GLOBAL REGISTRATION OF NON STATIC 3D LIDAR POINT CLOUDS: SVD FACTORISATION AND ROBUST GPA METHODS

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ABSTRACT:

The paper reports two analytical methods capable to reliably perform the simultaneous global registration of non static 3D LiDAR point clouds, and investigates their applicability by analysing the results of some preliminary numerical examples. The first method, proposed by Xiao (2005), and Xiao et al. (2006), apply a direct SVD factorisation to non static 3D fully overlapping point clouds characterised by target points. The factorisation is applied to a matrix, sequentially containing by rows the coordinates of the corresponding targets present in the cloud scenes. Besides the rigid transformation parameters, a number of shape bases is determined for each point cloud, whose linear combination describes the dynamic component of the scenes. A linear closed-form solution is finally obtained, enforcing linear constraints on orthonormality of the rigid rotations and on uniqueness of the linear bases. The second method analysed is the so called “Robust Generalised Procrustes Analysis”, recently proposed by the authors. To overcome the lack of robustness of Generalised Procrustes Analysis, a progressive sequence inspired to the “forward search” was developed. Starting from an initial partial point cloud configuration satisfying the LMS principle, the configuration is updated, point by point, till a significant variation of the registration parameters occur. This reveals the presence of non stationary points among the new elements just inserted, that are therefore not included in the registration process. Both methods are capable to correctly determine the registration parameters, when compared to the commonly applied “two steps method”, where the registration of deformable shapes is biased by non - rigid deformation components.

1. INTRODUCTION

In some papers published a few years ago (*e.g.* Beinat and Crosilla, 2001), the authors proposed the Generalised Procrustes Analysis to perform a high precision simultaneous registration of multiple partially overlapping 3D point clouds acquired with terrestrial laser scanning devices. The proposed technique requires for each point cloud the matching of a sufficient number of artificial targets, eventually pre-signalised on the object surface to survey. Furthermore, the same authors have recently proposed (Beinat, Crosilla, Sepic, 2006) an automatic registration technique that does not require any manual matching of the target points, but that instead uses the morphological or the radiometric local variations on the surveyed surface. The method, by studying the differential properties of the sampled point surface, computes at first the local values of the Gaussian curvature, then applies a topological research to define for each point cloud the corresponding zones characterised by the same curvature values. By applying an SVD algorithm, it is possible to automatically solve a coarse registration followed by an Iterative Closest Point (ICP) global refinement.

Both registration approaches can be correctly applied if the object does not change its shape during the survey of the complete sequence of point clouds. That is, the registration problem consists in the definition of the correct similarity transformation parameters for each point cloud. On the other hand, registration and modelling of dynamic point cloud scenes is a prominent problem for robot navigation, for reconstruction of deformable objects, and for monitoring environmental phenomena. The recovery of the resulting shapes can be regarded as a combination of rigid similarity transformations of the 3D point clouds and unknown non - rigid deformations. In the literature (e.g. Dryden and Mardia, 1999), the problems solution is usually carried out in two consecutive steps. The first step registers the point clouds by similarity transformation, considering the deformable shapes as contaminated by Gaussian noise. The second step determines the linear deformable model of the registered shapes by applying Principal Component Analysis (PCA) to the registration residuals. Proceeding in this way, the registration of deformable shapes is biased by non - rigid deformation components. It is therefore necessary to apply some procedures that make possible to reliably estimate the roto-translation components, and the deformable shapes.

The paper synthetically describes two methods recently proposed in the literature, and analyses the results obtained for the registration of a 3D scene characterised by static and dynamic elements. The first method, introduced by Xiao (2005), solves the combined problem of registration and dynamic shape modelling by a direct factorisation of the points coordinate matrix, containing by rows for each acquired scene the 3D sampled model point coordinates. The method works well when the dynamic object shape can be described by a linear combination of a small number of shape bases, that, together with the similarity transformation parameters for each cloud, are the unknown elements of the joint registration and shape modelling problem. The second method proposed (Crosilla, Beinat; 2006) represents a robust solution of the Generalised Procrustes problem. The described algorithm derives from the Robust Regression Analysis based on the Iterative Forward Search approach proposed by Atkinson and Riani (2000), and Cerioli and Riani (2003). The procedure starts from a partial point configuration only containing stationary points. At each iteration, the transformation parameters are determined, and the initial dataset is enlarged by one or more new points, till a significant variation of the transformation parameters occur. At this point the method allows to identify in the various configurations the remaining non stationary points that represent the dynamic component of the scene.

2. JOINT REGISTRATION AND SHAPE MODELING BY SVD FACTORISATION

The method proposed by Xiao (2005) and Xiao et al. (2006) is based on the fact that the shape $\mathbf{S}_i$ of a deforming object, or of a non static scene at epoch i (i = 1…N), can be modelled as a linear combination of k shape bases $\mathbf{B}_k$ (k= 1…K). Each basis is a (D×P) matrix, where D is the space coordinate dimension and P is the number of the points. According to these positions, we can write that:

$$\mathbf{S}_i = \sum_{k=1}^{K} l_{ik}\mathbf{B}_k \tag{1}$$

where l_{ik} is a coefficient to apply to the $\mathbf{B}_k$ basis in order to define the shape **S** at epoch i. Of course, every shape $\mathbf{S}_i$ can be measured from a different point of view, (eventually) with a different scale, and the coordinates of each shape model may be defined with respect to a different coordinate system. Therefore, the measured shape $\mathbf{W}_i$ can be considered as a similarity transformation of the shape $\mathbf{S}_i$, that is

$$\mathbf{W}_i = c_i\mathbf{R}_i\mathbf{S}_i + \mathbf{t}_i1' \tag{2}$$

where c_i is a non zero scalar, $\mathbf{R}_i$ is a (D×D) rotation matrix, $\mathbf{t}_i$ is a translation vector and **1** is a unit vector. Combining formula (1) and (2), matrix $\mathbf{W}_i$ can be expressed as

$$\mathbf{W}_i = \begin{pmatrix}\mu_{i1}\mathbf{R}_i & \cdots & \mu_{ik}\mathbf{R}_i & \mathbf{t}_i\end{pmatrix}\begin{pmatrix}\mathbf{B}_1' & \cdots & \mathbf{B}_k' & \mathbf{1}\end{pmatrix}'$$

Now, considering all the N measurement epochs, a (DN×P) matrix **W** can be defined. Of course matrix **W** can be considered as a result of the following matrix expression

$$\mathbf{W} = \mathbf{MB} + \mathbf{t1}' \tag{3}$$

Where **M** is a (DN×DK) matrix

$$\mathbf{M} = \begin{pmatrix}\mu_{11}\mathbf{R}_1 & \cdots & \mu_{1k}\mathbf{R}_1 \\ \vdots & & \vdots \\ \mu_{N1}\mathbf{R}_N & \cdots & \mu_{Nk}\mathbf{R}_N\end{pmatrix}$$

B is a (DK×P) basis matrix $\mathbf{B} = \begin{pmatrix}\mathbf{B}_1' & \cdots & \mathbf{B}_k'\end{pmatrix}'$ and **t** is a (DN×1) translation vector $\begin{pmatrix}\mathbf{t}_1' & \cdots & \mathbf{t}_N'\end{pmatrix}'$.

The joint registration and shape modelling problem proposed by Xiao et al. (2006), considers at this point a direct factorization of matrix **W**. Before doing so, all the point coordinates are translated into a barycentral system, so to neglect the translation components. From now on, let **W** be the coordinate matrix, where the coordinates of each sampled point cloud are referred to the corresponding centroid. Next step is to proceed to the Singular Value Decomposition (SVD) of **W**, so to obtain a factorization that can be written as $\mathbf{W} = \tilde{\mathbf{M}}\tilde{\mathbf{B}}$.

The rank of **W** is min(DK, DN, P). Since generally DN>DK and P>DK, the SVD of **W** makes possible to determine K, that is the number of shape bases required to describe the shape variation model. In fact, if rank(**W**)=DK, than K=rank(W)/D.

Furthermore, the SVD of **W** allows to obtain at first a (DN×DK) matrix $\tilde{\mathbf{M}}$, and a (DK×P) matrix $\tilde{\mathbf{B}}$. Matrices **M** and **B**, containing the unknown terms of the problem reported in Equation (3), can be determined by applying a further unknown linear "corrective transformation" (DK×DK) matrix **G** to the matrices $\tilde{\mathbf{M}}$ and $\tilde{\mathbf{B}}$, that is

$$\mathbf{M} = \tilde{\mathbf{M}}\mathbf{G} \qquad \mathbf{B} = \mathbf{G}^{-1}\tilde{\mathbf{B}} \tag{4}$$

in order to satisfy:

$$\mathbf{W} = \tilde{\mathbf{M}}\mathbf{G}\mathbf{G}^{-1}\tilde{\mathbf{B}} = \mathbf{MB}$$

Matrix **G** can be partitioned into the following K sub-matrices of size (DK×D):

$$\mathbf{G} = (\mathbf{G}_1 \ldots \mathbf{G}_K)$$

Sub-matrices $\mathbf{G}_k$ (k=1...K) satisfy the following property:

$$\tilde{\mathbf{M}}\mathbf{G}_k = \begin{pmatrix}\tilde{\mathbf{M}}_1 \\ \vdots \\ \tilde{\mathbf{M}}_N\end{pmatrix}\mathbf{G}_k = \begin{pmatrix}\mu_{1k}\mathbf{R}_1 \\ \vdots \\ \mu_{Nk}\mathbf{R}_N\end{pmatrix} \tag{5}$$

Now, let $\mathbf{Q}_k = \mathbf{G}_k\mathbf{G}_k'$ be a (DK×DK) matrix. Then, from Eq. (5) it is possible to consider the following general condition:

$$\tilde{\mathbf{M}}_i\mathbf{Q}_k\tilde{\mathbf{M}}_j' = \mu_{ik}\mu_{jk}\mathbf{R}_i\mathbf{R}_j' \qquad (i,j = 1\ldots N) \tag{6}$$

that has to be considered in order to satisfy two fundamental constraints:

1. orthonormality of the rotations
2. uniqueness of the shape bases.

The first constraint is satisfied by the following condition:

$$\tilde{\mathbf{M}}_i\mathbf{Q}_k\tilde{\mathbf{M}}_i{}' = \mu_{ik}^2\mathbf{I}_{(dxd)} \qquad \text{(i=1..N)} \tag{7}$$

As an example for D =3, since $\mathbf{Q}_k$ is symmetric, and due to the presence of the unknown term μ_{ik}^2, for each submatrix $\tilde{\mathbf{M}}_i$ (i=1...N), condition (7) generates the following system of linear equations

$$\begin{aligned}&\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_i^{(1)}{}' - \tilde{m}_i^{(2)}\mathbf{Q}_k\tilde{m}_i^{(2)}{}' = 0\\&\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_i^{(1)}{}' - \tilde{m}_i^{(3)}\mathbf{Q}_k\tilde{m}_i^{(3)}{}' = 0\\&\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_i^{(2)}{}' = 0\\&\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_i^{(3)}{}' = 0\\&\tilde{m}_i^{(2)}\mathbf{Q}_k\tilde{m}_i^{(3)}{}' = 0\end{aligned} \tag{8}$$

where $\tilde{m}_i^{(1)}$, $\tilde{m}_i^{(2)}$, $\tilde{m}_i^{(3)}$ are the first, second and third row of the (D×DK) sub-matrix $\tilde{\mathbf{M}}_i$.

Enforcing the orthonormality constraints alone, is not enough in the case in which a deformation of the point clouds occurs. It is

therefore necessary to enforce also the second constraints that guarantee the uniqueness of the bases.
The problem can be solved analysing the independence properties of the measured shape random samples. That is, it is necessary to determine K measured shapes that contain independent deformable shapes. This can be done by measuring the condition number for all the possible permutation sets of (DK×P) sub-matrices of **W**, and by choosing the set that minimizes that value (Xiao et al., 2006). Smaller condition number means higher independence. The deformable shapes contained in the selected K measurement shapes are considered as the unique bases. Since scaling does not influence the independence of the shapes, the scalars μ_{ik}^2 are absorbed into the bases, and then the chosen K measurements are simply the rotated bases.
Denoting the K selected basis measurements as the first K measurements in the barycentral coordinate matrix **W**, it follows that $\mathbf{W}_i = \mathbf{R}_i\mathbf{B}_i$ (i=1...K). The corresponding coefficients are thus:

$$\begin{aligned} &\mu_{ii} = 1 \qquad (i = 1\ldots \mathrm{K}) && (9)\\ &\mu_{ij} = 0 \qquad (i,j = 1\ldots \mathrm{K}\ ;\ i \neq j) \end{aligned}$$

According to Equations (7), and (9) the uniqueness of the bases is satisfied by the following conditions

$$\tilde{\mathbf{M}}_i\mathbf{Q}_k\tilde{\mathbf{M}}_j{}' = 0_{(dxd)} \qquad (i = 1..\mathrm{K}\ ; j = 1..\mathrm{N};\ i \neq k) \qquad (10a)$$

$$\tilde{\mathbf{M}}_i\mathbf{Q}_k\tilde{\mathbf{M}}_j{}' = \mathbf{I}_{(dxd)} \qquad (i=j=k) \qquad (10b)$$

As in the previous example for D=3, for each matrix product reported in (10a), we can write the following set of linear equations

$$\begin{aligned} &\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_j^{(1)}{}' = 0\\ &\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_j^{(2)}{}' = 0\\ &\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_j^{(3)}{}' = 0\\ &\tilde{m}_i^{(2)}\mathbf{Q}_k\tilde{m}_j^{(2)}{}' = 0\\ &\tilde{m}_i^{(2)}\mathbf{Q}_k\tilde{m}_j^{(3)}{}' = 0\\ &\tilde{m}_i^{(3)}\mathbf{Q}_k\tilde{m}_j^{(3)}{}' = 0 \end{aligned} \qquad (11)$$

While, for each matrix product reported in (10b) it follows the following equations

$$\begin{aligned} &\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_j^{(1)}{}' = 1\\ &\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_j^{(2)}{}' = 0\\ &\tilde{m}_i^{(1)}\mathbf{Q}_k\tilde{m}_j^{(3)}{}' = 0\\ &\tilde{m}_i^{(2)}\mathbf{Q}_k\tilde{m}_j^{(2)}{}' = 1\\ &\tilde{m}_i^{(2)}\mathbf{Q}_k\tilde{m}_j^{(3)}{}' = 0\\ &\tilde{m}_i^{(3)}\mathbf{Q}_k\tilde{m}_j^{(3)}{}' = 1 \end{aligned} \qquad (12)$$

Systems (8), (11), and (12) enlarged for all possible indexes i and j make possible to find an inconsistent system of linear equations in the unknown terms of the symmetric matrix $\mathbf{Q}_k$ upper triangle that can be solved by least squares.

Once $\mathbf{Q}_k$ is determined, to compute $\mathbf{G}_k$, it is necessary to apply an SVD to matrix $\mathbf{Q}_k$, since $\mathbf{Q}_k = \mathbf{G}_k\mathbf{G}'_k$. This decomposition allows to determine matrix $\mathbf{G}_k$ apart for an arbitrary (D×D) orthonormal transformation **F**, since $\mathbf{G}_k\mathbf{F}\mathbf{F}'\mathbf{G}'_k = \mathbf{Q}_k$. This ambiguity is due to the fact that matrices $\mathbf{G}_k$ (k=1...K) are independently estimated under different coordinate systems (Xiao et al., 2006). Therefore matrices $\mathbf{G}_k$ (k=1...K) have to be transformed under a unique reference system. Before doing so, it is necessary to determine for each k the rotation matrices $\mathbf{R}_i$ relating to each scene.
Remembering that $\tilde{\mathbf{M}}_i\mathbf{G}_k = \mu_{ik}\mathbf{R}_i$ (i=1...N), since $\mathbf{R}_i$ is orthonormal, i.e. $\|R\| = 1$, than $\mathbf{R}_i = \pm\dfrac{\mathbf{M}_i\mathbf{G}_k}{\|\mathbf{M}_i\mathbf{G}_k\|}$.
In this way K sets of rotation matrices $\mathbf{R}_i$ (i=1...N) are computed. Specifying one of the sets as the reference one, an Ordinary Procrustes Analysis (OPA) is applied to all the other sets so to align them to the selected one. The result furnished by OPA makes also possible to transform $\mathbf{G}_k$ (k=1…K) under a common coordinate system, and in this way the searched transformation matrix **G** is achieved.
The coefficients are then computed by (5), and the shape bases **B** are recovered by (4). In this way the shape of a non static scene at epoch i can be finally determined by (1).

3. ROBUST GENERALISED PROCRUSTES ANALYSIS

Generalised Procrustes Analysis (GPA) is a well known multivariate technique used to provide multiple and simultaneous L.S. similarity transformations of $\mathrm{M} \geq 2$ data sets composed of P corresponding D-dim points, whose coordinates are referred to $\mathrm{M} \geq 2$ different reference frames, and characterised by measurement noise. The following least squares objective function has to be satisfied:

$$\begin{aligned} \mathrm{S} = \mathrm{tr}\sum_{i<j}^{\mathrm{M}}&\left[\left(\mathrm{c}_i\mathbf{X}_i\mathbf{R}_i + \mathbf{1}\mathbf{t}'_i\right) - \left(\mathrm{c}_j\mathbf{X}_j\mathbf{R}_j + \mathbf{1}\mathbf{t}'_j\right)\right]' \cdot\\ &\cdot\left[\left(\mathrm{c}_i\mathbf{X}_i\mathbf{R}_i + \mathbf{1}\mathbf{t}'_i\right) - \left(\mathrm{c}_j\mathbf{X}_j\mathbf{R}_j + \mathbf{1}\mathbf{t}'_j\right)\right] = \min \end{aligned} \qquad (13)$$

under the orthogonality condition $\mathbf{R}'\mathbf{R} = \mathbf{I}$; where $\mathbf{X}_1 \ldots \mathbf{X}_\mathrm{M}$ are $\mathrm{M} \geq 2$ data matrices of size ($\mathrm{P} \times \mathrm{D}$), each one containing the coordinates of the same set of P corresponding points defined in M different reference frames; **1** is the (P×1) auxiliary unitary vector; $\mathbf{t}_j$, $\mathbf{R}_j$ and c_j are the unknowns (j=1 … M), *i.e.* the (D×1) j^{th} translation vector, the (D×D) j^{th} rotation matrix, and the j^{th} isotropic scale factor, respectively.
The solution of Equation (13) represents the GPA problem described by Kristof and Wingersky (1971), Gower (1975), ten Berge (1977), and Goodall (1991).
This problem has an alternative formulation. Said $\mathbf{X}_i^{\mathrm{p}} = \mathrm{c}_i\mathbf{X}_i\mathbf{R}_i + \mathbf{1}\mathbf{t}'_i$, the following measures:

$$\sum_{i<j}^{\mathrm{M}}\left\|\mathbf{X}_i^{\mathrm{p}} - \mathbf{X}_j^{\mathrm{p}}\right\|^2 = \sum_{i<j}^{\mathrm{M}}\mathrm{tr}\left(\mathbf{X}_i^{\mathrm{p}} - \mathbf{X}_j^{\mathrm{p}}\right)'\left(\mathbf{X}_i^{\mathrm{p}} - \mathbf{X}_j^{\mathrm{p}}\right) \qquad (14)$$

$$\mathrm{M}\sum_{i}^{\mathrm{M}}\left\|\mathbf{X}_i^{\mathrm{p}} - \mathbf{H}\right\|^2 = \mathrm{M}\sum_{i}^{\mathrm{M}}\mathrm{tr}\left(\mathbf{X}_i^{\mathrm{p}} - \mathbf{H}\right)'\left(\mathbf{X}_i^{\mathrm{p}} - \mathbf{H}\right) \qquad (15)$$

are perfectly equivalent (e.g. Borg and Groenen, 1997), where **H** is the unknown centroid. Therefore Eq. (15), instead of Eq. (14), can be minimised so to determine the unknowns $\{c, \mathbf{R}, \mathbf{t}\}_j$ (j= 1…M) that make it possible to iteratively compute the final $\mathbf{X}_i^{\mathrm{p}}$ (i=1…M).

Matrix $\hat{\mathbf{H}} = \frac{1}{\mathrm{M}}\sum_{i=1}^{\mathrm{M}} \mathbf{X}_i^{\mathrm{p}}$ represents the LS estimate of **H**. Note that $\mathbf{H} + \mathbf{E}_i = \mathbf{X}_i^{\mathrm{p}}$, where $\mathrm{vec}(\mathbf{E}_i) : N\{0, \mathbf{\Sigma} = \sigma^2(\mathbf{Q}_n \otimes \mathbf{Q}_k)\}$ and σ has a factored structure.

In the current algorithm implementation of the Robust Generalised Procrustes problem solution, the procedure starts from a partial point configuration containing only stationary data. At each iteration, the initial dataset is enlarged by one or more points, till a significant variation of the transformation parameters occurs.

In order to define the initial configuration subset X^i of X, *i.e.* the one containing stationary data, it is necessary to compute the LS estimate of the corresponding centroid H^i, and consequently determine the similarity transformation parameters for all the j = 1 … M data sub-matrices X_j^i:

$$\hat{H}^i = \frac{1}{\mathrm{M}}\sum_{j=1}^{\mathrm{M}}\left(c_j^i X_j^i \mathbf{R}_j^i + \mathbf{1}\mathbf{t}_j^{i\,\prime}\right) \tag{16}$$

where $\hat{H}^i$ corresponds to the LS estimate of the unknown H^i. This procedure is repeated for every i = 1 … $\binom{\mathrm{P}}{\mathrm{S}}$ possible configuration subset X^i, where S is the number of points forming the subset.

Now, the global pseudo-centroid is computed by applying the transformation parameters, relative to the i-th data submatrix X_j^i, to the full corresponding $\mathbf{X}_j$, obtaining $\mathbf{X}_j^{P(i)}$:

$$\tilde{\mathbf{H}}^i = \frac{1}{\mathrm{M}}\sum_{j=1}^{\mathrm{M}}\left(c_j^i \mathbf{X}_j^i \mathbf{R}_j^i + \mathbf{1}\mathbf{t}_j^{i\,\mathrm{T}}\right) = \frac{1}{\mathrm{M}}\sum_{j=1}^{\mathrm{M}} \mathbf{X}_j^{P(i)} \tag{17}$$

To define the initial subset X^i containing stationary points, the least median of squares (LMS) principle is applied (Rousseauw, 1984). As well known, this regression method can normally reach a break down point as high as 50%: among all the possible configuration subsets X^i, the one satisfying the following LMS condition is chosen as the initial one:

$$\mathrm{med}\,\mathrm{diag}\sum_{j=1}^{\mathrm{M}}\left(\mathbf{X}_j^{P(i)} - \tilde{\mathbf{H}}^i\right)\left(\mathbf{X}_j^{P(i)} - \tilde{\mathbf{H}}^i\right)^{\mathrm{T}} = \min \tag{18}$$

This initial subset is then enlarged joining up the point for which:

$$\mathrm{diag}\sum_{j=1}^{\mathrm{M}}\left(\mathbf{X}_j^{P(i)} - \tilde{\mathbf{H}}^i\right)\left(\mathbf{X}_j^{P(i)} - \tilde{\mathbf{H}}^i\right)^{\mathrm{T}} = \min \tag{19}$$

selected from the remaining ($\mathrm{P}-\mathrm{S}$) points of the configuration, not belonging to the initial subset.

The LS estimate of the enlarged partial centroid $H^{i(+1)}$, and the S-transformation parameters for the M sub-matrices $X_j^{i(+1)}$, are computed again as:

$$\hat{H}^{i(+1)} = \frac{1}{\mathrm{M}}\sum_{j=1}^{\mathrm{M}}\left(c_j^{i(+1)} X_j^{i(+1)} \mathbf{R}_j^{i(+1)} + \mathbf{1}\mathbf{t}_j^{i(+1)\mathrm{T}}\right) = \frac{1}{\mathrm{M}}\sum_{j=1}^{\mathrm{M}} X_j^{i,\mathrm{P}[i(+1)]} \tag{20}$$

Now, Procrustes statistics (Sibson, 1979; Langron and Collins, 1985) is applied to verify whether a significant variation of the S-transformation parameters occurs by enlarging the original selected data subset. To this aim, the total distance between the partial centroid $\hat{H}^i$ and the M sub-matrices $X_j^{i,\mathrm{P}[i(+1)]}$, obtained by applying to the original X_j^i the S-transformation parameters relating to the i(+1) dataset, is computed:

$$\mathbf{G} = \sum_{j=1}^{\mathrm{M}} \mathrm{tr}\left(X_j^{i,\mathrm{P}[i(+1)]} - \hat{H}^i\right)^{\mathrm{T}}\left(X_j^{i,\mathrm{P}[i(+1)]} - \hat{H}^i\right) \tag{21}$$

The following distances are also computed:

$$\mathbf{G}_t = \sum_{j=1}^{\mathrm{M}} \mathrm{tr}\left(X_j^{i,\mathrm{P}[i(+1)]} + \mathbf{1}d\mathbf{t}_j^{\mathrm{T}} - \hat{H}^i\right)^{\mathrm{T}}\left(X_j^{i,\mathrm{P}[i(+1)]} + \mathbf{1}d\mathbf{t}_j^{\mathrm{T}} - \hat{H}^i\right) \tag{22a}$$

$$\mathbf{G}_{tR} = \sum_{j=1}^{\mathrm{M}} \mathrm{tr}\left(X_j^{i,\mathrm{P}[i(+1)]} d\mathbf{R}_j + \mathbf{1}d\mathbf{t}_j^{\mathrm{T}} - \hat{H}^i\right)^{\mathrm{T}}\left(X_j^{i,\mathrm{P}[i(+1)]} d\mathbf{R}_j + \mathbf{1}d\mathbf{t}_j^{\mathrm{T}} - \hat{H}^i\right) \tag{22b}$$

$$\mathbf{G}_{tRc} = \sum_{j=1}^{\mathrm{M}} \mathrm{tr}\left(d\mathrm{c}_j X_j^{i,\mathrm{P}[i(+1)]} d\mathbf{R}_j + \mathbf{1}d\mathbf{t}_j^{\mathrm{T}} - \hat{H}^i\right)^{\mathrm{T}}\left(d\mathrm{c}_j X_j^{i,\mathrm{P}[i(+1)]} d\mathbf{R}_j + \mathbf{1}d\mathbf{t}_j^{\mathrm{T}} - \hat{H}^i\right) \tag{22c}$$

after having taken care of the fact that the translation components relating to the i(+1) subset must be previously reduced by the difference between the centroids of $\hat{H}^{i(+1)}$ and $\hat{H}^i$. These distances are residual distances after a Procrustes transformation. In particular G_t is the residual distance after a translation, G_{tR} is the residual distance after a translation and a rotation, and G_{tRc} is the residual distance after a translation, a rotation, and a scaling.

Assuming a proper first kind error α, and the proper degrees of freedom df_1 and df_2, the rejection of the null hypothesis for the following tests (Langron and Collins 1985):

$$\left\{\frac{\mathrm{G}-\mathrm{G_t}}{\mathrm{G_{tRc}}};\ \frac{\mathrm{G_t}-\mathrm{G_{tR}}}{\mathrm{G_{tRc}}};\ \frac{\mathrm{G_{tR}}-\mathrm{G_{tRc}}}{\mathrm{G_{tRc}}}\right\} > F_{1-\alpha,\,\mathrm{df}_1,\,\mathrm{df}_1} \tag{23}$$

indicates a significant variation of some or of all the transformation parameters at this step, due to the possible entering into the $X_j^{i(+1)}$ datasets of non stationary data.

If the null hypothesis for all the tests is accepted instead, the iterative process continues with the insertion of a further new point $X_j^{i(+2)}$, satisfying Equation 19 within the remaining ones of the dataset.

4. ALGORITHM IMPLEMENTATION AND TESTING

The SVD factorisation, and the Robust GPA methods were implemented in Matlab™, in order to test their capability to correctly register models by using both static and non-static tie-point configurations. The experiments, related to simulated environments, let us to introduce variably modulated measurement noise in the tie-point coordinates.

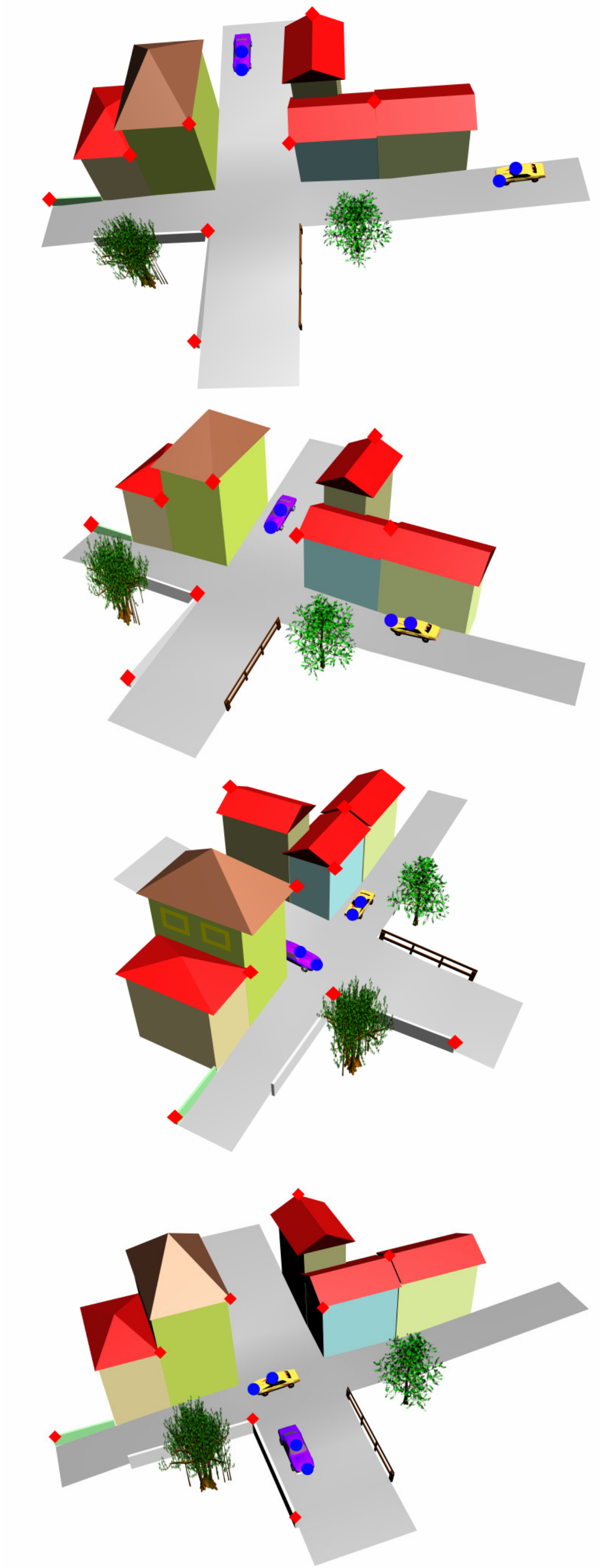

Figure 1a,b,c,d: Simulated models of an urban environment, and static and non-static tie-points for model registration.

As example, we report one of these tests. Figures 1a to 1d depict four models (or scenes) of one reconstructed urban environment, in different reference systems (or poses). Of the 12 tie-points employed for the model alignment, 8 identify static entities (buildings, roofs, walls, roads), and 4 relate to moving objects (cars). Static tie-points are evidenced by red diamond symbols, non static ones by blue circles.

Figure 2: Global registration by Robust GPA

Figure 2 shows the result of the global alignment of the four models of Figure 1, performed by Robust GPA. The method identifies all the non-static tie-points, and treats them as outliers: the global registration is then achieved by way of the largest static tie-point subset, common to all the models.

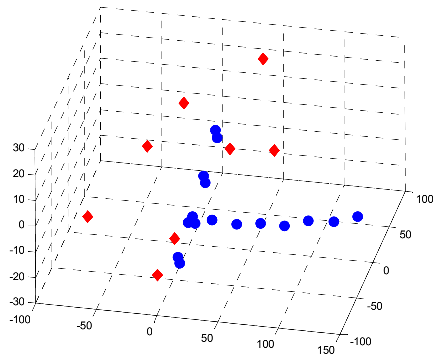

Figure 3: Tie points distribution after Robust GPA registration: static ones (red diamonds) appear precisely overlapped.

Figure 3 shows the tie-point distribution after the registration: non-stationary points are automatically detected, and outlined by blue circle symbols, while static ones are marked by overlapping red diamonds.

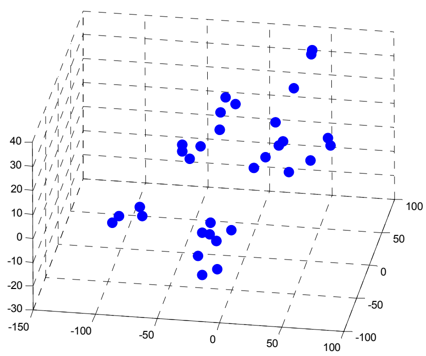

Figure 4: Tie points distribution after Ordinary GPA registration: static tie-points are not overlapped, and the high value of the residuals represents a distorted reconstruction

Figure 4 represents, for comparison, the results obtained performing the registration by an ordinary GPA: non-static tie-points heavily contaminate the registration accuracy by a quantity proportional to their number, displacement length and relative position (leverage effect).
Several experiments were performed varying tie-points location, number, and accuracy. A detailed analysis of the numerical experiments will be presented in a future work.

5. CONCLUDING REMARKS

Our investigations concerning the registration methods for non static models are still under development, nevertheless some clear considerations regarding the methods discussed here can be expressed.
As commonly known, ordinary GPA, although very efficient, may fail in achieving an acceptable registration accuracy due to the presence of outliers or non-stationary data.
On the contrary, the SVD factorisation method (Xiao et al., 2006) reported in the paper, does not exclude, but is capable to employ the non-stationary points for a correct registration process. Moreover it furnishes the geometric bases to reconstruct the deformable shapes. But this method, although robust against measurement noise, introduces a restrictive operative condition: the shape deformations, in whole, must span all the model space dimensions. As mentioned in Section 2, the shape of a deformable object can be regarded as a linear combination of a selected number of shape bases. When at least three points simultaneously move along three different fixed directions in the 3D space, their trajectories form a deformation basis of rank 3. If two points move along fixed directions within a 2D plane, their trajectories form a rank-2 shape basis. If finally one point moves along a fixed direction, its trajectory forms a rank-1 basis. Non-degenerate bases of a 3D non rigid shape are characterized by a full rank 3 and, according to what reported in Section 2, a closed form solution exists enforcing linear rotation, and basis constraints. Degenerate deformations often occur in practice, i.e. some bases are of rank 1 or 2. Relating to the reported example, cars moving independently on a straight plane road refer to rank-1 deformation of the scene. Cars moving along two differently oriented straight plane roads refer to a rank-2 deformation of the scene. Finally, cars moving on two differently oriented straight and slope roads refer to rank-3 non-degenerate deformation of the scene. The solution of degenerate deformations could require further and computationally heavy constraints, or may not exist (Xiao and Kanade, 2004).
Robust GPA overcomes the drawbacks due to insufficient rank deformations providing a correct model registration. If the number of non-static tie-points is less than the LMS breakdown limit of 50%, and the number of the static tie-points is at least equal to the model space dimensions, Robust GPA can represent a valid complement, or a valuable alternative, to SVD factorisation for the deformable shape registration, and for the relative non-stationary components detection.

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