AUTOMATIC MORPHOLOGICAL PRE-ALIGNMENT AND GLOBAL HYBRID REGISTRATION OF CLOSE RANGE IMAGES

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ABSTRACT:

The point cloud alignment problem has always attracted the interest of the researchers. Two are the procedures usually applied for the close range surveys: the ICP method with all its variants, and the method based on the use of tie points identified by reflecting targets properly located on the overlapping part of two range images. The two methods are not concurrent. ICP requires an initial sufficiently accurate pre-alignment and does not work with regular surfaces; the tie points method requires the materialization and the recognition of the corresponding points, task that it is not always feasible and realizable in economical terms. This paper proposes a new hybrid technique to automatically execute the alignment of close range point clouds by evidencing the corresponding morphological singularities in the various scanning models. The point recognition is based first on the study of the local surface Gaussian curvature values, second by running a clustering procedure of laser points having extreme curvature values, and third by determining the centroids of each cluster. The computation of the local Gaussian curvature is carried out by applying to each sampled point a Taylor's expansion local nonparametric algorithm with respect to its surrounding points. This makes it possible to locally estimate the surface function value, and its first and second order partial derivatives. The computation of the Weingarten map matrix elements, from second order Taylor's expansion differential terms, allows to easily determine the Gaussian curvature for each point. For each point cloud, the above defined centroids, generate a vertex configuration. The punctual correspondence with the analogous vertices of an adjacent point cloud are automatically defined, according to the analysis of the respective adjacency matrices. From these sets of pairs, the pre-alignment rototranslation parameters are computed by a SVD algorithm. The final alignment is completed with an ICP method. The experimental results obtained for the alignment of the various parts of one of the well known Stanford models are shown. The paper also reports a general model, derived from the Generalized Procrustes Analysis, to obtain the simultaneous global registration of a set of point clouds. The method is able to simultaneously consider pairs of correspondent points automatically obtained by the ICP algorithm, pairs of tie points manually defined, and control points.

1. INTRODUCTION

Laserscanning surveys of large and complex 3D objects require the execution of many point clouds acquired from sensor locations properly distributed in space. This allows the view and the measurements of the different parts composing the complete object surface. Since the point clouds acquired from different sensor locations are expressed in their own independent reference frame, it is necessary to join and match the various partial point clouds, so to seamlessly recompose the surveyed object. This is analytically carried out transforming the surveyed points coordinates into a unique reference system. The definition of the best transformation parameters constitutes the core of the registration problem.

The laser point cloud registration problem has been considered by many authors. A recent general overview of the various proposed solutions is reported by Gruen and Akca (2005). According to Goshtasby (1998), and to the common sense, the various methods can be catalogued into two main families: surface matching and feature matching.

Surface matching techniques have been developed in the robotics and computer vision sectors. After an initial approximated manual alignment, these methods iteratively improve the registration, gradually reducing the distance between the points belonging to overlapping areas of adjacent scans. The most famous and implemented method is the so-called ICP – Iterative Closest Point – proposed by Besl e McKay (1992), soon improved by Chen and Mediani (1992). In the following, many authors have proposed various improvements so to make it more robust and efficient. Turk and Levoy (1994) for instance, neglect the points falling along the borders of the overlapping areas, while others as Godin et al (1994), Dorai et al. (1997), Masuda et al. (1996), Godin et al. (2001) do not consider, or differently weight, the pairs of corresponding points whose geometric or qualitative attributes – colour, reflectivity, curvature, surface normal direction, incident angle of the beam with respect to the surface etc. - are mutually very dissimilar. Other authors (e.g. Zinßer et al., 2003; Greenspan et al., 2000; Greenspan and Godin 2001) have developed more efficient procedures to speed the search of the closest point, that is the most expensive task in terms of computation time. An interesting critical overview about the variants of the ICP method is reported in Pulli (1999) and, mainly in Rusinkiewicz e Levoy (2001). Although these updates, the method still presents some limitations: it requires an accurate manual pre-alignment; it does not work for regular surfaces, and the solution can converge to local minima.

Feature matching techniques are conceptually the same as those largely employed in photogrammetry, and they correspond to the procedures that use tie and control points. A series of links among pairs of manually, or automatically defined correspondent points are established. According to these points the unknown transformation parameters for the optimal matching of the various point cloud models can be obtained. This approach is largely applied in the practice of terrestrial laser scanning, where often exists the possibility to fix some artificial target points to the object. The targets are constituted by geometric signals (e.g. spheres), or by proper adhesive stickers collocated on the object surface (i.e. on the building walls), easily and accurately identifiable on the point clouds by their particular shape, or by the high reflectivity of the composing material. Not always it is possible or convenient to

signalize the object, like in the case, for instance, of a very large building, or of a rock wall subjected to rock falls. In these cases some morphological details, univocally detectable among the various point clouds, are used as tie points.
This operation, if manually carried out, is tedious and not always easy to do. Furthermore, the number and the level of accuracy with which the tie points are defined, directly conditions the geometric quality of the registration. For this reason (see e.g. Audette et al., 1999; Bae and Lichti, 2004), since the end of the last decade, methods for the automatic detection of significant morphological details, allowing to establish correspondences between partially overlapped adjacent point clouds, have been suggested.

2. THE PROPOSAL

This work reports a completely automatic registration technique, that does not require the knowledge of approximated initial orientation parameters, manual point clouds pre alignment, prior search of corresponding points, pre-signalized surveyed objects.
The method is based on the study of physical characteristics variations of the object surface; the only requirement is that the surface presents at least one unambiguous time and space invariant property, variable from point to point of the surface. The parameter can be the morphology, like in the case of this paper, or one of its attributes like colour, normalized reflectivity, or both. The solution is useful for a lot of laboratory or real world applications.
The method proposed is composed of the following main steps:
- extraction from the different point clouds of the most significant morphological details (singular feature detection);
- determination of the most probable morphological correspondences between clouds partially overlapped (feature labelling);
- pre alignment of the clouds by feature matching and SVD algorithm;
- final alignment by surface matching ICP algorithm;
- global alignment by iterative hybrid feature and surface matching.

All these operations are carried out in a completely automatic way and are described in the following.

2.1 Gaussian curvature analysis and feature detection

A set of significant morphological details allowing the link of adjacent point clouds, are identified at first.
The automatic association of the corresponding features is based on the study and mapping of the surface local Gaussian curvature. Some authors have already devoted their attention to this invariant surface feature (eg. Besl and Jain, 1986; Thirion, 1993; Boulanger and Cohen, 1994). Our proposal to compute the local Gaussian curvature is based on the application of a non parametric local polynomial regression. This approach, carried out by a Taylor's expansion extended to a prefixed number of points round the studied one, makes it possible to locally estimate the value of the surface function and its k-order derivatives. A proper weighting of the contribution of the various points, according to their distance to the considered one, is also taken into account. From differential geometry, the curvature parameters can be detected from the elements of the Weingarten map matrix, that can be locally computed from estimated non parametric Taylor's expansion parameters.
Let us consider at first the analytical model $\mathrm{z}(\mathbf{x}) = \mu(\mathbf{x}) + \varepsilon$ for a certain function value z acquired at the generic position $\mathbf{x} \in D$, ($D \subset \Re^2$), where $\mu(\mathbf{x})$ is the true value of $z(\mathbf{x})$, ε is a random variable with $E(\boldsymbol{\varepsilon}) = 0$ and $\mathrm{var}(\boldsymbol{\varepsilon}) = \sigma_\varepsilon^2$. If $\mu(\mathbf{x})$ admits partial derivatives, then according to the Taylor's expansion, $\mu(\mathbf{x})$ can be approximated by a polynomial of order k in the neighbourhood of **x**. The resulting Taylorized model, approximated to the second order terms, is:

$$z_j = z_{0_i} + \left(\frac{\partial z}{\partial x}\right)_i (x_j - x_i) + \left(\frac{\partial z}{\partial y}\right)_i (y_j - y_i) + \left(\frac{\partial^2 z}{\partial x^2}\right)_i \frac{(x_j - x_i)^2}{2} + \left(\frac{\partial^2 z}{\partial y^2}\right)_i \frac{(y_j - y_i)^2}{2} + \left(\frac{\partial^2 z}{\partial x \partial y}\right)_i (x_j - x_i)(y_j - y_i) + \varepsilon_j \tag{1}$$

$$\beta = \begin{bmatrix} z_{0_i} & \left(\frac{\partial z}{\partial x}\right)_{x_i} & \left(\frac{\partial z}{\partial y}\right)_{y_i} & \left(\frac{\partial^2 z}{\partial x^2}\right)_{x_i} & \left(\frac{\partial^2 z}{\partial y^2}\right)_{y_i} & \left(\frac{\partial^2 z}{\partial x \partial y}\right)_{x_i y_i} \end{bmatrix}^T \tag{2}$$

where x_i, y_i and x_j, y_j are the 2-D Cartesian coordinates of the generic point *i*, and of its *j* neighbour point, respectively.
From differential geometry, is well known that to determine at a certain point *i* the values of the Gaussian curvature (invariant with respect to the reference system), it is necessary to locally compute the Weingarten map of the surface (Do Carmo, 1976). For a surface S, parametrized according to Formula (1), the Weingarten map matrix A is given by:

$$\mathbf{A} = -\begin{bmatrix} \mathrm{e} & \mathrm{f} \\ \mathrm{f} & \mathrm{g} \end{bmatrix} \begin{bmatrix} \mathrm{E} & \mathrm{F} \\ \mathrm{F} & \mathrm{G} \end{bmatrix}^{-1} \tag{3}$$

where E, F, G are the coefficients of the "first fundamental form" $\mathrm{I}_i(w) = \langle w, w \rangle_i = |w|^2 \geq 0$ in the basis:

$$\{x_u, x_v\} = \left\{ \begin{bmatrix} 1 & 0 & \frac{\partial z}{\partial x_i} \end{bmatrix}^T, \begin{bmatrix} 0 & 1 & \frac{\partial z}{\partial y_i} \end{bmatrix}^T \right\} \tag{4}$$

of the tangent plane $T_i(S)$ to the regular surface S at point *i*. They are given as:

$$E = \langle x_u, x_u \rangle = 1 + \left(\frac{\partial z}{\partial x_i}\right)^2 \tag{5}$$

$$F = \langle x_u, x_v \rangle = \left(\frac{\partial z}{\partial x_i}\right)\left(\frac{\partial z}{\partial y_i}\right) \tag{6}$$

$$G = \langle x_v, x_v \rangle = 1 + \left(\frac{\partial z}{\partial y_i}\right)^2 \tag{7}$$

The terms e, f, g are the coefficients of the "second fundamental form" $\mathrm{II}_i(w) = -\langle dN_i(w), w \rangle_i = |w|^2 \geq 0$ of S at *i*, in the basis $\{x_u, x_v\}$, where dN_i is the differential of the normal vector to the tangent plane $T_i(S)$. The coefficients are given as:

$$e = -\langle dN(x_u), x_u \rangle = \frac{\frac{\partial^2 z}{\partial x_i^2}}{\sqrt{\left(\frac{\partial z}{\partial x_i}\right)^2 + 1 + \left(\frac{\partial z}{\partial y_i}\right)^2}} \tag{8}$$

$$f = -\langle dN(x_u), x_v \rangle = \frac{\frac{\partial^2 z}{\partial x_i \partial y_i}}{\sqrt{\left(\frac{\partial z}{\partial x_i}\right)^2 + 1 + \left(\frac{\partial z}{\partial y_i}\right)^2}} \tag{9}$$

$$g = -\langle dN(x_v), x_v \rangle = \frac{\frac{\partial^2 z}{\partial y_i^2}}{\sqrt{\left(\frac{\partial z}{\partial x_i}\right)^2 + 1 + \left(\frac{\partial z}{\partial x_i}\right)^2}} \quad (10)$$

The Gaussian curvature K corresponds to the determinant of matrix **A**, that is:

$$K = \frac{eg - f^2}{EG - F^2} \quad (11)$$

The main advantage of computing the terms of the Weingarten map matrix **A** from the differential terms of the Taylor's expansion is that the computation of the invariant curvature parameters can be carried out in the original reference frame, without any transformation in the so called "Monge coordinate system" (Cazals and Pouget, 2003) characterized by a diagonal Hessian matrix. The latter is defined, for the point *i* of the surface S, by the principal directions of the tangent plane, and of the normal vector.

Once the Gaussian curvature values are determined in correspondence of all the cloud points, a region growing algorithm is applied to cluster the points having homogeneous geometrical characteristics. Among the identified clusters, only those points having greatest Gaussian curvature value are selected. These are the best candidates, from the geometrical point of view, to represent tie points. For each cluster having extreme Gaussian curvature values, the corresponding centroid coordinates, obtained as a mean of the 3D point cluster coordinates, are computed. The set of centroids constitute the first point configuration to submit to a correspondence search.

This process is repeated for all the point clouds that have to be aligned, defining in this way a set of centroid configurations.

2.2 Automatic Feature Matching and Labelling

The centroids identified above, form a set of possible candidates to be homologous points of adjacent point clouds. The next step consists in the recognition of topological relationships among the clusters (labelling problem).

Let us consider two partially overlapped point clouds, from which two sets **p** and **q,** respectively constituted by *m* and *n* tie points (the centroids), have been identified. The problem consists in defining the intersection **p**∩**q,** and in automatically finding out, within the intersection, the probable correspondences between the tie points of the sets **p** and **q.**

With respect to other methods already proposed in the past (i.e. Beinat, Crosilla e Sossai, 2004), in this case, no scale variation between the coordinate systems of **p** and **q** are considered. This simplified hypothesis is correct according to the purpose of this operation. The implemented method runs in the following way:

a. Let us consider **p** = {p_1 … p_m} the arbitrary *m* corresponding point configuration. To this configuration, the *m*×*m* symmetric adjacency matrix $\mathbf{D}^p$ is associated, whose elements are the Euclidean distance $d^p_{i,j} = \|p_i - p_j\|$ between the points p_i e p_j;
b. Analogously, once the arbitrary *n* point configuration **q** = {q_1 … q_n} is defined, it is possible to associate to it the *n*×*n* symmetric adjacency matrix $\mathbf{D}^q$, whose elements are the $d^q_{k,l} = \|q_k - q_l\|$ Euclidean distance between the points q_k and q_l;
c. The row of maximal asymmetry $\mathbf{d}^p_{i\max}$ is searched in $\mathbf{D}^p$ (or in $\mathbf{D}^q$), whose distinct elements, ordered in terms of magnitude, present the "maximal minimal difference", that is: $\mathbf{d}^p_{i\max} := \left\{ d^p_{i,j} \geq d^p_{i,j+1};\ \max_{i=1\ldots m}\left[\min_{i,j=1\ldots m-1}\left(d^p_{i,j} - d^p_{i,j+1}\right)\right] \in \mathbf{d}^p_{i\max} \right\}$
This search minimizes the ambiguity of the geometrical configuration.
d. In the row $\mathbf{d}^p_{i\max}$ of maximal asymmetry, the greatest element $d^p_{i,j\max} = \max\left[\mathbf{d}^p_{i\max}\right]$ is selected.
Next step consists in searching in $\mathbf{D}^q$ the values $d^q_{k,l} := \left\{ \left| d^p_{i,j\max} - d^q_{k,l} \right| \leq \varepsilon; \forall k, \forall l < k \right\}$, where ε is a prefixed tolerance. The values *k*,*l* satisfying the previous relationship are stored into an array of pointers, that represents the list of the possible pairs (q_k,q_l) corresponding to $(p_i,p_j)_{max}$. If this set is empty, the search is repeated considering the next component to $d^p_{i,j\max}$ in terms of magnitude;
e. The various pairs of possible correspondences (q_k,q_l) are orderly considered. For each row of $\mathbf{D}^q$, where one of the correspondence pairs is present, the equivalence of the remaining elements of the same row, with respect to possible elements of $\mathbf{d}^p_{i\max}$, within a fixed tolerance, is verified. This allows to generate a binary table, of size *m*×*n*, where the elements express the potential correspondence among the points of **p** and of **q,** according to the initial choice for *i* max and *k*, respectively.
f. If this table has at least two not null elements, a cross-validation of all the possible correspondences is carried out. This is performed verifying the equivalence among all the remaining distances defined by point pairs of **p** and the point pairs of **q** inserted into the table.
g. This process is repeated for each pair (q_k,q_l) identified at step (d.), adopting the pair generating the largest number of correspondence pairs valid between **p** and **q.**

The procedure here described is schematic. A set of implemented tests, makes it possible to solve ambiguous situations. According to current literature, to evaluate the correspondence point degree, the use of some attributes associable to the points, e.g. curvature, will be applied in future.

2.3 SVD pre-alignment

Thanks to the pairs of tie points identified and linked by the preceding phases, two matrices $\mathbf{X}_i$ and $\mathbf{X}_j$ of *k* corresponding point coordinates are obtained. The translation **t** and rotation **R** parameters to transform the coordinates of a point cloud onto another one are directly determined by applying the SVD method (Schoenemann, 1966). An alternative procedure consists in computing also an isotropic scale factor c (Schoenemann and Carrol, 1970), and to use it as a registration quality index. By computing the SVD of the following product:

$$\mathbf{X}_i^T \left[\mathbf{I} - \mathbf{jj}^T \big/ \left(\mathbf{j}^T\mathbf{j}\right)\right] \mathbf{X}_j = \mathbf{VD}_s\mathbf{W}^T \quad (12)$$

where **I** is the identity matrix, and **j** is a *k*×1 predefined auxiliary unitary vector,

- the rotation matrix: $\mathbf{R} = \mathbf{VW}^T$,
- the global scale factor:

$$c = \mathrm{tr}\left[\mathbf{R}^T\mathbf{X}_i^T\left(\mathbf{I} - \frac{\mathbf{jj}^T}{\mathbf{j}^T\mathbf{j}}\right)\mathbf{X}_j\right] \Big/ \mathrm{tr}\left[\mathbf{X}_i^T\left(\mathbf{I} - \frac{\mathbf{jj}^T}{\mathbf{j}^T\mathbf{j}}\right)\mathbf{X}_j\right],$$

- and the translation vector: $\mathbf{t} = \left(\mathbf{X}_j - c\mathbf{X}_i\mathbf{R}\right)^T \mathbf{j} \big/ \left(\mathbf{j}^T\mathbf{j}\right)$,

can be directly computed.

2.4 Pairwise registration refinement by ICP

Once the pre-alignment is obtained, the pairwise registration process is completed by ICP. In our case, the basic version of the method proposed by Besl and Mc Kay (1992), has been implemented, updated by some improvements proposed in the literature (e.g. Rusinkiewicz and Levoy, 2001).

2.5 Global registration by Generalised Procrustes

The proposed method, up to now, is employed to mutually align pairs of 3D point clouds. For redundant point clouds, or for scans spatially disposed so to define a sequence of clouds connecting the first and the last one, there exists the necessity or the possibility to distribute the alignment errors among the various composing 3D clouds.

It must be stressed how the original global alignment model proposed by the authors for the feature matching solution (Beinat and Crosilla, 2001) can be easily adopted within the ICP, offering to the latter method the possibility of using control points. In fact, it is only necessary to consider the various matrices $\mathbf{X}_i$ of homologous points, associated to each 3D point cloud, like constituted by tie points manually defined, and by automatically identified correspondences within the ICP, with the option to discriminate the two sets by proper weights. All this without introducing any variation to the solving scheme, even better emphasizing the common and extended use of the SVD function by the two algorithms.

The simultaneous global registration problem requires the definition of the transformation parameters c_i, $\mathbf{t}_i$ and $\mathbf{R}_i$ ($i = 1 \ldots$ m), for each homologous point coordinate matrix $\mathbf{X}_i$, so to minimize the following objective function:

$$\begin{aligned} &\operatorname{tr}\sum_{i<j}^{m}\left[\left(c_i\mathbf{X}_i\mathbf{R}_i+\mathbf{j}\mathbf{t}_i^{\mathrm{T}}\right)-\left(c_j\mathbf{X}_j\mathbf{R}_j+\mathbf{j}\mathbf{t}_j^{\mathrm{T}}\right)\right]^{\mathrm{T}}\cdot \\ &\qquad\left[\left(c_i\mathbf{X}_i\mathbf{R}_i+\mathbf{j}\mathbf{t}_i^{\mathrm{T}}\right)-\left(c_j\mathbf{X}_j\mathbf{R}_j+\mathbf{j}\mathbf{t}_j^{\mathrm{T}}\right)\right]=\min \end{aligned} \tag{13}$$

The satisfaction of the previous condition represents the Generalized Procrustes problem solution (Gower, 1975; Goodall, 1991). Once $\mathbf{X}_i^{\mathrm{p}} = c_i\mathbf{X}_i\mathbf{R}_i + \mathbf{j}\mathbf{t}_i^{\mathrm{T}}$ is defined, it is possible to prove the equivalence of the following formulations (i.e Borg e Groenen, 1997):

$$\sum_{i<j}\left\|\mathbf{X}_i^{\mathrm{p}}-\mathbf{X}_j^{\mathrm{p}}\right\|^2=\sum_{i<j}\operatorname{tr}\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{X}_j^{\mathrm{p}}\right)^{\mathrm{T}}\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{X}_j^{\mathrm{p}}\right) \tag{14}$$

$$\mathrm{m}\sum_{i}\left\|\mathbf{X}_i^{\mathrm{p}}-\mathbf{C}\right\|^2=\mathrm{m}\sum_{i}\operatorname{tr}\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{C}\right)^{\mathrm{T}}\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{C}\right) \tag{15}$$

From this correspondence, it is possible to iteratively minimize Formula (15) instead of (14), obtaining for each configuration $\mathbf{X}_i$ ($i = 1 \ldots$ m), the unknown {c, $\mathbf{R}$, $\mathbf{t}$}$_i$ that transform it in $\mathbf{X}_i^{\mathrm{p}}$, in such a way to find the simultaneous least squares matching among the all configurations. For this solution, the term

$$\hat{\mathbf{C}}=\frac{1}{\mathrm{m}}\sum_{i=1}^{m}\mathbf{X}_i^{\mathrm{p}} \tag{16}$$

Represents the least squares estimate of the unknown term C, geometrical centroid of the m aligned configurations of corresponding points. $\mathbf{C}$ is composed by the matched point coordinates expressed in a mean common reference system. Please note that every $\mathbf{X}_i^{\mathrm{p}}$ corresponds to $\mathbf{C}$, unless a random error component $\mathbf{E}_i$, that is: $\mathbf{C}+\mathbf{E}_i=\mathbf{X}_i^{\mathrm{p}}$.

The parameters {c, **R, t**}$_i$**,** relative to the various configurations $\mathbf{X}_i$ of homologous points, are applied to the remaining points of the 3D model, performing in this way the corresponding alignment.

In real world applications, each pair of point clouds is partially and differently overlapped, therefore the GPA model has to be further extended to account for missing correspondences among point clouds.

This can be done considering coordinate matrices $\mathbf{X}_i^{\mathrm{p}}$ ($i = 1 \ldots$ m) pre-multiplied by a proper real diagonal weight matrix $\mathbf{D}_i$

The equivalence of the measurements, expressed by Formula (15) becomes now:

$$\begin{aligned} &\sum_{i=1}^{m}\operatorname{tr}\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{C}\right)^{\mathrm{T}}\mathbf{D}_i\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{C}\right)= \\ &=\sum_{i<j}^{m}\operatorname{tr}\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{X}_j^{\mathrm{p}}\right)^{\mathrm{T}}\mathbf{D}_i\mathbf{D}_j\left(\sum_{k=1}^{m}\mathbf{D}_k\right)^{-1}\left(\mathbf{X}_i^{\mathrm{p}}-\mathbf{X}_j^{\mathrm{p}}\right) \end{aligned} \tag{17}$$

while , the estimate of the geometric centroid $\mathbf{C}$ becomes in that case:

$$\hat{\mathbf{C}}=\left(\sum_{k=1}^{m}\mathbf{D}_k\right)^{-1}\sum_{i=1}^{m}\mathbf{D}_i\mathbf{X}_i^{\mathrm{p}} \tag{18}$$

This formulation allows to handle the missing homologous points among the various models, like happens in practice. A binary diagonal matrix $\mathbf{M}_i$ is introduced, with the terms equal to 1 where the corresponding points of $\mathbf{X}_i$ are effectively present, and 0 in the other cases (Commandeur, 1991). $\mathbf{P}_i$ (optional), is the so called diagonal weight matrix.

$$\mathbf{D}_i=\mathbf{M}_i\mathbf{P}_i=\mathbf{P}_i\mathbf{M}_i \tag{19}$$

This method presents some significant advantages: it does not require the solution of equation systems and does not require the computation of approximate values of the unknowns. However, the most relevant aspect is that it furnish the mathematical model to implement the global alignment, both for tie points model or with ICP. Conceptual investigation and details of the procedure, are reported in Beinat and Crosilla (2001) and Crosilla and Beinat (2002).

3. ONE EXAMPLE

The proposed method has been implemented in C^{++} in a specific software for multipurpose Lidar data processing.

We report the results of the automatic alignment of the point clouds composing "The dragon", the famous Chinese symbol, one of the case studies in The Stanford 3D Scanning Repository (see: http://graphics.stanford.edu/data/3Dscanrep/).

The full dataset is composed by 70 point clouds, produced by a Cyberware 3030 optical range-finding system. In operation, the head of this scanner emits a low-intensity laser beam on the object surface to create a lighted profile. A high-quality video sensor captures this profile from two viewpoints (see: http://www.cyberware.com/products/scanners/3030.html). For the aim of the test we processed a subset of 30 point clouds, for a total amount of 912000 points, relative to the "stand" and to the "up" scan sequences, enough to provide a redundant coverage of the entire object surface.

The results are explained graphically. In Figure 1, two overlapping point clouds (red and green) are represented in their proper initial reference system. Figure 2 shows the mapping of the areas containing equal Gaussian curvature values, evidenced by an arbitrary colour scale. This analysis has been performed for each point cloud. Figure 3 and 4 put in evidence, for two overlapping point clouds, the possible tie point detection carried out by centroid computation of the points clusters having extreme Gaussian curvature values. The number of correspondences is variable and depends on the extreme values interval considered. An excessive number of possible tie points slows down the automatic feature matching and labelling step.

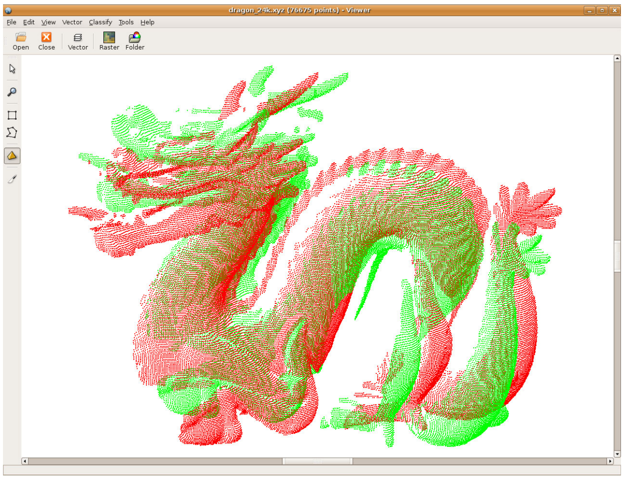

Figure 1. Two overlapping point clouds before registration.

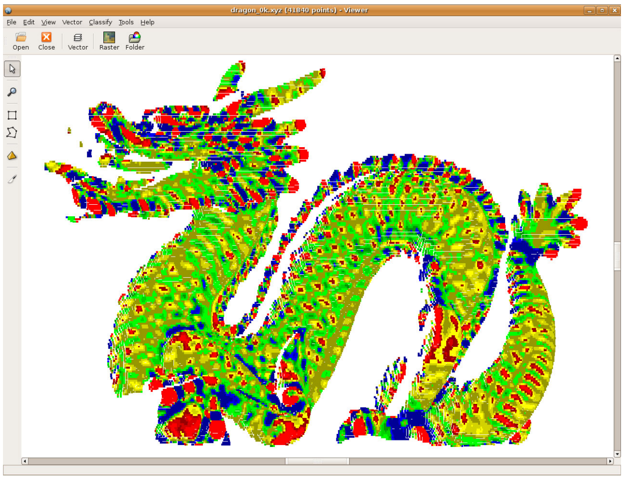

Figure 2. Mapping the areas of equal Gaussian curvature values (evidenced by a colour scale).

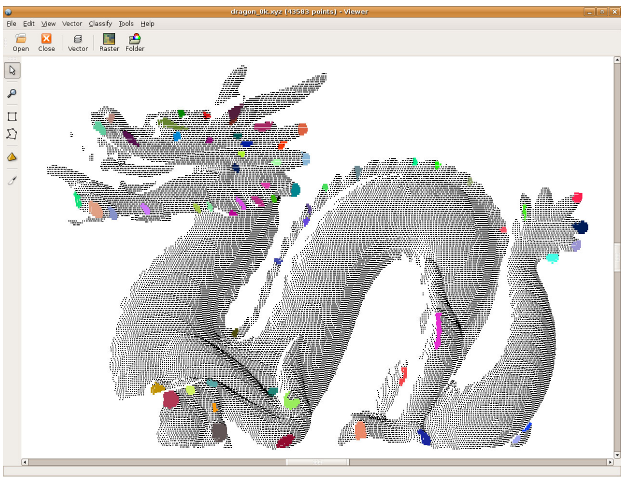

Figure 3. Tie point detection by centroid computation of the clusters of points having extreme Gaussian curvature values.

Figure 5 shows the two overlapping point clouds after the automatic pre-alignment, and the pairwise ICP registration. From the graphical point of view, there is no detectable difference between the pre-alignment and the ICP registration: i.e. the pre-alignment is close to the ICP solution. Figure 6, shows the 30 point clouds composing the dragon, evidenced by a different colour, merged together to compose the model.

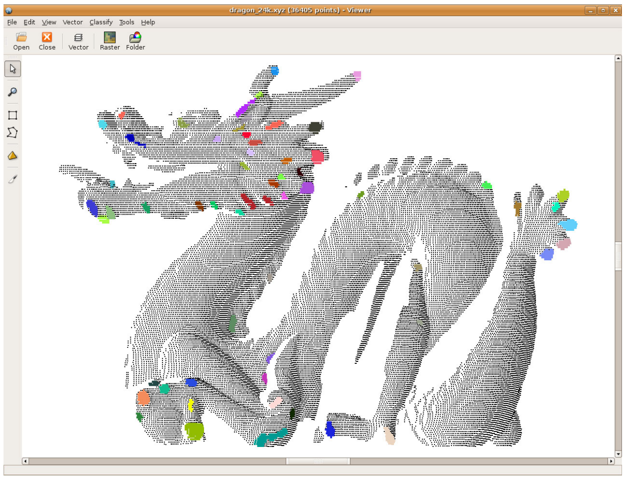

Figure 4. Same as Fig. 3, but relative to a different point cloud

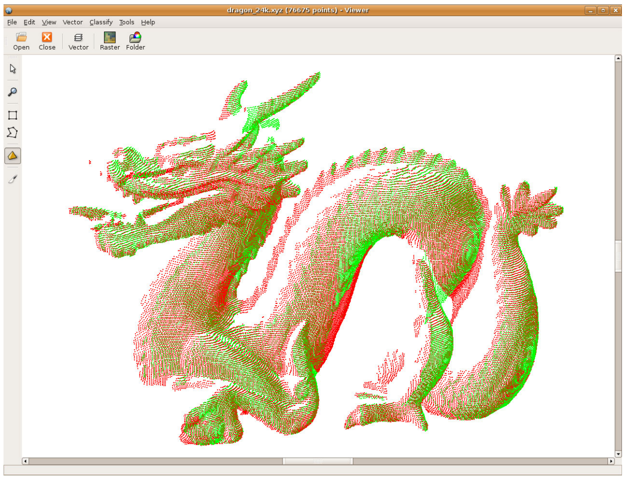

Figure 5. The two overlapping point clouds after the automatic pre-alignment and the pairwise ICP registration.

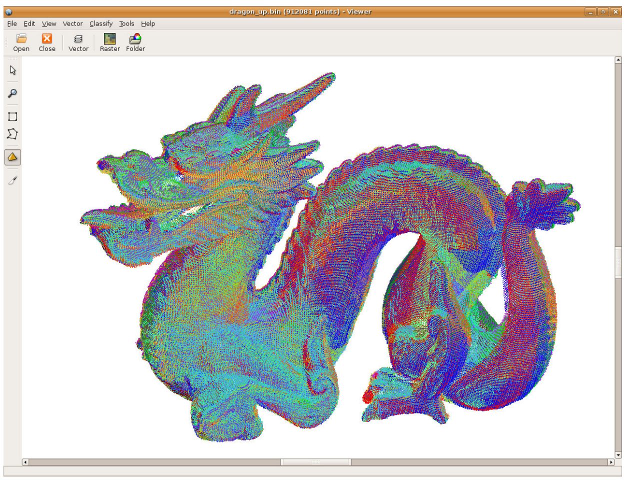

Figure 6: Thirty point clouds of the dragon, evidenced by a different colour, registered together to form the model.

3. CONCLUSIONS

A full automatic method for the global registration of laserscanning point clouds has been developed. The approach is hybrid, and acts along five main steps. At first, a tie point detection is performed, based on morphologic feature extraction by way of Gaussian curvature analysis. Then a labelling procedure is carried out in order to establish valid correspondences among tie points of the overlapping scans. From these tie points coordinates, the transformation parameters are directly computed, and the different point clouds pairwise aligned into the same reference frame. Furthermore, an ICP algorithm improves the pairwise alignment. Finally, the global registration of the full set of point clouds is obtained by merging together all the tie points of the different point clouds, by way of the Generalised Procrustes algorithm.

The method has been implemented, and successfully tested on laser point clouds of 3d objects. Current developments are addressed to improve the efficiency of the procedure, by optimizing the tie point labelling procedure, and the ICP algorithm, through a more efficient and robust closest point search, able to take advantage of the point attributes.

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