

# AN IMPROVED MODEL FOR ALONG TRACK STEREO SENSORS USING RIGOROUS ORBIT MECHANICS AND NAVIGATION DATA

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## ABSTRACT:

In this paper a major improvement of the generic rigorous sensor model for along track stereo optical satellite sensors which has been developed at UCL over the last three years is introduced, in detail. This improved model is implemented for satellite images where information of the orbit (state vectors) of the acquired satellite is provided, as is usual nowadays. The main achievement is that the precision of the solution is improved along with the accuracy, as the correlation between the exterior parameters is dramatically eliminated. Moreover, it is possible to carry out self calibration without correlation of the exterior orientation parameters with the interior orientation parameters (even for the calculation of the focal length). This model can be solved directly if the attitude information needed for a photogrammetric solution is given or after a calibration process using a small number of GCPs, in order to calculate missing information. The model is evaluated using SPOT5-HRS imagery.

## 1. INTRODUCTION

In recent years, CCD linear array sensors have been widely used to acquire panchromatic and multispectral imagery in pushbroom mode for remote sensing applications. Sensors like SPOT, IKONOS and Quickbird provide not only high resolution, but also along track stereo capability. Moreover these satellites have on-board instruments and autonomous system of positioning and attitude control that will enable high absolute location accuracy.

However, the metadata provided together with the images are focus on solving the orientation of the sensor using specific models which are not compatible with the well known photogrammetry solution based on collinearity equations. As an example, IKONOS and Quickbird use the rational function polynomial model for orientation while SPOT uses the 'viewing geometry model'. These models are something like 'black box' for the researchers because mainly the interior orientation of the sensors is not known at all or is not well established. Moreover, because of the high correlation between the exterior and interior orientation parameters in a photogrammetric solution it is not possible to achieve a precise and more importantly an accurate solution without knowledge of the interior orientation parameters.

But, why a photogrammetric solution is the subject? In a few words, it is a very well known and examined procedure along with the ability of producing more accurate and precise results than the other procedures using less reference data. As an example for SPOT-HRS viewing model it is needed to be known the viewing angles of each of 12000 CCDs while in a photogrammetric solution the rotation angles of the central CCD is enough. Moreover, is more general model and can be used in every optical sensor. Finally, if it is well establish can be used in in-flight and in self-calibration processes.

In this paper an attempt is done in order to achieve a photogrammetric solution which can be used for every optical pushbroom sensor even in case where the interior orientation parameters are not known. This means that we try to develop a model where it is possible not only to calculate the exterior orientation but also the interior orientation of the satellite with acceptable precision and accuracy. It can be done only in case where high accurate navigation data (state vectors) are provided during the acquisition of the images. It is an improvement of

the UCL model (Michalis and Dowman, 2005) which has been developed in UCL since 2002.

## 2. MODEL DESCRIPTION

The simulation of the pushbroom sensor is more complicated than the frame camera model. The scanning effect on the ground is due to the motion of the satellite. The pushbroom model is a kinematic model. A single image consists of a number of framelets which are independent one-dimensional images with their own exterior orientation parameters (Dowman and Michalis, 2003). Thus, in a rigorous sensor model the satellite motion in space should be described as accurately as possible. In other words, a rigorous sensor model should describe the state of the satellite during the acquisition time of the images. Six parameters are enough to establish the state of the satellite at an epoch (time stamp), which are the state vector associated with position and velocity vectors. This final statement leads to the conclusion that for a single pushbroom sensor the simplest model has nine unknown parameters; six to describe the state and three the attitude of the satellite.

On the other hand the model should be a generic one capable to be used in various sensors. Thus, it is based on the collinearity equations which are generic equations relating the image space with the ground space. However, for the simulation of the acquisition geometry of pushbroom images the collinearity equations should be modified as it has already been mentioned.

### 2.1. Modified collinearity equations of pushbroom scanners

A pushbroom image consists of a number of consecutive framelets which are acquired due to the satellite motion. Thus, the collinearity equations are modified in a way where the ground coordinates and the rotations of the perspective center are modelled as a function of time.

$$\begin{bmatrix} 0 \\ y - y_0 \\ -c \end{bmatrix} = \lambda M(t) \begin{bmatrix} X - X_c(t) \\ Y - Y_c(t) \\ Z - Z_c(t) \end{bmatrix}$$

where

$c$  is the focal length

$t$  is the acquisition time of a framelet which is defined in terms of image coordinates

$X, Y, Z$  are the ground coordinates of a point  
 $X_c(t), Y_c(t), Z_c(t)$  are the ground coordinates of the framelet perspective center as a function of time  
 $\lambda$  is a scale factor which varies from point to point  
 $M(t)$  is a 3x3 rotation matrix which brings the ground coordinate system parallel to the framelet coordinate system as a function of time.  
 $y$  is the y-framelet coordinates of the corresponding point  
 $y_0$  is a small offset from the perspective center origin

## 2.2. Fundamental point in the sensor modeling development

In this section the fundamental point in the sensor modelling research is introduced. If a thorough examination is taken, to the form of the second order polynomials (e.g.  $X_c(t) = X_0 + \alpha_1 t + b_1 t^2$ ), it is clearly understood that because the results should be in meters, the units of the coefficient  $\alpha_1$  should be in meters/sec and the  $b_1$  units should be in meters/sec<sup>2</sup>. Definitely this means that the first order coefficient represents the velocity of the satellite on the reference axis and in the same way the second order represents the acceleration on the same axis. For the same reason, the first and the second order coefficients in the rotation angles polynomials, represent the angular velocity and the angular acceleration, accordingly.

As a conclusion, using the notation of equation 1 the  $X_c(t), Y_c(t), Z_c(t)$  should be at least, first order polynomials, representing the position and the velocity of the base point (state vector) while the polynomials  $\omega_c(t), \varphi_c(t), \kappa_c(t)$  at least, constant.

## 2.3. UCL model in Inertial Coordinate system

In this paragraph the UCL sensor model is described in a few words as its results along with the improved model is involved in the evaluation process. The UCL sensor model is developed in Inertial Coordinate where the collinearity equations are modified and combined with orbit determination-propagation methods (Michalis and Dowman, 2005). The fundamental assumption is that Kepler motion is maintained for the acquisition time of the along track images. Different versions of the model are developed based on different orbit determination-propagation methods. In the evaluation process only the version which is based on the Kepler problem (orbit propagation) is used. Using Kepler's equation is the future state of a satellite can be found given the last known position and velocity vectors at a particular time (Bate et al., 1971). In the sensor modeling this equation can be used as follows: Assuming that, the state vector of the principal point of the base framelet of the first image of the along track images sequence is known. Then, using this equation the state vector of the base framelet of the first image is related to the state vectors of principal point of the base framelet of each along track images and even more to the state vectors of all the other framelets in every image. The number of unknown parameters is reduced, as only the state vector of the first along track image should be defined.

## 2.4. Improved UCL model in Geocentric Coordinate system

In the improved model the orbit of the satellite is simulated based on the provided navigation data. The initial research achievements what the improved sensor model should be reach are the following:

- Fitting of the satellite orbit as accurate as possible. The simulation should be carried out in a geocentric coordinate system in order to avoid distortions caused by earth curvature and map projection and to facilitate integration with sources of metadata information that may be available.
- The acquisition time of sensor line is constant. It should be examined if it is possible to calculate it.

- Calculation of the rotation angles of the sensor expressed within the Satellite Coordinate System. As the simulation of the satellite is a geocentric coordinate system the attitude of the satellite regarding this earth system is known.

- The order of the rotations are decided based on a work assumption which is introduced in § 3.2.

- Calculation of the focal length.

- Calculation of the  $y_0$  (offset from the perspective center).

## 2.5. Methodology of the solution of the improved model

### 2.5.1. Exterior orientation

The solution of the model regarding the exterior orientation should be done in two steps.

In the first step the orbit of satellite is simulated taking into consideration the ephemeris of the satellite where the state vectors are given in constant time intervals before, during and after the acquisition time of the images with sufficient accuracy (better than a meter), based on DORIS system (SPOT IMAGE, 2002, page 10) in ITRF90 Coordinate system which is almost identical to WGS84 (for now on WGS84 and ITRF90 means the same). In this step the images themselves are not involved in the solution. The mathematical model of the simulation is based initially in Kepler equation, although the state vectors in ITRF90 are used instead of Inertial Coordinate System. This is a second order equation which is extended in the improved model in sixth order equation trying to involve the perturbations of the orbit along with the difference in motion from Inertial to ITRF90 system. When the Kepler equation is used by itself in ITRF90 the accuracy of the position of the satellite after 90 seconds, which the time interval of the acquisition of the HRS stereo images, is about 3000m (Michalis, 2005), while with the modified model is better than one centimeter. At this moment it should be mentioned here that in Inertial Coordinate System the accuracy of Kepler model is about 30m (Michalis 2005). To conclude the calculated orbit of the satellite in the improved model simulates the

- The Keplerian motion
- Perturbations of the orbit
- The motion on ITRF90 Coordinate system (not Inertial).

After this process the position and the velocity of each line of each image along with the attitude of the navigation satellite system in WGS84 are known.

This means that only the rotation angles of the sensor expressed within the Satellite Coordinate System images are unknowns in the solution. This means also that the well known correlation between X,Y position coordinates and phi and omega rotations respectively does not exist because the position vectors are already known from the above procedure.

The second step is the calculation of the rotation angles. In this process there are two alternatives. The first one is to take the rotation angles directly from the metadata file if the attitude information given can be converted to the rotation angles needed for a photogrammetric solution. In the alternative, GCPs should be used in order to calculate the rotation angles. The number of GCPs is perpendicular to the order of the rotation polynomials.

As an example, in case of SPOT HRS data, the metadata information focuses on solving a specific direct orientation method which is proposed by SPOT IMAGE. Some important information is missing in order to establish a direct

photogrammetric solution. This information comprises the offsets and the rotation angles from the navigation coordinate system to the framelet coordinate system.

### 2.5.2. Interior orientation

On the other hand, having in mind that in the first step the position vector of each line of the along track image is known, it is possible to calculate the interior orientation parameters as it has already mentioned. In this process the focal length the offset of the principal point and the acquisition time of each line is calculated. It is included in the second part of the solution trying to avoid the correlation between the flight height of the satellite and the focal length does not existed, as the flight height is known. Moreover, the strong correlation between state vectors and the acquisition time is also eliminated. As a result a accurate solution of the interior orientation parameter can be established.

## 3. MODEL EVALUATION

In the evaluation process of the improved model SPOT5-HRS images are used. The High Resolution Stereoscopic instrument (HRS) has two telescopes and acquires stereopairs at a 90-second interval, of 120-km swath, with viewing angles of  $\pm 20^\circ$  along the track of the satellite, with a B/H ratio of about 0.8 (Bouillon and Gigord, 2004)

### 3.1. Data sets-Reference data

Two SPOT-HRS data sets are used for this evaluation which is provided under the SPOT Assessment Project (SAP) set up by CNES and ISPRS.

The first one covers an area located around Aix-en-Provence in SE France (Michalis and Dowman, 2004). The ground control points were originally provided by IGN for the OEEPE test of SPOT data and were mainly extracted from 1:25000 maps. A total of 33 reference points were measured in HRS images having a good distribution on the images (Figure 1). The image coordinates are measured manually in 2D. Twelve of them are used as Ground Control Points in this evaluation process while the remaining 21 are used as Check Points.

The second data set covers an area in Bavaria and Austria. A total of 81 points measured with surveying methods are provided where only 41 points have been identified in the images (Poli D, et al., 2005). The exact image coordinates of the points have been measured with unconstrained Least Square Matching, by measuring the points in the master image manually. The distribution on the images is shown in Figure 2.

### 3.2. First step-Orbit simulation accuracy

An accuracy assessment of the orbit simulation used in the improved model compared with the orbit which comes from the Kepler equation, is introduced. This evaluation process takes place in WGS84 coordinate system. It should be mentioned here that in order to meet the fundamental assumption of the Keplerian motion in Inertial Coordinate system should be used. However for convenience in the improved model, WGS84 coordinate system is chosen as all the navigation data are provided in this coordinate system and it does not needed to convert all the navigation data to Inertial Coordinate System, which is very tough procedure. Moreover the transformation of the GCPs to the Inertial Coordinate system is also avoided. The navigation data which are used in this calculation of the satellite orbit is the position and velocity vectors of the satellite measured by the DORIS system every 30 seconds with respect to ITRF90 (almost identical to WGS84). In the improved model, instead of Lagrange interpolation as suggested in the

SPOT SATELLITE GEOMETRY HANDBOOK, (SPOT, 2002), sixth order polynomials are used.

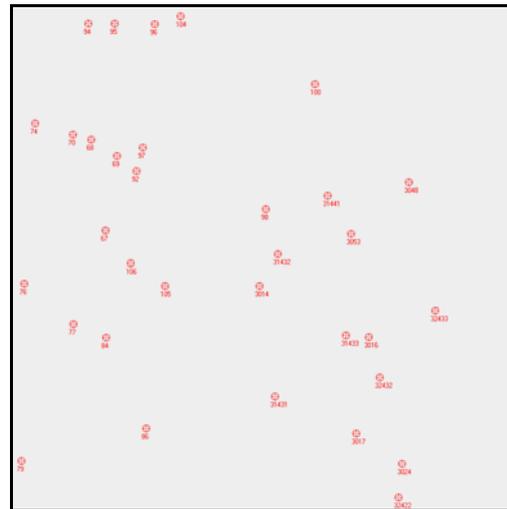


Figure 1. Distribution of Reference Points of Aix-en-Provence test site

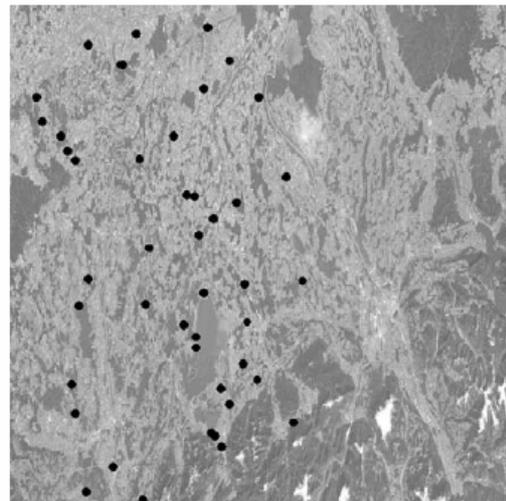


Figure 2. Distribution of Reference Points of Bavaria test site

For evaluation it is used four state vectors of both test sites metadata (90 sec time interval between first and four). Having the first vector as initial value the fourth state vector is calculated using the six order polynomial and the Kepler equation. In table 1 the results from Aix-en-Provence as the results of Bavaria test site are almost the same.

It is obvious that six polynomial model which is extracted for the navigation data themselves can represent very accurately the satellite orbit during the acquisition time than the Kepler model especially in WGS84 coordinate system which is an Earth fixed coordinate system. It should be mentioned again that in inertial coordinate system the difference from the true values in Kepler model is much better (close to 30 meters).

	Difference from the true values-Kepler model	Difference from the true values in improved model
X (m)	539.72	.001
Y (m)	3688.57	.001
Z (m)	-13.76	.005
Vx (m/sec)	12.756	.00002
Vy (m/sec)	81.060	.00001
Vz (m/sec)	-.226	.00003

Table 1. Kepler and six polynomial model accuracy for 90sec interval in WGS84 coordinate system

Having in mind that the difference on the values of the Kepler compared in WGS84 which is an Earth fixed and the Inertial coordinate system we conclude that the main phenomenon that is additionally simulated in the sixth order polynomials is the rotation of the earth relative to the satellite. This means that the rotation angles  $\omega$  and  $\varphi$  (table 1), which are changed due to this relative motion and brings the Earth system parallel to the satellite navigation system is involved in orbit simulation. From table 1 it seems that the  $\omega$  rotation which is correlated to Y coordinate has been influenced more than the other rotations. This statement will be used in the next paragraph where the order of the rotation angles is decided.

### 3.3. Second step-Rotation angles and interior orientation calculation

#### 3.3.1. Rotation angles order

As it has already been mentioned, in the previous paragraph the rotation angle which is influenced more in the is the  $\omega$  rotation. On the other hand, it is known that SPOT satellite is a heavy and quite stable satellite platform which is not need to rotate itself in order to take images in different angles. Moreover, it seems that the HRS subsystem is not rotated during the aquisition of the stereo images. Thus, the following work assumption is done trying to keep the solution as uncorrelated as possible. The assumption is that the rotation angles which bring the navigation satellite system parallel to the framelet coordinate system are constant instead of the rotation  $\omega$  which is a first order rotation.

#### 3.3.2. Unknown parameters

At this stage the images themselves are involved in the solution in order to calculate the unknown parameters. The unknowns are:

- Four coefficients represent the rotation angles ( $\varphi, \kappa$  constant while  $\omega$  is represented with first order polynomial).
- The focal length
- The acquisition time of each line which is assumed constant
- The  $y_0$  offset (across the track).

Thus, the unknown parameters are 7 in total. Four GCPs is needed at least in order to have a solution.

#### 3.3.3. Correlation study of interior orientation parameters

Before going further it is important to make a correlation study of the unknown parameters. For this reason the model is solved in both test sites using as more GCPs as possible. Thus, in case of Aix-en-Provence 33 GCPs are used while in case of Bavaria

39. The correlation coefficients of  $y_0$ ,  $f$  and time interval are shown in tables 2,3 and 4 respectively.

From table 2 it is obvious that the offset across the track is high correlated to the focal length and the rotations. It seems that can not be calculated accurately using this model without additional information.

Unknown (offset across track)	Aix-en-Provence test site (33 GCPs)		Bavaria test site (39 GCPs)	
	HRS-1	HRS-2	HRS-1	HRS-2
$f$	-0.9344	-0.8542	-0.9822	-0.9859
$\omega$	0.9966	0.9920	0.9995	0.9989
$\varphi$	-0.9999	-0.9999	0.9988	-0.9991
$\kappa$	-0.9985	-0.9999	-0.9986	-0.9999
1 <sup>st</sup> order $\omega$	0.3171	0.3137	0.1779	0.16771
Line interval	0.0258	0.0672	-0.0052	-0.0147

Table 2. Correlation coefficients of the across track offset

Unknown ( $f$ )	Aix-en-Provence test site (33 GCPs)		Bavaria test site (39 GCPs)	
	HRS-1	HRS-2	HRS-1	HRS-2
$\omega$	0.3592	0.3869	-0.0368	-0.0154
$\varphi$	0.0451	0.1147	-0.0175	-0.0227
$\kappa$	0.0042	0.0328	0.0076	0.0448
1 <sup>st</sup> order $\omega$	-0.3637	-0.3898	0.0230	0.0067
Line interval	-0.0254	-0.0825	-0.0009	-0.0004

Table 3. Correlation coefficients of the focal length

Unknown (time interval)	Aix-en-Provence test site (33 GCPs)		Bavaria test site (39 GCPs)	
	HRS-1	HRS-2	HRS-1	HRS-2
$f$	-0.0254	-0.0825	-0.0009	-0.0004
$\omega$	-0.0811	-0.2055	0.0347	0.0745
$\varphi$	-0.2456	-0.2515	0.1421	0.1135
$\kappa$	-0.3627	-0.3777	0.0231	0.0059
1 <sup>st</sup> order $\omega$	0.0808	0.2051	-0.3484	-0.0745

Table 4. Correlation coefficients of the time interval

On the other hand from tables 3 and 4, that the focal length and the time acquisition of one line are uncorrelated to the other unknown parameters. It seems that they can be calculated accurately.

#### 3.3.3. Accuracy of interior orientation parameters

In case of SPOT HRS it is known that the focal length of both lens are 580mm while the time interval is 0.752 msec. Trying to find if using this model is possible to calculate the focal length and the time interval together with the calculation of the rotation angles. In this evaluation the model is solved in both test sites using different number of GCPs. Thus, in case of Aix-en-Provence 3,4,6,12,20 and 33 GCPs are used while in case of Bavaria 4,6,8,9,12,39. The calculated value of the focal length along with the accuracy of this unknown parameter is shown in table 5 for Aix-en-Provence test site and in table 6 for Bavaria test site.

The calculated value of the time interval along with the accuracy of this unknown parameter is shown in table 7 for Aix-en-Provence test site and in table 8 for Bavaria test site.

In both cases the results are quit well. The calculated values are close enough to the true values. The calculated accuracy of

unknown parameters defines this approximation. However the following remarks could be made:

- In Aix-en-Provence test site the calculated values are more accurate and more stable than in Bavaria test site.
- Also the accuracy of the calculated values is not perpendicular to the number of the GCPs in general.

Having in mind that in Bavaria test site the GCPs are not distributed in the whole image we reach the conclusion that in order to calculate accurately an unknown parameter a few good but well distributed GCPs in whole image are needed.

No of GCPs	HRS-1 focal length (mm)		HRS-2 focal length (mm)	
	value	accuracy	value	accuracy
3	579.852	-	580.115	-
4	579.752	0.118	580.075	0.058
6	579.746	0.083	580.141	0.037
12	579.793	0.119	580.167	0.060
20	580.004	0.059	580.078	0.074
33	579.935	0.071	580.169	0.080

Table 5. Calculated value and accuracy of focal length with various combination of GCPs in Aix-en-Provence test site

No of GCPs	HRS-1 focal length (mm)		HRS-2 focal length (mm)	
	value	accuracy	value	accuracy
4	580.056	0.144	580.326	0.470
6	579.958	0.216	580.271	0.265
8	580.148	0.119	580.244	0.212
9	580.114	0.119	580.239	0.191
12	580.203	0.139	579.837	0.129
39	580.251	0.075	579.986	0.075

Table 6. Calculated value and accuracy of focal length with various combination of GCPs in Bavaria test site

No of GCPs	Acquisition time of line (msec)		Acquisition time of line (msec)	
	value	accuracy	value	accuracy
3	0.752448	-	0.752552	-
4	0.752190	0.000462	0.724459	0.000144
6	0.752147	0.000269	0.752420	0.000133
12	0.752475	0.000285	0.752690	0.000186
20	0.752575	0.000287	0.753203	0.000350
33	0.752320	0.000163	0.752952	0.000184

Table 7. Calculated value and accuracy of time interval with various combination of GCPs in Aix-en-Provence test site

No of GCPs	Acquisition time of line ( $\mu$ m)		Acquisition time of line ( $\mu$ m)	
	value	accuracy	value	accuracy
4	0.752474	0.000200	0.752410	0.000656
6	0.752777	0.000387	0.724279	0.000369
8	0.752497	0.000157	0.752414	0.000282
9	0.752497	0.000156	0.752417	0.000251
12	0.752418	0.000204	0.752349	0.000192
39	0.752248	0.000102	0.752236	0.000103

Table 8. Calculated value and accuracy of time interval with various combination of GCPs in Bavaria test site

### 3.3.3. Accuracy of Independent Check Points

In table 9 the RMSE of Independent Check Points (ICP), in Bavaria test site using various combination of GCPs is introduced in order to compare with other scientists which are use the same data set. This improved model produced better results from the

No of GCPs	4	6	12
X (m)	5.75	5.36	5.02
Y (m)	7.21	4.92	4.68
Z (m)	6.32	6.28	4.53

Table 9. RMSE of Independent Check Points (ICP) in WGS84

Daniela Poli (Poli, 2004) which is use the same test site and also the same GCPs (but perhaps not the same combination) gives better results than the above in her paper. However about 19 unknown parameters are involved in the solution while in this model the unknowns are just 6. This leads to the conclusion that the accuracy of the ICP could be improved more as the self-calibration could be involved in the solution.

## 4. CONCLUSIONS-FURTHER WORK

In this paper a methodology based on a photogrammetry based model which can be used for every optical pushbroom sensor even in case where the interior orientation parameters are not known, is introduced. It can be done only in case where high accurate navigation data (state vectors) are provided during the acquisition of the images. It is an improvement of the UCL model (Michalis and Dowman, 2005) which has been developed in UCL since 2002.

This means that we develop a model where it is possible not only to calculate the exterior orientation but also the interior orientation of the satellite with good accuracy.

The next step is in-depth investigation of the self-calibration process. Another very important step is to test the model on the ALOS-PRISM high resolution three line scanner where it is believed that the stability of the solution is increased with the three line geometry. This model will also be tested with Cartosat images.

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