photo of size 64×64. In order to evaluate selected wavelet basis, different wavelet functions included coiflet1 and symmlet4 were used while applying Mallat's algorithm. Furthermore, in order to decrease the artifacts in the results, the *a trous* algorithm with the Gaussian lowpass filter was used to decompose the image.

The wavelet coefficients were obtained from the logarithmically transformed image using *a trous a*lgorithm. The maximum number of wavelet decomposition levels was 2. Then, these wavelet coefficients were modeled using alpha-stable and Gaussian distribution functions. Figure (2) illustrates the modeling of the wavelet coefficients of the second decomposition level.

To evaluate the results, two criteria including the signal to noise ratio and correlation were used. The signal to noise ratio (*S/N*) is defined by:

$$S/N \approx S/mse = 10\log_{10}\left(\sum_{i=1}^{k} s_i^2 / \sum_{i=1}^{k} (\hat{s}_i - s_i)^2\right) \tag{11}$$

where *S* and $\hat{S}$ represent the original (before adding noise) and denoised images, respectively. The correlation measure (β) is computed using:

$$\beta = \frac{\sum_{i=1}^{k} (\Delta s_i - \Delta\bar{s})(\Delta\hat{s}_i - \Delta\bar{\hat{s}})}{\sqrt{\sum_{i=1}^{k} (\Delta s_i - \Delta\bar{s})^2 \sum_{i=1}^{k} (\Delta\hat{s}_i - \Delta\bar{\hat{s}})^2}} \tag{12}$$

where $\Delta s, \Delta\hat{s}$ are high pass filtered of $s, \hat{s}$ using Laplacian filter. The correlation value is close to one when the edges are optimally preserved in the image.

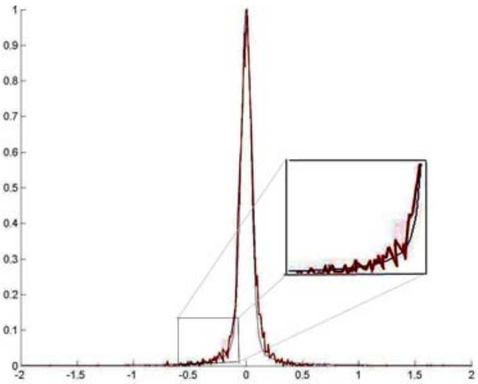

Figure 2. Modeling of the second level of wavelet transforms using $S\alpha S$ and Gaussian distributions. The estimated parameters α_s, γ_s and σ_e are 0.6469, 0.0774 and 0.0707, respectively.

Table (3) presents the quantitative results of different noise reduction methods. It can be observed that the proposed method using *a trous* algorithm has the best results from the signal to noise ratio and edge preservation points of view.

Table 3. Results of different speckle noise reduction methods.

Speckle Noise Reduction Method	***Signal to Noise Ratio***	***Edge Preservation***
Bayesian *a trous* algorithm	25.796	0.843
Bayesian Mallat algorithm-sym	25.279	0.840
Bayesian Mallat algorithm-coif	25.452	0.843
Wiener Filter	24.652	0.833
Hard-Thresholding	24.924	0.787
Garrote-Thresholding	24.595	0.794
Soft-Thresholding	22.814	0.773
Frost Filter	23.388	0.821
Median Filter	23.342	0.688
Lee Filter	22.838	0.764
Gamma Filter	22.404	0.791
Mean Filter	20.427	0.608

Figure (4) illustrates the results of some currently used speckle reduction techniques on the same aerial image. As can be observed, the proposed method has performed better in reducing the speckle noise while preserving edge information.

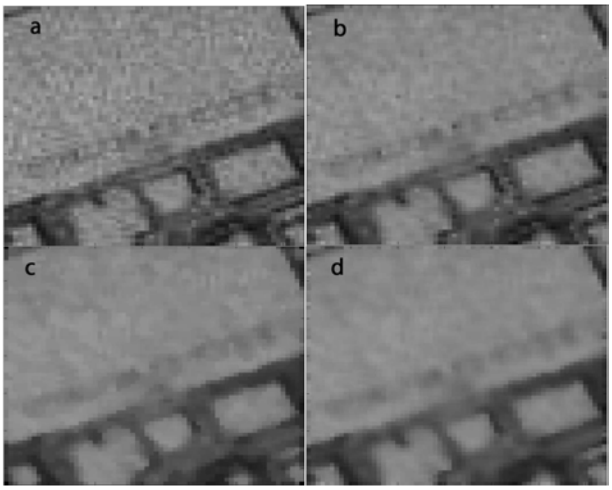

Figure 4. (a) Original noisy image, (b-d) Results of the proposed method, median filtering and Wiener filtering.

Figure (5) shows the results of applying three different speckle noise reduction techniques including the proposed wavelet-based method, Wiener filtering and median filtering on a noisy SAR image.

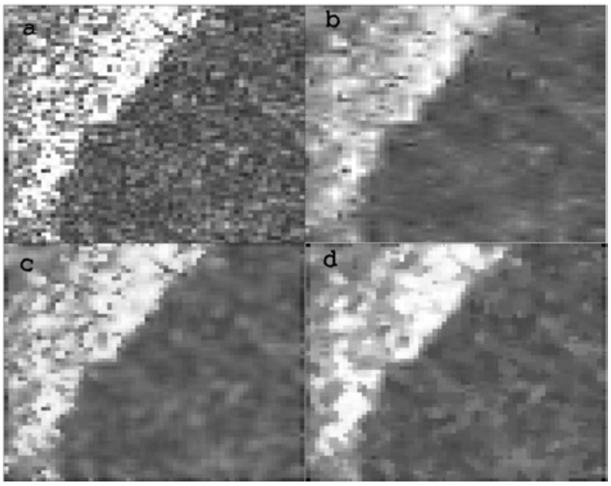

Figure 5. (a) Original noisy SAR image, (b-d) Results of the proposed wavelet-based method, Wiener filtering and median filtering.

4. CONCLUDING REMARKS

A speckle noise reduction method in wavelet domain using *a trous* algorithm was introduced. Major contributions of this work were the optimal selection of the wavelet function and the algorithm to be used for wavelet decomposition of the image. The best model fitted to speckle pattern was found to be a circularly symmetric Gaussian function. Therefore a two-dimensional Gaussian function was proposed as the best wavelet basis to be used. In order to achieve complete reconstruction of the image using processed wavelet coefficients, and to obtain shift invariancy property of the wavelet transform, the *a trous* algorithm was selected as the most appropriate for wavelet decomposition.

A Bayesian estimator was used for estimating the denoised wavelet coefficients. This estimator uses *a priori* knowledge on probability distribution of the signal and noise wavelet coefficients. This estimator performs like a feature detector, preserving the features that are clearly distinguishable in the speckled data such as lines and edges.

The whole algorithm is computationally expensive. Particularly, parameter estimation of the signal and noise distributions is the most time consuming part of the algorithm. More efficient parameter estimation algorithms may reduce the computational cost of this part. Further improvements to this algorithm may be achieved using knowledge-based information such as image texture or PDF of radar cross section (RCS). Integrating these different kinds of information may be performed using Neural Networks.

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