SPATIAL AND DYNAMIC MODELING TECHNIQUES FOR LAND USE CHANGE DYNAMICS STUDY

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ABSTRACT

This paper presents a prototype of a simulation model based on cellular automata (CA), and multi criteria evaluation, integrated with Geographic Information System (GIS). Multi criteria evaluation procedure is used to derive behavior-oriented transition rules. The CA model is built within a grid-GIS system of ARC/INFO GIS using arc macro language to facilitate easy access to GIS databases for constructing the constraints. A suitability-based cellular automata model has been developed to simulate land use change dynamics through the concepts of 'probability of happening of the dynamic phenomenon' and 'suitability of the land for the dynamic phenomenon'. Land degradation is the dynamic phenomenon that has been modeled in the present study. It can be used as a useful planning tool to test the effects of different land use change scenarios.

1. INTRODUCTION

Geographic Information Systems (GIS) provide rich spatial databases but have been traditionally static. The coupling of dynamic models to GIS provides an insight to the evolution of spatial phenomena as discussed by Grossman and Eberhardt (1992). Differential equations and partial differential equations have been the mathematical tools of choice for most of the dynamic models that have been developed. Toffoli (1984) and Toffoli and Margolus (1987) proposed cellular automata models to replace differential equation models.

The theory of cellular automata was first introduced by John (1966). One of the best known and pioneering studies in this area was done by John et al. (1982), Gardener (1974), which emerged in Conway's Game of life. While this work was essentially abstract, it demonstrated that the repeated application of very simple rules to some random initial state could generate interesting, and recurring patterns as the state of the system evolved. In recent years they have been increasingly used in the simulation of complex systems such as biological reproduction, chemically self-organizing systems, propagation phenomenon, and human settlements. A series of urban models based on CA techniques have been reported (Batty and Xie, 1994; White and Engelen, 1993; Wu 1998; Li and Yeh, 2001). There are numerous studies on the detection of land use change using remote sensing and GIS (Howarth 1986, Jensen et al. 1995, Li and Yeh, 1998). However, there is a general lack of studies on the simulation of land use changes because of their complexities. Lo and Xiaojun (2002) have studied the drivers of land use / land cover changes in Atlanta using remote sensing data and employed a process-based CA model to simulate the urban growth and landscape changes. Li and Yeh (2002) presents a new method to simulate the evolution of multiple land uses based on the integration of neural networks and cellular automata using GIS.

Traditionally GIS, means a system capable of storing, manipulating, analyzing and displaying spatial data. It lacks the ability to model a dynamic phenomenon in spatial-temporal domain. But it can act as a platform on which further modeling capabilities can be built. In the present study, an attempt has been made to enhance the spatial modeling capability of a GIS to address spatial dynamic modeling problem, through Cellular Automata and Multi-Criteria Evaluation procedures. A suitability-based cellular automata model has been developed, which can evolve an organized global pattern from locally defined behavior, because of the interaction between a site and its neighborhood. State transitions are governed by transition rules, which are universally applied and are defined through multi-criteria evaluation procedures. This can act as a generic framework, which can handle any kind of spatial dynamic phenomenon. The method has been tested and evaluated by modeling land degradation process.

2. METHODOLOGY

2.1 Cellular Automata

A Cellular Automata system usually consists of four elements – cells, states, neighborhoods and rules. Cells are the smallest units, which manifest adjacency or proximity. The state of a cell can change according to transition rules, which are defined in terms of neighborhood functions and other suitability criteria. CA are cell-based methods that can model two-dimensional space. Because of this underlying feature, it becomes easy to use CA to simulate land use change, urban development and other changes of geographical phenomena.

Most current GIS techniques have limitations in modeling changes in the landscape over time, but the integration of CA and GIS has demonstrated considerable potential (Itami 1988, Deadman et al. 1993). The limitations of contemporary GIS include, its poor ability to handle dynamic spatial models, poor performance for many operations, and poor handling of the temporal dimension (Park and Wagner 1997). In coupling GIS with CA, CA can serve as an analytical engine to provide a flexible framework for the programming and running of dynamic spatial models. Masanao and Couclelis (1997) addresses a generalized modeling formalism of CA, which is extended with Geo-algebra capable of expressing a variety of dynamic spatial models within a common framework.

2.2 State-based Cellular Automata

In a standard CA model, the state is usually used as the main attribute to describe the development of a cell. Any cell cannot take on more than one state simultaneously, although the state can change from one to another in different periods. In land degradation simulation, the most general state for a cell is degraded or not degraded. The essence of CA is that the states of the neighboring cells influence the state of the central cell. The notion of neighborhood is central to the CA paradigm (Couclelis 1997), but the definition of neighborhood is rather relaxed. A simple model is to project the state of a central cell using a 3X3 window to count the distribution of states in its neighboring cells. Land use classes were grouped into three categories: degraded, degradable or prone to degradation and non-degradable, which becomes the state of a cell.

2.3 Suitability-based Cellular Automata

More sophisticated CA systems have been further developed to simulate urban growth through the concepts of 'development probability' and 'development suitability' (White et al. 1997). This kind of simulation assumes a relation between the states (developed or not), development probability and development suitability:

$$S^{t+1}\{x,y\} = f\,(P^t\{x,y\})$$

$$P^t\{x,y\} = f'(DS^t\{x,y\})$$

Where $S\{x,y\}$ is the state at location $\{x,y\}$; $P\{x,y\}$ is the probability of transition to the state S at the location; and $DS\{x,y\}$ is the suitability of conversion to the state S. f and f′ are transition functions. Suitability-based cellular automata differs from state-based cellular automata, in which the state of a cell not only depends on the state of its neighborhood, but also checks for its degree of suitability (DS) for development, which in turn is based on a number of terrain-related factors.

This logic has been extended in the present study to model land degradation dynamics. It is obvious that the CA simulation heavily depends on the calculation of suitability score based on neighborhood configuration. The suitability of a cell for degradation is usually evaluated according to location factors and site properties. The conversion criterion is that cells with high degree of suitability will be first selected for degradation. Much work has been done on the evaluation of land suitability, which usually involves multi criteria evaluation techniques (Novaline et al., 2001).

Land suitability, which describes the potential of a cell for a specific type of land use, can act as an important constraint in the CA model. For example, we may allow faster land degradation in dry land area and more restricted or slower degradation in vegetated area. Therefore, suitability plays an important role in affecting the state or the transfer of the state of a cell in an idealized situation. Suitability scores should be re-computed in each iteration to achieve compatible land use. The model may be expressed as a two-dimension model, including states S(t) and suitability DS(t):

$$(S^{t+1}, DS^{t+1}) = f\,(S^t, DS^t, N)$$

where N, is the neighborhood providing input values for the transition function f.

2.3.1 Multi Criteria Evaluation technique for Land Suitability analysis: Multi criteria decision-making (MCDM) problems involve a set of alternatives that are evaluated on the basis of a set of evaluation criteria. The multi criteria decision analysis has recently received considerable attention in GIS. Combining different factors, some exclusionary and some expedient, requires a weighting factor. Alternate approaches to GIS-based multi criteria analysis have been suggested to overcome the problem of weighting and data integration. Analytic Hierarchy Process (AHP) has been identified as a weighting strategy and Compromise Programming (CP) technique has been identified for data integration (Novaline et al. 1996, Deekshatulu et. al. 1999).

AHP is an approach that can be used to determine the relative importance of a set of activities or criteria. The first step of the AHP is to form a hierarchy of objectives, criteria and all other elements involved in the problem. Once the hierarchical structure has been formed, comparison matrices are to be developed. These are evaluations made by the decision-makers on the intensity of difference in importance, expressed as a rank number on a given numerical scale, for each level in the hierarchy. From these weights, priorities are determined. An expert would be asked to make pair wise comparisons between two factors at a time, decide which factor is more important, then specify the degree of importance on a scale between 1 and 9 in which 9 is most important. These evaluations would result in reciprocal matrices of the components of each level against the items in the level above. Consistency of the matrix has to be checked and eigen value of the matrix has to be found. Upon normalization of the eigen vector corresponding to maximum eigen value, each factor coverage would have only one weight associated with it.

Another important problem in GIS is how to efficiently integrate data from various sources. Weighted linear additive model is the one that is widely used for data integration and is done with the help of algebraic functions available in any commercial GIS package. In this, a total compensation between criteria is assumed, meaning that a decrease of one unit on one criterion can be totally compensated by an equivalent gain on any other criteria. Ideal Point Analysis, a Compromise Programming technique, is a method to arrive at non-compensatory solution. It measures the deviations from the ideal point in each data layer and a min-max rule is applied wherein minimum of the maximum weighted deviations are sought for getting a composite layer.

The best compromise solution is defined as that which is at the minimum distance from the theoretical ideal.

Driver variables for land degradation process include: terrain-related variables like drainageJ slopeJ soil textureJ dry land areaJ neighborhood agglomeration; climatic variables like rainfallJ aridity index; socio-economic variables like populationJ livestock and tube well density. Neighborhood agglomeration layer was derived by counting the number of degraded cellsJ within a circular neighborhood with radius of 5 cells from each cell and assigning the sum to the center cell. IRS LISS III data was used to interpret dry land areaJ soil texture and drainage combined with ground truth. Slope was generated from the contours at 1:50J000 scale toposheet. Aridity index was calculated from the meteorological data on temperatureJ potential evapo transpiration and rainfall. Socioeconomic data on populationJ livestock and tube well density were gathered from census data.

These driver variables acted as the land degradation criterion in the multi criteria evaluation procedure. Each of them was given appropriate weight adopting AHP procedure.

<u>Weights given for driver variables</u>

Neighborhood Agglomeration	: 100
Drainage	: 90
Soil Texture	: 80
Slope	: 70
Dry land area	: 65
Aridity Index	: 60
Rain fall	: 50
Population density	: 40
Livestock density	: 30
Tube well density	: 10

For exampleJ neighborhood agglomeration was given higher preferenceJ because in a cellular automata modelJ the state of a cell would depend on the state of its neighborhood. Next preference goes to terrain-related variables: drainageJ soil textureJ slope and dry land areaJ owing to their obvious influence on land degradation; followed by climatic variables: aridity indexJ rainfall and socio-economic variables: populationJ livestockJ tube well density. To derive the areas suitable or prone to degradation based on the said criteriaJ all the criterion maps were integrated adopting Ideal Point AnalysisJ a Compromise programming technique. Suitability score DS is computed using the distance metric as below:

$$DS = \left[\sum_{i=1}^{n} \beta_i^p (x_i^* - x_{ik})^p \right]^{1/p} \qquad \text{Equation (1)}$$

where i is the map layerJ β is the criterion preference J x_i^* is the ideal point J x_{ik} is the cell value in k_{th} cell for i_{th} parameter and p is the factor which leads to non-compromising solution. p can take values from 1 to infinity. Different values for p were tried and p was set at 4 (Jose & LucienJ 1993). Climatic and socio-economic data were simulated every year using their growth rate value computed per year. And the land degradation suitability was recomputed every year.

2.4 Probability-based Cellular Automata

TraditionallyJ CA simulation only uses a binary value to address the status of conversion based on the calculation of probability. The probability of conversion is calculated based on some kind of neighborhood function. UsuallyJ the probability is further compared with a random value to decide whether a cell is converted or not (1 for converted and 0 for non-converted). In our modelJ the status of cell has a continuous suitability value between 0 and 1 to represent the stepwise selection or conversion process. A cell will not be suddenly selected or converted.

A stochastic disturbance term is added to represent unknown errors during the simulation. This can allow the generated patterns to be closer to reality. Suitability values are converted into probability values by introducing a stochastic disturbance parameter α. Thus this rule defines the probability of site selection in terms of land suitability. Since the neighborhood is used in evaluationJ land suitability here is dynamicJ which means that the maximum score of land suitability is changing over simulation time. While transforming the evaluation score into development probabilityJ one can use the maximum score of evaluation during each simulation time as a benchmark because it represents a relative availability at the time when the decision is made. The probability is defined in a nonlinear form to the evaluation score:

$$P^t_{xy} = \begin{cases} \exp\left[\alpha\left((DS^t_{xy} / DS^t_{max}) - 1\right)\right] & \text{if } DS^t_{xy} \neq 0 \\ 0 & \text{if } DS^t_{xy} = 0 \end{cases} \qquad \text{Equation (2)}$$

where P^t_{xy} is the probability of land conversion from degradable to degraded land at the location xy at time t; DS^t_{xy} is the land suitability score at the same location at time t; DS^t_{max} is the maximum score of land suitability at the simulation time t of calculation; and α is the dispersion parameter to be input through the first rule. The higher the value of αJ the more stringent is the site selection process. The exponent function in the equation (2) makes α to behave in the required formJ likeJ if you decrease α probability increasesJ thereby introducing stochastic disturbance in the simulation.

See for exampleJ how the probability value changes for a suitability value of 0:

when $\alpha = 4$ J exp(-4) = 0.018
when $\alpha = 1$ J exp(-1) = 0.3678
when $\alpha = 10$J exp(-10) = 0.000045 (which is almost equal to 0)
If $DS^t_{xy} = DS^t_{max} = 1$ J then exp(0) = 1
(i.e) if suitability is high and equals 1J then probability = 1J irrespective of any value of α

ThereforeJ α can take a value between 0 and 10. The (– 1) term in $[((DS^t_{xy} / DS^t_{max}) - 1)]$J in the equation (2) makes the probability value range between 0 and 1.

Because of time limits and information barriersJ the best site is not always chosen. Less desirable sites still have a chance of being degraded. ThusJ this rule introduces stochastic disturbance to the system. Various values of α were tried ranging between 1 and 10 and α was set at 4 in the present study based on the calibration analysis. A flow chart describing the methodology is shown in figure 1.

3. RESULTS AND DISCUSSION

The CA model is built within a grid-GIS system of ARC/INFO GIS using arc macro language. The model was applied on parts of degradation-prone district in Andhra Pradesh, India covering an area of 6410 square kilometers. Land degradation maps corresponding to the years 1989, 1997 and 2002 were provided by National Remote Sensing Agency, India. 1989 data was used as the seed and 1997 data was used for calibration. 2002 data was used for validation (figure 2). Prediction of the land degradation process was done for the next 10 years till 2012 (figure 3). Further prediction can also be done by appropriately predicting the growth rate of climatic and socio-economic variables that were used in the model. A simple formula for calculating growth rate is $g_t = ((x_t / x_{t-n}) - 1) * 100$, where g_t is the growth rate in period t, x is the variable being examined and n is the time period of interest.

For each iteration (corresponding to one year), cells beyond certain probability are selected. The threshold value is learnt through the calibration process. During the calibration phase, the model was also tested with different values of α and was finally set at 4. The preference value given for the driver variables were also changed and the corresponding results were checked during the calibration phase.

For the calibration data set corresponding to the year 1997, the percentage of correctly predicted cells is 78.27%. For the validation data set corresponding to the year 2002, the percentage of correctly predicted cells is 77.68% (figure 2).

Calibration report for 1997	Validation report for 2002
Correctlypredicted:5568209 cells	Correctlypredicted:5532171cells
Commission error:784343 cells	Commission error :815829 cells
Omission error:761481 cells	Omission error:773801cells

Correctly predicted cells include, degraded and non-degraded cells present in both original land degradation map and the predicted land degradation layer. Commission error indicates cells, which were not found as degraded in original land degradation map, but has been predicted as degraded. Omission error indicates cells, which were found as degraded in original land degradation map, but has not been predicted as degraded.

Dynamic terrain-related processes are complicated in nature as number of factors plays a role in reality. Some of the drivers, which could have played a role in the degradation process, could have been omitted, possibly because they could not be recorded or monitored.

In the present study a probability-based cellular automata has been implemented. A state-based cellular which is based on the neighborhood configuration alone has evolved into suitability-based cellular automata and then into a probability-based cellular automata with the inclusion of suitability score and stochastic disturbance factors. In state-based cellular automata, the state of a cell will depend on the state of the neighboring cells. It becomes more logical to include the land suitability for degradation score as another factor contributing to the degradation process in addition to neighborhood configuration. Also, the stochastic disturbance factor helped in creating some randomness and took care of some of the unknown errors in the simulation.

4. CONCLUSION

Integrated CA-GIS approaches can enhance the current poor spatial dynamic modeling capability of GIS (Park and Wagner 1997). CA models can be completely developed within GIS for easily accessing the information stored in the GIS database during the modeling processes. Constraints for modeling can be defined using GIS and remote sensing data. Remote sensing can be used to obtain land use and other land-related data, and this data can be transformed, so that it can be used in GIS for analysis and modeling. Therefore, the development of CA within GIS greatly enhances the ability of dynamic spatial modeling within GIS.

Achieving prediction accuracies of the order of 78% is a significant task, as dynamic terrain-related processes are complicated and there could be a possible omission of some of the driver variables, which could not be recorded or monitored. The integrated CA-GIS framework proves to be a promising environment, wherein a variety of spatial-dynamic phenomena can be modeled.

The model developed for simulating the spatial dynamic process can be used as a planning tool to test the effects of different land use change scenarios. Cellular Automata are seen not only as a framework for dynamic spatial modeling, but also as a paradigm for thinking about complex spatial-temporal phenomena and an experimental laboratory for testing ideas.

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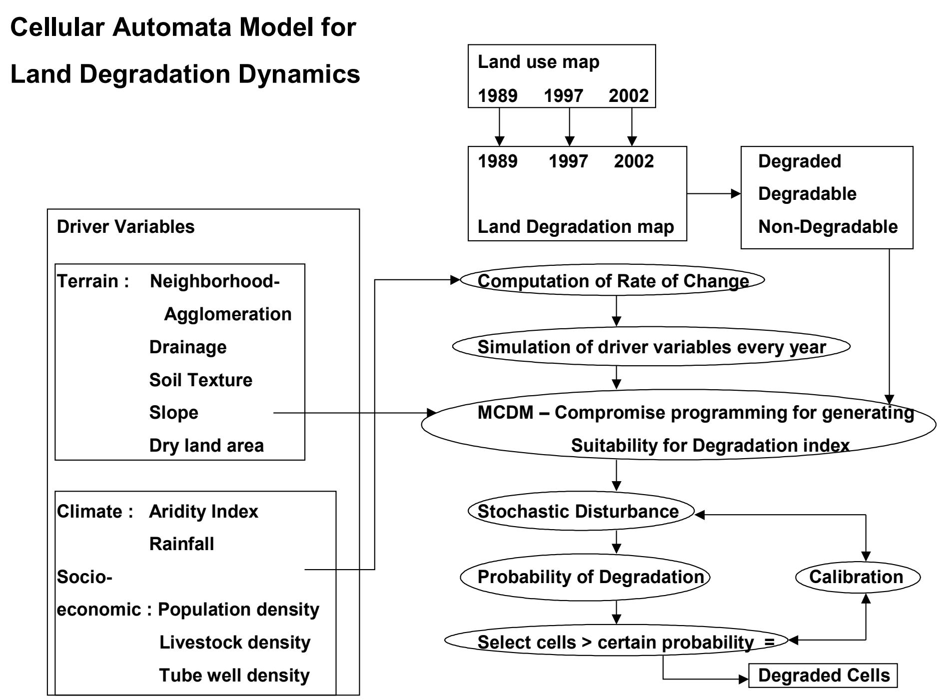

Figure 1. Flow Chart describing the Cellular Automata methodology

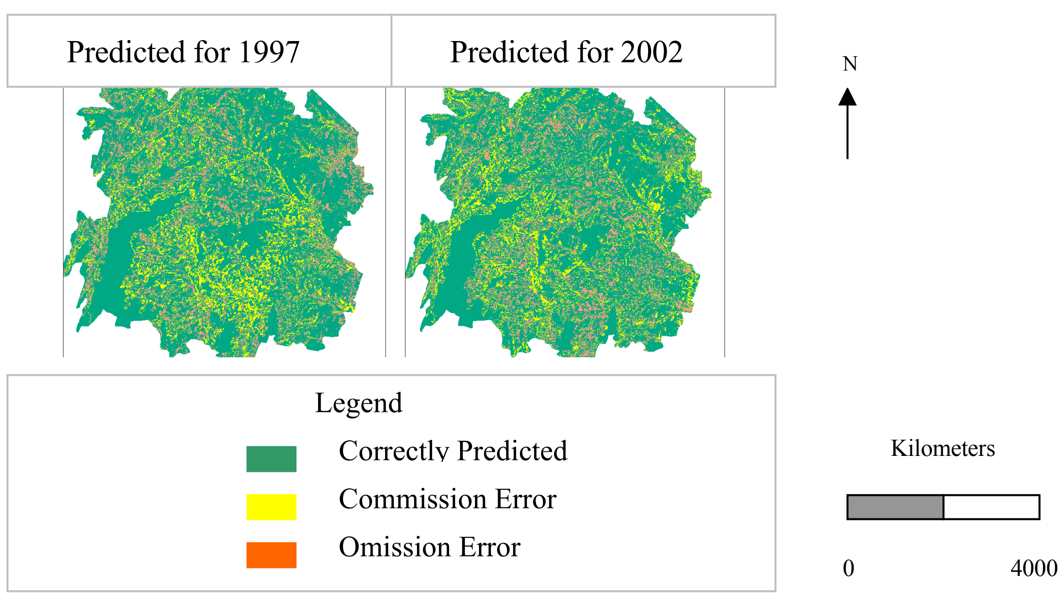

Figure 2. Validation results for Cellular Automata simulation of Land Degradation

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