

## RECOVERING PLOT-SPECIFIC DIAMETER DISTRIBUTION AND HEIGHT-DIAMETER CURVE USING ALS BASED STAND CHARACTERISTICS

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**KEY WORDS:** Airborne laser scanning, forest inventory, Weibull, Korf, Scots pine

### ABSTRACT:

Density, diameter distribution and height-diameter (H-D) pattern of a forest stand are of primary importance in deriving various stand characteristics, but measuring diameters and heights of a tree stock is rather time-consuming. That is why theoretical diameter distribution and H-D models are usually employed. We examine the prediction of them for Scots pine sample plots using information obtained with airborne laser scanning (ALS). We propose a parameter recovery approach, where such values for the parameters of assumed diameter distribution and H-D models are determined, that satisfy the mathematical relationships between the predicted plot-specific characteristics. If the solution for the formulated system of equations exists, it is always compatible with the predictions of stand characteristics. The method is developed and tested with a dataset of 213 Scots pine stands. A solution was found for all but 2 plots. The proposed method appears to be a reasonable alternative for predicting stand structure from ALS data.

### 1. INTRODUCTION

Information on tree diameters and heights from a forest stand can be used for deriving various stand characteristics, such as basal area, volume and timber assortments, which are of primary interest in forest management. Since measuring standing tree diameters and heights is rather time-consuming, a theoretical model for the structure of the growing stock is usually employed in growth and yield simulators. We call such a model *stand description*. A stand description characterizes stand structure with only a few parameters. The level of detail in a stand description varies according to information needs of the application. In this study, we use a triplet including stand density (i.e. number of trees per hectare), diameter distribution, and height-diameter (H-D) curve. Assuming that tree volume and taper curve are known functions of tree diameter and height, such a stand description is sufficient for computing timber assortments. In a mixed species stand, such a description would be needed for all tree species, but this study will consider only single species stands.

Numerous approaches for describing the diameter distribution of a stand have been presented, including beta, Weibull and Johnson SB functions, as well as percentile-based and non-parametric approaches (Bailey and Dell, 1973; Loetsch et al., 1973; Hafley and Schreuder, 1977; Borders et al., 1987; Maltamo and Kangas, 1998). Correspondingly, two main methods have been used to predict parameters of an assumed theoretical distribution, namely the parameter prediction method (PPM) and the parameter recovery method (PRM) (Hyink and Moser, 1983). In PPM, field measured stand variables, such as basal area and mean diameter, are used as predictors in regression models that are applied in predicting the diameter distribution for a target stand. In PRM, stand variables, moments or percentiles of diameter distribution are predicted or measured for the target stand, and parameters of an assumed diameter distribution are then

recovered using mathematical relationship between them and the utilized characteristics (see Knoebel and Burkhardt, 1991).

This study generalizes the idea of PRM to recovery of a stand description of an assumed form. Instead of recovering the parameters of a diameter distribution, we will simultaneously recover the parameters of diameter distribution and H-D curve. In addition to stand density, as many predicted stand variables are needed as the assumed stand description has stand-specific parameters to make recovery possible.

The development of small footprint and discrete return airborne laser scanning (ALS) technology has offered possibilities for accurate prediction of forest variables, such as standing tree volume. Numerous studies have shown that both the recognition of individual trees and plot level utilization of the characteristics of canopy height distributions can produce highly accurate predictions of forest variables (e.g. Næsset, 1997; Hyyppä et al., 2001; Persson et al., 2002; Lim et al., 2003; Holmgren, 2004; Næsset et al., 2004; Hopkinson et al. 2006; Maltamo et al., 2006). ALS-based variables have usually been stand mean and sum characteristics, but in Norway and Finland, ALS data have also been used to predict the parameters of assumed diameter distribution models (Gobakken and Næsset, 2004; 2005; Maltamo et al., 2006; Bollandsås and Næsset, 2007). These studies have employed either Weibull function or percentile-based distributions, and have applied parameter prediction method. Recently, Maltamo et al. (2007) applied Weibull distribution, PPM, and calibration estimation (see Deville and Särndal, 1990; Kangas and Maltamo, 2000) to predict distributions that are compatible with ALS based stem number, basal area and stand volume.

The aim of this study was to apply the parameter recovery method to estimate diameter distribution and H-D curve of Scots pine (*Pinus sylvestris*) by using ALS based stand variables. First, stem number, stand volume, basal area median diameter, and the corresponding tree height were

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regressed on independent variables based on ALS data, and the obtained models were applied for prediction in the modelling data. A parameter recovery system, developed for this study, was then utilized to recover such values for the parameters of two parameter Weibull distribution and Korf's height curve that are compatible with these predictions. The system was validated by calculating the proportion of plots where recovery was possible, and RMSE and bias of volume predictions for trees above given diameter limits.

## 2. STUDY MATERIAL

### 2.1 Field data

The Juuka test area (about 10 000 hectares) in eastern Finland is a typical Finnish managed boreal forest area. The field data were acquired during summers 2005 and 2006. A total of 506 circular sample plots with a radius of 9 metres were established on the area. Sample plots were located rather systematically to the young, middle-aged and mature forests; sapling stands were left out. Subsequently, the Global Positioning System (GPS) was used to determine the position of the centre of each of the 506 plots to the accuracy of about 1 meter. The diameter at breast height (*DBH*), tree and storey class, and tree species were recorded for all trees with *DBH* over 5 cm, and the height of one sample tree of each species in each storey class was measured on each plot. For prediction of heights for other trees, a Näslunds height model with a random constant for each plot was fitted to the data of measured heights (Aki Suvanto, personal communication). The model with predicted plot effects was utilized to predict heights for trees without height measurement. The volume models of Laasasenaho (1982) were used to compute tree volumes. Finally, the stand characteristics of interest were scaled up to per hectare level. Of those 506 plots, pure Scots pine plots (the proportion of Scots pine over 90 % of volume) were selected to the study data of 213 plots (Table 1).

	mean	min	max	sd
Volume, m <sup>3</sup> ha <sup>-1</sup>	122.8	14.7	317.8	61.8
Number of stems, ha <sup>-1</sup>	903.8	196	2122	377.3
Basal area median diameter, cm	18.1	9.4	40.0	4.8
Height of a basal area median tree, m	14.0	6.0	23.4	3.3

Table 1. Mean characteristics of the study data. Sd is standard deviation.

### 2.2 Laser scanning

Georeferenced point cloud data were collected from Juuka on 13<sup>th</sup> July 2005 using an Optech ALTM 3100C laser scanning system. The test site was measured from an altitude of 2000 m above ground level using a field of view (FOV) of 30 degrees. This resulted in a swath width of approximately 1050 m and a nominal sampling density of about 0.6 measurements per square meter. The divergence of the laser beam (1064 nm) was set at 0.26 mrad. Optech ALTM 3100C laser scanner captures 4 range measurements for each pulse, but here the measurements were reclassified to represent first and last pulse echoes.

In order to generate a digital terrain model (DTM) from the laser scanner data, the points reflected from objects, e.g. from trees, were classified as vegetation hits. The laser point

clouds were first classified to ground points and other points (the method is explained by Axelsson, 2000). Then a DTM raster with a cell size of 2.5 meters was created by computing the mean of the ground points within each raster cell. Values for raster cells with no data were derived using Delaunay triangulation and bilinear interpolation.

Laser canopy height at a given location was calculated as the difference between the *z* value of laser hit and the estimated DTM raster. Points having canopy height over 0.5 meters were classified as vegetation hits. Different height metrics were calculated using the vegetation hits of each sample plot. Percentiles for the canopy height were computed for 5, 10, 20, ... , 90, 95 and 100 % ( $h_5, \dots, h_{100}$ ) (see Næsset, 2004), and proportional canopy densities were calculated for each of these quantiles ( $p_{05}, \dots, p_{100}$ ). For example,  $h_{10}$  means the height where 10% of all vegetation hits are accumulated, and  $p_{10}$  the proportion of laser hits that is accumulated at the height of 10%. Moreover, the standard deviation ( $h_{std}$ ), mean ( $h_{mean}$ ), and proportion of vegetation hits (*veg*) were computed. In addition to height and density metrics the intensity of reflection of backscattered laser pulse was utilized. Intensity variables were calculated as percentiles ( $i_{10}, \dots, i_{90}$ ) within a plot using vegetation hits only. All these characteristics were calculated for both first (*f*) and last (*l*) pulse data.

## 3. METHODS

### 3.1 Modeling stand characteristics using ALS data

Regression models were constructed for standing tree volume (*V*), stem number (*N*), basal area median diameter (*D*) and height of a basal area median tree (*H*) using ALS based characteristics as independent variables. The models were fitted using ordinary least squares. A stepwise procedure was applied in the choice of independent variables. The obtained models were applied for prediction in the modelling data.

### 3.2 Recovering the stand description

In parameter recovery, a stand description of an assumed form was determined that is compatible with all four predicted stand variables. The stand description includes stand density, diameter distribution and H-D curve. The stand density is directly obtained as the predicted number of stems, whereas the other components are recovered using the other three predicted stand variables. We assume that the growing stock is fully described with the stand variables we have, and PRM is used to find a compatible stand description.

Denote the predicted values of stand variables by  $\hat{V}$ ,  $\hat{G}$  and  $\hat{H}$  and the values based on an assumed stand description by  $V(\theta, \hat{N})$ ,  $D(\theta)$  and  $H(\theta)$ , which depend on the parameters of the stand description,  $\theta$ , and the predicted stand density,  $\hat{N}$ . A stand description that is compatible with all the predicted stand variables is obtained as a solution to the following system of equations:

$$\begin{cases} V(\theta, \hat{N}) - \hat{V} = 0 \\ D(\theta) - \hat{D} = 0, \\ H(\theta) - \hat{H} = 0 \end{cases} \quad (1)$$

The system is infeasible if no stand description of the assumed form exists that complies with  $\hat{V}$ ,  $\hat{D}$ ,  $\hat{H}$  and  $\hat{N}$ .

Denote tree diameter by  $x$ , height by  $h$  and volume by  $v$ . We assume a diameter distribution with density  $f(x|\alpha, \beta)$ , height  $h(x|\gamma)$  for a tree with diameter  $x$ , and volume  $v(x, h)$  for a tree with diameter  $x$  and height  $h$ , where the parameters of the stand description are  $\theta = (\alpha, \beta, \gamma)$ . Thus, the diameter distribution is characterized by two plot-specific parameters,  $\alpha$  and  $\beta$ , and the H-D curve with one plot-specific parameter,  $\gamma$ . A general volume function with no stand-specific parameters is assumed for all trees. The stand variables based on such a stand description are

$$V(\theta, \hat{N}) = \hat{N} \int_0^{\infty} f(x|\alpha, \beta) v(x, h(x|\gamma)) dx$$

$$D(\theta) = F_G^{-1}(0.5|\alpha, \beta)$$

and

$$H(\theta) = h(\hat{D}|\gamma),$$

where  $F_G^{-1}(p|\alpha, \beta)$  is the inverse (quantile) function of the basal-area weighted diameter distribution

$$F_G(x|\alpha, \beta) = \int_0^x f_G(u|\alpha, \beta) du,$$

where

$$f_G(x|\alpha, \beta) = \frac{x^2 f(x|\alpha, \beta)}{\int_0^{\infty} x^2 f(x|\alpha, \beta) dx}$$

is the density of basal-area weighted diameter distribution (see e.g. Gove and Patil, 1998).

We described the diameter distribution with the two-parameter Weibull distribution. It has density

$$f(x|\alpha, \beta) = \left(\frac{\alpha}{\beta}\right) \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right),$$

where  $\alpha$  is a shape and  $\beta$  is a scale parameter. As the H-D curve, we used the model of Mehtätalo (2005)

$$h(x|\gamma) = \max\left[1.4, \exp(\gamma - \delta y(x))\right].$$

The independent variable is a transformation of tree diameter

$$y(x) = \frac{(x+7)^c - (\hat{D}+17)^c}{(17)^c - (37)^c},$$

where  $c = 0.98 + 0.058\hat{D}$ . This transformation gives interpretations to the two parameters:  $\gamma$  is the logarithmic height of trees with diameter  $\hat{D}+10$  and  $\delta$  is the expected difference in logarithmic height between diameters  $\hat{D}$  and  $\hat{D}+10$ . We allowed only  $\gamma$  vary between plots in recovery and  $\delta$  depended on basal area median diameter according to  $\delta = 0.62 - 0.027\hat{D} + 0.00094\hat{D}^2$  (Mehtätalo, 2005). As the volume function, we used that of Laasasenaho (1982),

$$v(x, h) = 0.036x^{2.01} 0.997^x h^{2.07} (h-1.3)^{-1.07}.$$

Using the functions presented above, we solved the system of equations given in (1) for  $\theta = (\alpha, \beta, \gamma)$ . The estimation was carried out in two stages: we first searched initial estimates for the parameters by minimizing function  $(V(\alpha, \beta, \gamma, \hat{N}) - \hat{V})^2 + (D(\alpha, \beta) - \hat{D})^2 + (H(\gamma) - \hat{H})^2$  for  $\theta = (\alpha, \beta, \gamma)$ , using the algorithm of Nelder and Mead with  $\theta = (5, \hat{D}, 3)$  as initial guesses. The obtained estimates were used as initial guesses for the Newton-Raphson algorithm, which was used for finding the final estimates. The required integrals and differentials were evaluated numerically. The basal-area weighted quantile function was solved using a simple up and down algorithm. R-software (R Development core team 2006) was used for computations.

### 3.3 Evaluation of results

The results were first evaluated using the proportion of plots where the system of equations was feasible. For plots with feasible solution, the obtained stand description was used to compute the total volumes for trees with diameter above a predefined lower diameter limit  $L$ . The volume was computed as

$$\hat{V}_L = \hat{N} \int_L^{\infty} f(x|\hat{\alpha}, \hat{\beta}) v(x, h(x|\hat{\gamma})) dx.$$

We used values 10, 15 and 20 cm as the values for  $L$ . As criteria of comparison, we used the bias and RMSE of these volumes, together with the bias and RMSE of the predicted stand characteristics. In addition, we evaluated the predictions visually and report graphs of selected sample plots of our data.

## 4. RESULTS

### 4.1 Prediction models for stand characteristics.

The models for plot volume, number of stems, basal area median diameter and corresponding height are:

$$\ln V = 0.134 + 1.202 \ln(f - h_{50}) + 0.198 \sqrt{f - veg} + 0.114 \ln(l - veg)$$

( $R^2 = 0.924$ , s.e. = 0.155, relative s.e. 16.3%)

$$\ln N = 7.803 - 1.027 \ln(f_{-h_{95}}) + 0.251 \sqrt{f_{-veg}}$$

$$+ 7.988 \left( \frac{1}{f_{-i_{50}}} \right) - 0.319 \ln(l_{-p_{20}})$$

(R<sup>2</sup> = 0.497, s.e. = 0.311, relative s.e. 30.8%)

$$\ln H = -26.075 + 5.747 \ln f_{-h_{95}} + 3.581 \ln f_{-h_{40}}$$

$$+ 38.371 \frac{1}{f_{-h_{60}}} + 0.605 \ln l_{-veg} + 4.907 \ln l_{-h_{50}}$$

(R<sup>2</sup> = 0.783, s.e. = 0.119, relative s.e. 12.9%)

$$\ln D = 2.697 - 2.605 \frac{1}{f_{-i_{50}}} - 37.812 \frac{1}{f_{-p_{20}}}$$

$$+ 2.004 \ln f_{-h_{50}} - 1.231 \ln f_{-h_{40}}$$

(R<sup>2</sup> = 0.860, s.e. = 1.230, relative s.e. 8.7%)

where *f* or *l* denotes the laser pulse type (first or last pulse), *h<sub>p</sub>* denotes the height at which *p*% of the height distribution has accumulated, *veg* is the proportion of vegetation hits, *i<sub>50</sub>* is the 50<sup>th</sup> percentile of intensity reflection, and *p<sub>20</sub>* is the proportion of laser hits which is accumulated at the height of 20%.

The RMSEs of predicted volume, number of stems, basal area median diameter and the corresponding height (Table 2) were at the same level as they have been in previous studies with laser scanning data (Holmgren, 2004; Næsset, 2002; 2004; Maltamo et al. 2007). The predictions of the number of stems and total volume are slightly downward biased.

	RMSE		Bias	
	Absolute	%	Absolute	%
<i>H</i> , m	1.22	8.70	0.00	-0.01
<i>D</i> , cm	2.35	12.96	-0.15	-0.80
<i>N</i> , ha <sup>-1</sup>	279.8	31.00	-35.30	-3.91
<i>V</i> , m <sup>3</sup> ha <sup>-1</sup>	20.02	16.29	-1.64	-1.33

Table 2. RMSE and bias of predicted stand characteristics in the data of feasible solutions (211 plots).

#### 4.2 Recovery of stand description

A stand description of the assumed form could be found for almost all stands. In only two out of the 213 plots, the system of equations was infeasible. In addition, the resulting distribution was highly peaked (recovered shape parameter  $\alpha > 20$ ) for 2 stands.

The accuracy of predicted volume above diameters 10, 15 and 20 cm is given in table 3. The RMSE of volume above 10 cm is slightly lower than that of total volume in absolute terms, but slightly higher in relative terms. For higher diameter limits, the RMSE gets higher both in absolute and relative terms, being 24.33 m<sup>3</sup>ha<sup>-1</sup> (42%) for the volume of trees above 20 cm in diameter. The volumes are slight underestimates in all cases; an expected result because the predictions of stand characteristics were underestimates, too.

	RMSE		Bias	
	Absolute	%	Absolute	%
<i>V</i> <sub>10</sub> , m <sup>3</sup> ha <sup>-1</sup>	19.70	16.67	-0.79	-0.67
<i>V</i> <sub>15</sub> , m <sup>3</sup> ha <sup>-1</sup>	22.20	22.70	-2.20	-2.25
<i>V</i> <sub>20</sub> , m <sup>3</sup> ha <sup>-1</sup>	24.33	42.76	-1.72	-3.02

Table 3. RMSE and bias of volumes above 10, 15 and 20 cm diameter limit in the data of feasible solutions (211 plots).

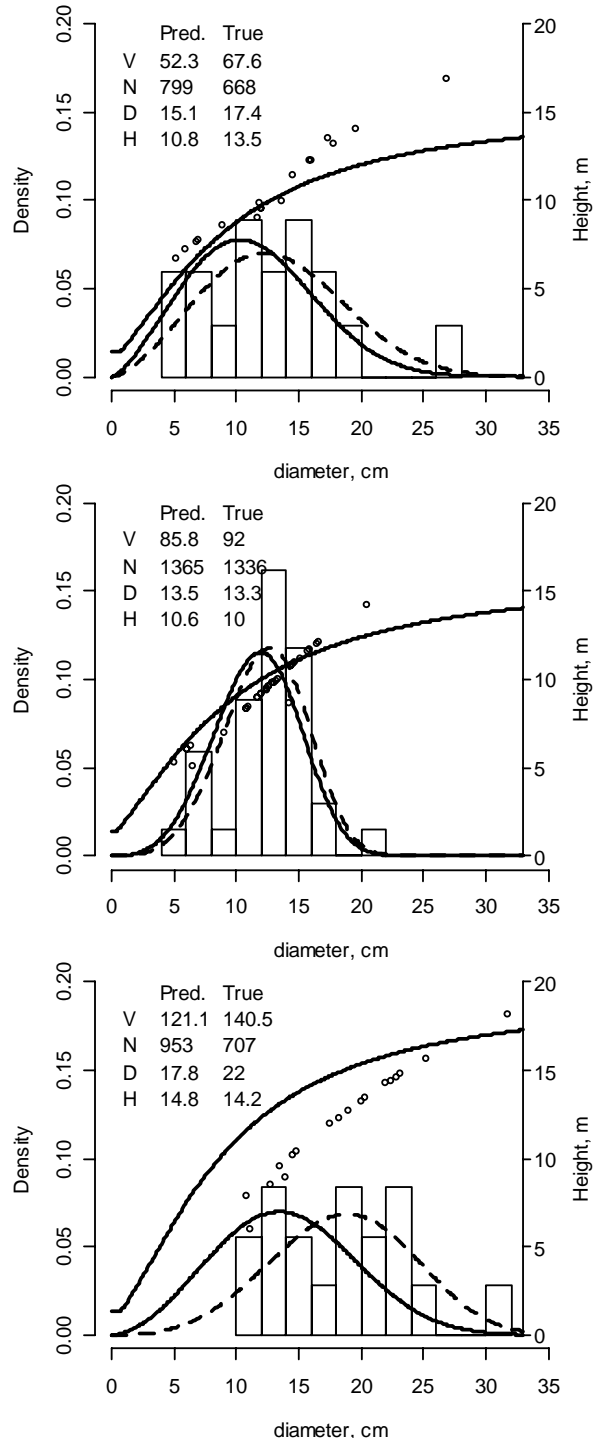


Figure 1. Examples of true and recovered stand descriptions. The histogram shows the observed diameter distribution and the open circles the tree heights. The dashed line shows a Weibull-distribution fitted to the observed data using ML. The solid lines show the recovered diameter distribution and H-D curve.

Visual evaluation of predictions showed that they are, in most cases, fair (the two uppermost stands in Figure 1). However, especially errors in predicted basal area median diameter made the location of the distribution inaccurate, causing large errors in the predicted structure of the growing stock (The lowest plot in Figure 1). The recovered H-D curve did not agree very well with the “true” heights, which are based on plot-specific Näslund’s curve. These differences result from different model forms.

## 5. DISCUSSION

This study generalized the parameter recovery of diameter distribution to the recovery of stand description, including stand density and models for diameter distribution and H-D curve. Such a method is useful in laser scanning approaches, where accurate predictions are obtained for characteristics that depend on both tree diameter and height. In the proposed parameter recovery approach, diameter distribution and H-D curve of assumed forms are recovered so that the obtained stand description is compatible with predicted stand characteristics. The approach appears to be a reasonable alternative for obtaining estimates of stand structure using laser data.

With the stand description of this study, the parameter recovery would be possible with any four stand characteristics, given that they are mathematically related to the stand description and at least one of them depends on tree height. In addition, the parameters of the assumed stand description can further be increased if the number of predicted parameters is increased. Total volume, stand density, basal area median diameter and corresponding height were selected as the variables being predicted because they can be rather accurately predicted using laser data. However, attention was also paid to the correlation of the prediction errors. Using two accurately predicted variables with highly correlated prediction errors (e.g. volume and basal area) might have lead to worse results than using variables with lower accuracy but less correlated prediction errors (e.g. volume and the number of stems). However, analysis of this requires further efforts.

A problem with the proposed approach is that a solution to the system of equations cannot necessarily be found. In our data, this happened with two plots. For those plots, a stand description that simultaneously satisfies the three equations does not exist. This may result from too restricted form of the assumed stand description: three parameters are not enough to realistically describe all possible forms of diameter distribution and H-D curve. A solution would be to assume a more flexible stand description, including more parameters and, for example, allowing bimodal diameter distributions. However, the number of parameters in the stand description would be increased, and that would also require more ALS-predicted stand variables. Another possible cause for infeasibility is an illogical set of predictions. Especially, if the models are based on small plots, sampling errors may cause the modeling data to include such combinations of stand variables that cannot be described with the simple description we used. This may affect also to the prediction models, making them it more likely for them to give illogical predictions. To prohibit this effect, the plot size of the modeling data should be large enough. However, the above solutions do not guarantee feasibility, and for operational utilization, predictions are required for all plots. One

alternative would be using the solution that minimizes the sum of squared distance between the predicted values and the values based on the stand description. Appropriate weights should be used for different terms to make them comparable. The solution would no more be compatible but, however, as compatible as possible.

The approach presented in this study would allow conversion of old inventory results, where stand mean characteristics are predicted by using ALS data (e.g. Næsset, 2002, 2004; Holmgren, 2004; Suvanto et al.; 2005), to stand descriptions as used in this study. Compared to earlier ALS data based diameter distribution studies (Gobakken and Næsset, 2004;2005; Maltamo et al. 2006;2007) the benefit of the proposed method is that no additional modelling is needed since the approach recovers possible parameter values without the use of tree level data. Finally, this study also confirms the result by Maltamo et al. (2007) that in the case of ALS data, basal area diameter distributions are not needed to obtain accurate volume estimates.

This study considered stand description in Sots pine stands. It was assumed that it is known prior to prediction that these forests are pure pine stands (pine proportion of volume over 90%). In practical applications, information concerning species proportions could be obtained by using non-parametric species-specific forest inventory approach which utilises both ALS data and digital aerial photographs (Packalén and Maltamo, 2007). This kind of method would also recognise stands where pine exists but it is not the main tree species. The usability of stand description approach of this study could be worse in some of those stands, i.e., the system of equations might be infeasible due to inconsistency with mean characteristics of pine. Crude information about species proportions could also be obtained from old stand register data or site fertility class. Photo interpretation could also be used for estimation of species proportions, as it was done in Norway (e.g. Næsset, 2004)

This study expected that the diameter distribution of a pure pine stand can be characterized with a unimodal function. In managed stands, this is, in fact, true in most cases. In unmanaged stands, conservation areas, or forest reserves, the situation would be more complex since fire ecology and gap dynamics may result in diameter distributions of a multimodal shape (Esseen et al. 1997).

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