# CCD CAMERA CALIBRATION BASED ON THE SUN'S IMAGES 

U. Ethrog<br>Department of Transportation and Geo-Information Engineering<br>Faculty of Civil and Environmental Engineering<br>Technion - Israel Institute of Technology<br>Technion City, Haifa 32000, Israel<br>cvuzi@tx.technion.ac.il

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#### Abstract

A Nikon D70s Digital camera with resolution of 6 M , equipped with a narrow angle Nikkor $85 \mathrm{~mm} \mathrm{f} / 1.8 \mathrm{D}$ lens and suitable filters; was calibrated successfully, by two sets of the sun's images. Each set was taken while the camera's orientation was held unchanged for more than a hour. During this time the sun's image cross the camera's field of view from left to right and thirty photographs were taken at fixed known time intervals, controlled by a GPS receiver that was used as an intervalometer. The camera was held so that the sun's trajectory during the first set of images was an arc approximately at the direction of the sensor's lines. The sun's trajectory in the second set was nearly perpendicular to it. The two trajectories pass close to the sensor center. The elements of the camera's interior orientation and its tilt angles in each set of photography were determined simultaneously by least squares adjustment procedure. Due to the big number of the sun's images located in good positions, the elements in the variance-covariance matrix were small. In addition the location of the sun's image was determined accurately on the digital photograph, by an edge detection techniques. As a result the RMS values of the interior orientation parameters were small.


## 1. Introduction

Photogrammetric measurements are used in several Hi Tec industries like building simulators for low altitude air navigation. People who are experts in mathematics or computer sciences but not in photogrammetry prefer to do almost all of the photogrammetric works by themselves, using a full automatic procedure, based on computer programs for the all photogrammetric works, from conjugate point measurement, till DEM and orthophoto generation. Usually digital images taken with 6 M to 12 M non-metric digital cameras, from low altitude air carrier equipped with GPS and IMU systems are perfect for their needs. The help of experts in photogrammetry is needed, in this process, only for cameras calibration. In some cases the camera that is used has a narrow angle lens and should be focused to infinity.
All the methods of camera calibration determine the image coordinates of the photo perspective center: $(x 0, y 0, c)$ and at least some of the coefficients $\mathrm{A} 1, \mathrm{~A} 2, \mathrm{~A} 3, \ldots$ and $\mathrm{P} 1, \mathrm{P} 2$, $\mathrm{P} 3, \ldots$. of the radial and decentering distortions polynomial given by brown (1966, and 1971) and Freyer and Brwon(1986). Sometimes there is a need to determine simultaneously other parameters like the image exterior orientation by a process of least squares adjustment Their RMS values that are determined by the product of $\sigma^{2}$ the mean square error of an observation with $\mathrm{N}^{-1}$ the inverse of the coefficient matrix of the normal equations. The motivation of this work is to find a calibration method, suitable for digital cameras, that enable small values for both, $\sigma^{2}$ and $\mathrm{N}^{-1}$ and that is suitable also for non-expert in photogrammetry, who prefer automatic procedure of photogrammetric measurements.
The combination of digital camera, narrow angle, relatively big distortions and focous to infinity cause some difficulties in camera calibration, as a result none of the existing camera calibration techniques is perfectly suitable for this purpose.. The ideal size of a target on a digital photograph for manually measurement is 3 pixels and it is hard to measure its image coordinates with standard deviation of less then half a pixel. The automatic feature base methods are much
more accurate but need also bigger target. For this reason the laboratory methods which are usually ideal for camera calibration that are focused to the infinity, are not suitable in this case. The goniometer method, described by Hakkarainen (1972), is not suitable for any CCD sensor. The multicollimator method, described by Light(1992), used thirty three collimators that project small thin crosshair on the photograph.. These crosshairs are too small to be measured accurately, certainly not by automatic procedures. Furthermore, the angle between two successive collimators is $7^{\circ} 30^{\prime}$, suitable for a wide angle photograph. The projection of no more than three collimators will appear in a $17^{\circ}$ narrow angle lens, not enough for camera calibration. Potentially this method can be excellent for the calibration of digital narrow angle cameras, but only after its construction is change and the collimators reticles will be replaced.
The stellar method of camera calibration, presented by Fritz \&Schmid (1974), and brown (1966) has also some of these problems. The star light is very week and small and there is always fear that only few of them will appear inside the field of view of the $17^{\circ}$ narrow angle camera, not enough for camera calibration. Anyhow the stellar method is not suitable for non-experts. The Field methods are the most suitable for digital cameras calibration. There are two main techniques, the test field method and the self calibration method. The first one uses a test field with many spatial control points that their spatial coordinates are measured precisely. It can be inside a big room as was done by Altan (1986) and Paquette (1990) or outside on the walls of a big building as described by Krause (1997). Usually a single photograph of the test field is sufficient for the calibration. The self calibration method is described by Fraser(1997), Fryer(1992), Ghosh, Rahimi \& Shi(1988) and many others. It is similar to the test field method but there is not need to measure the 3D coordinates of the control points. They are determined in the bundle adjustment procedure together with the other parameters. On the other hand the test field should be photographed from at least three different angles. It is hard to find a appropriate place for a test field suitable for the calibration of narrow angle cameras that are focused to
nfinity, especially inside a room or a corridor. Furthermore, in these methods the exterior orientation parameters of the photograph are determined simultaneously together with the interior orientation parameters. The results are big values for the elements along the main diagonal of the variance covariance matrix. In theory they are growing proportional to forth power of the cameras principal distance.
The method of calibration presented in this paper is based on the known location of the sun at any moment. It has some important disadvantages:

1. It can be used only in sunny days.
2. It can be used only when the sun height is above $25^{\circ}$, to avoid big effect of atmospheric refraction.
3. A special filter is needed for taking digital images of the sun.
4. Its success depends on the ability to keep the camera's orientation unchanged for an hour or two. But it has also some important advantages.
5. The observations, (measuring the position of the sun on the digital image) are done by automatic fast and accurate feature based procedures with standard deviation of sub-pixel.
6. The elements along the diagonal of the variance covariance matrix are relatively small; as a result the RMS values of the calibrated parameters are small.

## 2. The method's principles and the photography

For a person who is looking on the sky, the sun's trajectory seems to be a circular arc. Actually,, Its motion relatively to the earth is in a plane almost parallel to the equator. The sun's trajectory upon the camera's sensor is a hyperbolic arc, close to a straight line. The idea behind the proposed method of calibration is to hold the camera's orientation unchanged for the time needed for the sun to pass from one side of the camera's frame to the other and to capture snap shots at fixed time intervals, of 60 seconds. The zenithal angle of the camera axis is chosen so that the sun's trajectory passes near the frame center. Two sets of images are taken. The sun's trajectory at one of them is almost parallel to the sensor lines; and perpendicular at the other.
By this way we obtain many images like that illustrated in Fig.1. The sun's image at each of them is a good target at the right place on the camera's frame. The accurate exposure time of each of them, determined by the intervalometer, enable to determine the accurate values of the corresponding sun's declination and Greenwich Hour Angle.


Figure 1 A sun's image

The equipment which is needed for that purpose is
a. A good massive tripod witch enable us to put the camera at any wanted orientation and yaw and to keep them unchanged for the time of a set photography, more than an hour
b. An intervalometer which cause the camera to make an exposures at constant known time intervals. In the experiment which is described in this paper we use a GPS receiver
c. A special remote cord and a special electronic box to connect the camera and the receiver as it is illustrated in Fig 2.
d. Two special optical filters for the camera, (cokin gray nd8 and cokin UV154), so that the sun's image will be clear.


Figure 2 The equipment

## 3. Coordinates systems and collinearity equations

. Two spherical coordinates systems are used in this paper; they are illustrated in Fig 3. We assume that the radius of the earth is very small relative to the distance to the sun, so that it is neglected. Therefore point E in Fig 3 , represent the earth ,which is the celestial sphere center. Point S in Fig. 3 represents the sun which is lying on this sphere. The poles of this sphere are the points where it is cut by the prolongation of the earth axis. Z and C are the cut points of the zenith line and the camera axis, respectively with this sphere.
The "natural" coordinates system is the local spherical system in which the direction of the camera axis determined by its azimuth $\mathrm{A}_{0}$ and its zenithal angle $\zeta_{0}$. When the sensor lines are parallel to the horizon, the unit vectors of the camera's axes at this coordinates system are:


Figure 3. The two spherical coordinate systems
$\bar{r}_{\text {sample }}=\cos \mathrm{A}_{0} \bar{i}+0 \bar{j}-\sin \mathrm{A}_{0} \bar{k}$
$\bar{r}_{\text {line }}=\sin \mathrm{A}_{0} \cos \underline{\zeta}_{0} \bar{i}+\sin \underline{\zeta}_{\underline{0}} \bar{j}+\cos \mathrm{A}_{0} \cos \underline{\zeta}_{0} \bar{k}$
$\bar{r}_{z}=\sin \mathrm{A}_{0} \sin \zeta_{0} \bar{i}-\cos \zeta_{0} \bar{j}+\cos \mathrm{A}_{0} \sin \zeta_{0} \bar{k}$
The zenithal angle of the sun is $\zeta$ and its azimuth is $A$. Its unit vector is:
$\bar{r}=\sin \mathrm{A} \sin \zeta \bar{i}-\cos \zeta \bar{j}+\cos \mathrm{A} \sin \zeta \bar{k}$
From the scalar product of the this vector with each of the vectors in eq. 1 we obtain the collinearly equations:
$Q_{1}=\bar{r} \cdot \bar{r}_{\text {sample }}=\sin \zeta \sin \Delta A$
$Q_{2}=\bar{r} \cdot \bar{r}_{\text {line }}=\cos \zeta_{0} \sin \zeta \cos \Delta A-\cos \zeta \sin \zeta_{0}$
$Q_{2}=\bar{r} \cdot \bar{r}_{l z}=\sin \zeta_{0} \sin \zeta \cos \Delta A+\cos \zeta \cos \zeta_{0}$
semple $-x_{0}+\delta_{x}+\Delta_{x}=c \frac{Q_{1}}{Q_{3}}$
line $-y_{0}+\delta_{y}+\Delta_{y}=c \frac{Q_{2}}{Q_{3}}$
where: $\quad \Delta A=A-A_{0}$

Where:
$x_{0,} y_{0}$ - the image coordinates of the photo principal point c - the camera's principal distance
$\delta_{x}, \delta_{y}$-corrections for radial and decentering distortions.
$\Delta_{x}, \Delta_{y}$ - corrections to the photo coordinates because of atmospheric refraction and sun parallax.

The corrections for radial and decentering distortions as given by Brown (1966 and 1971) and Fryer and Brown (1986) are:

$$
\begin{align*}
\delta x= & \bar{x}\left[k_{1} r^{2}+k_{2} r^{4}+k_{3} r^{6}+\ldots \ldots .\right]+ \\
& +\left[P_{1}\left(r^{2}+2 \bar{x}^{2}\right)+2 P_{2} \bar{x} \bar{y}\right]\left[1+P_{3} r^{2}+\ldots \ldots .\right] \\
\delta y= & \bar{y}\left[k_{1} r^{2}+k_{2} r^{4}+k_{3} r^{6}+\ldots \ldots .\right]+  \tag{4}\\
& +\left[P_{2}\left(r^{2}+2 \bar{y}^{2}\right)+2 P_{1} \bar{x} \bar{y}\right]\left[1+P_{3} r^{2}+\ldots \ldots .\right]
\end{align*}
$$

where:
$\bar{x}=x-x 0$
$\bar{y}=y-y 0$
$r^{2}=\bar{x}^{2}+\bar{y}^{2}$

The values of the sun's azimuth and zenithal angle at any moment of exposure are unknown but can be calculated by the trigonometric connections in the spherical triangle NZS in Fig 3. For that, we have to know the sun's declination $\boldsymbol{\delta}$, its Local Hour Angle, and the exact latitude of the point of exposure. The last two together with the constant time error of all the exposures, because of the clock error and the camera delay, are unknown. Together with the camera orientation parameters we obtain six orientation unknown parameters which is to much.
$\mathrm{Q}_{1}, \mathrm{Q}_{2}, \mathrm{Q}_{3}$ in Eq. 3 can be written also as Matrixes product:

$$
\left(\begin{array}{l}
Q_{1}  \tag{5}\\
Q_{2} \\
Q_{3}
\end{array}\right)=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \sin \zeta_{0} & \cos \zeta_{0} \\
0 & -\cos \zeta_{0} & \sin \zeta_{0}
\end{array}\right)\left(\begin{array}{ccc}
\sin \Delta A & 0 & -\cos \Delta A \\
0 & 1 & 0 \\
\cos \Delta A & 0 & \sin \Delta A
\end{array}\right)\left(\begin{array}{c}
\sin \zeta \\
-\cos \zeta \\
0
\end{array}\right)
$$

Since the sensor lines are usually not horizontal the values of $\mathrm{Q}_{1}, \mathrm{Q}_{2}, \mathrm{Q}_{3}$ should be:
$\left(\begin{array}{l}Q_{1} \\ Q_{2} \\ Q_{3}\end{array}\right)=\left(\begin{array}{ccc}\cos \kappa & \sin \kappa & 0 \\ -\sin \kappa & \cos \kappa & 0 \\ 0 & 0 & 1\end{array}\right)\left(\begin{array}{ccc}1 & 0 & 0 \\ 0 & \sin \zeta_{0} & \cos \zeta_{0} \\ 0 & -\cos \zeta_{0} & \sin \zeta_{0}\end{array}\right)\left(\begin{array}{ccc}\sin \Delta A & 0 & -\cos \Delta A \\ 0 & 1 & 0 \\ \cos \Delta A & 0 & \sin \Delta A\end{array}\right)\left(\begin{array}{c}\sin \zeta \\ -\cos \zeta \\ 0\end{array}\right)$
The alternative is to choose the geographical coordinate system as it is illustrated in Fig 3. We replace $\kappa, \zeta, \Delta \mathrm{A}$, and $\zeta_{0}$ in Eq. 6 by $\theta, 90-\delta, G . H . A+\lambda$, and $90-\varphi$ respectively and obtain:

$$
\left(\begin{array}{l}
Q_{1}  \tag{7}\\
Q_{2} \\
Q_{3}
\end{array}\right)=\left(\begin{array}{ccc}
\cos \theta & \sin \theta & 0 \\
-\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{array}\right)\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \varphi & \sin \varphi \\
0 & -\sin \varphi & \cos \varphi
\end{array}\right)\left(\begin{array}{ccc}
\sin (G \cdot H \cdot A+\lambda) & 0 & -\cos (G \cdot H \cdot A+\lambda) \\
0 & 1 & 0 \\
\cos (G \cdot H \cdot A+\lambda) & 0 & \sin (G \cdot H \cdot A+\lambda)
\end{array}\right)\left(\begin{array}{c}
\cos \delta \\
-\sin \delta \\
0
\end{array}\right)
$$

There are usually eight unknowns in equation 7: the three coordinates of the digital image perspective center, two parameters of the radial distortion (usually only K1 and K2) and three parameters of the camera orientation, The camera's yaw respective to the equator, its latitude and the combined value of its longitude, the clock error and the intervalometer delay. The first five are common to all sets of sun's exposures. The last three are unique to each camera orientation and set of exposures.

## 4. Corrections for celestial refraction and sun's parallax

The sun's zenith distance is decreased by the air refraction. The accurate value of the needed correction depends on the air pressure, temperature and humidity but for the purpose of camera calibration the approximate value $\Delta_{1}=58^{\prime \prime} \tan \zeta$ is quite good. The suns parallax is caused because of the ratio between the earth radius and the distance to the sun. It is not zero as we assume when we choose the geographical coordinates system. Its accurate value varied with the distance to the sun, maximum in January and minimum in July. An average value of $\Delta_{2}=-8.8^{\prime \prime} \sin \zeta$ is quite good for our purpose.
The correction in radians is therefore
$\Delta=0.0002812 \tan \zeta-0.00004266 \sin \zeta$ This correction effect the image coordinates.
Refering to eqation 3 and 7 we obtain:
$\Delta_{x}=\frac{\text { osample }}{\partial z} \Delta=c \frac{\frac{\partial Q_{1}}{\partial \zeta} Q_{3}-Q_{1} \frac{\partial Q_{3}}{\partial \zeta}}{Q_{3}^{2}} \Delta$
$d_{y}=\frac{\partial \text { line }}{\partial z} \Delta=c \frac{\frac{\partial Q_{2}}{\partial \zeta} Q_{3}-Q_{2} \frac{\partial Q_{3}}{\partial \zeta}}{Q_{3}^{2}} \Delta$
where $: \Delta=0.0002812 \tan \zeta-0.00004266 \sin \zeta$
The values of these corrections are few pixels in direction perpendicular to the trajectory of the sun's image on the digital photograph. They are almost constant and effects mainly the the exterior orientation parameters of the exposure set z 0 and $\kappa$. However the maximal value of the corrections in the trajectory direction is only a sub pixel but it is proportional to the distance of the sun's image from the photo center and therefore effect the cameras interior orientation elements, mainly the value of its principal distance.
Since usually the necessary information to determine these corrections, the camera's orientation and the zenithal distance of the sun at each moment of exposure is unknown, the process of observation adjustment must be carried out twice: The first step is done with the orginal sun's image coordinates. From this step we obtain wrong interior orientation elements but approximate values of the needed data. The corrections are calculated using this information together with the known approximate geographical coordinates of the point of exposure. Then the final step of observation adjustment is done, using the corrected image coordinates. From this step we obtain the final values of the camera's interior orientation.

## 5.Photogrammetric measurements

The sun's semi diameter is something between $15.8^{\prime}$ and 16.3 ', depend on the year season. The sun's image on the
digital photograph is much bigger. The average radius of the area with maximal grey level is about 19' and it is surrounded by a wide halo. It is not easy to find the sun's image border. Furthermore even if the sun is a sphere, its image in a distortionless photograph should be an ellipse. It is well known that the center of this ellipse is not the image of the sphere center. Moreover, the photograph has radial distortions which are not equal all around the sun's image. The distortions deform the sun's image to something like an ellipse which its major axes are not parallel to the sensor lines and columns. The right way to determine the image coordinates of the sun's center is:

1. 1 To measure the coordinates of points around its circumference
2. To correct them for radial and decentring distortions
3. To form a cone which its vertex is the camera perspective center and to find its symmetry axes.
4. To determine the photo coordinates of the sun's center by intersection of this line with the sensor surface.
As a result, the image coordinates are different at each iteration of the calibration solution.

## 6 Practical test and accuracies

The test was based on two sets of the sun's images. The sun's trajectory upon the camera's sensor, in the 192 images of the first set, was close to the sensor lines direction. In the 108 images of the other set, it was perpendicular to it. The photo coordinates of about 280 points around the sun's image at each photograph were determined automatically and an equation of ellipse was fitted to them. The mean square error of the deviations of these points from the best fit ellipse was 0.17 pixels. Therefore, the theoretical calculated m.s.e. of the coordinates of its center is about 0.02 pixel. The real accuracy of the sun's image coordinates is of course not so good. After the corrections for celestial refraction and sun's parallax and the final adjustment of the cameras orientation elements, the m.s.e. of the 300 residuals was 0.27 pixels. Possibly small movements of the camera or the tripod during the photography can explain the big difference between the theoretical and practical values of the mean square errors. Anyhow the value of the element along the inverse matrix diagonal which is corresponding to the camera's principal distance is 3.8 which is almost two orders of magnitude better then that obtained by the test field method for the same lens. As a result the mean square error of the camera's
principal distance is: $0.27 * \sqrt{3.8} \approx 0.5_{\text {pixels }}$

## 7.Summery and conclusions

The use of the sun for camera calibration purposes seams from first glance to be difficult for implementation.
Practically, with little experience, it became an ideal method for amateur cameras that need frequent calibration. The accuracy obtained by this method is very good. The method is suitable especially for narrow angular field lens since as far as the lens field is narrow the needed time for a set photography is shortened, about an hour to an hour an six minutes (depend on the year season) for each $15^{\circ}$ field angle. There is no need for knowledge in photogrammetry and surveying to use the method and it can be implemented at any place were the sun's altitude is more then $25^{\circ}$ for more than an hour.

## 8. References

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