

# STUDY ON CHROMATIC ABERRATION OF TWO FISHEYE LENSES

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### ABSTRACT:

Chromatic aberration is observable especially with wide angle lenses. This paper presents a study on chromatic aberration of two fisheye lenses. Both tested lenses produce images with obvious colour aberration. Four different mathematical models were tested to compensate the aberration of distinct colour channels; shift and scale, affine, projective and polynomial. The model parameters were obtained by registering the red and blue channels to the green one. The performed experiments revealed that shifting and scaling of the colour channels is adequate to compensate most of the chromatic aberration of tested fisheye lenses. It also turned out that same corrections can not be applied to different aperture and focus settings. Registering of the colour channels improved the image quality notably.

## 1. INTRODUCTION

A light beam entering a medium with different optical density will be refracted. This refraction follows the Snell's law

$$n_i \sin \theta_i = n_t \sin \theta_t \quad (1)$$

where  $n_i$  and  $n_t$  are the refraction indices and  $\theta_i$  and  $\theta_t$  are the incidence angles of the beams in different mediums. Refraction indices  $n_m$  for different mediums are frequency dependent (Hecht, 2002). Therefore different frequencies will have distinct refraction angles which leads to the occurrence of dispersion. A prism splitting white light to different colours is an example of dispersion. All mediums except vacuum are dispersive.

The fact that different colours travel distinct routes through photographic optics causes chromatic aberration (CA) on the image. As illustrated in Figure 1 different wavelengths have separate focal points. The axial distance between these points is called axial (or longitudinal) chromatic aberration. It appears all over the image and causes fringes around the objects. Axial chromatic aberration can be reduced by stopping down the lens (van Valree, 2007). Because of the dispersion two colours, for example red and blue, will end up to different locations on the image as depicted in Figure 2. The distance between these locations is called lateral chromatic aberration. Lateral aberration increases towards the image borders. Unlike with axial aberration stopping down the lens does not necessarily reduce the magnitude of the lateral chromatic aberration. In lens design chromatic aberration can be compensated using special glass and achromatic lens doublets. The purpose of this paper is to study the lateral chromatic aberration of fisheye lenses.

Typically the chromatic aberration is compensated by shifting and scaling the red and blue colour channels to match the green channel. Correction can be carried out manually in a commercial image processing software to gain satisfactory results. Experiments on manual correction of fisheye images can be found in (van den Heuvel et al., 2006). An automatic method for determining the shift and scale parameters is presented in (Kaufman et al., 2006). In latter the correction parameters for three different lenses (17, 29 and 50mm) were calculated based on point correspondences. In image based 3D-measurements one way to avoid the inconvenience of chromatic aberration is to handle the colour channels as separate images (Luhmann et al., 2006).

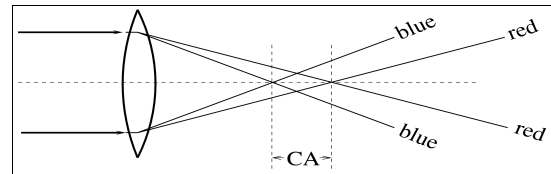


Figure 1. Axial chromatic aberration.

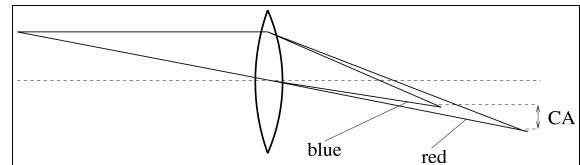


Figure 2. Lateral chromatic aberration.

In this paper four different correction models for fisheye images will be investigated; shift and scale, affine, projective and polynomial. The motivation to test also affine, projective and polynomial models is to see if they can improve the results obtained with the shifting and scaling. Also the effect of aperture and focus setting on the magnitude of aberration has been examined.

## 2. TESTED LENSES

The test lenses were Sigma 8mm 1:4 D EX Fisheye and AF DX Fisheye-Nikkor 10.5mm f/2.8G ED. For the test photographing both lenses were mounted to a Nikon D100 digital camera body. The sensor of Nikon D100 has 3008x2000 pixels. Sigma lens is originally designed for film SLR cameras. The diameter of the circular image produced by the Sigma lens was shorter than the camera sensor width but longer than the sensor height. So the left and right ends of the image were left blank and the top and bottom parts of the circular image were cut off (Figure 3). Sigma uses special low dispersion (SLD) lens elements to compensate chromatic aberration.

The tested Nikkor lens is a medium price lens for digital SLR cameras. It produces an image which covers the whole CCD sensor. The image diagonal corresponds to an opening angle of 180 degrees. Nikkor lens is also made of extra low dispersion (ED) glass.



Figure 3. Image taken with the Sigma lens.



Figure 4. Image taken with the Nikkor lens.

The existence of chromatic aberration is clear on both images. Figure 5 presents zoomed details of preceding figures. The black rectangular in the image taken with Sigma lens seems to have red and blue borders while in the other image taken with Nikkor lens it seems to have magenta and green borders. Two sets of colour profiles taken from the middle of the details presented in Figure 5 are illustrated in Figures 6 and 7. In both images the red channel seems to be shifted more than the blue channel. In the case of Nikkor lens the blue channel has shifted to the same direction as the red one.

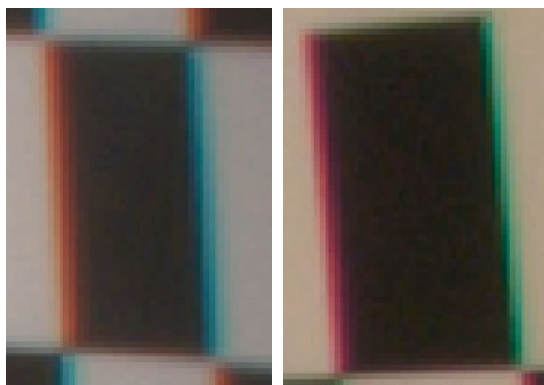


Figure 5. Zoomed details from Figure 3 and 4, respectively. Chromatic aberration is clearly visible.

### 3. CORRECTION MODELS

The green colour channels of the images were chosen to be the reference channels. The red and blue channels were registered to the green ones using four different geometrical models. The

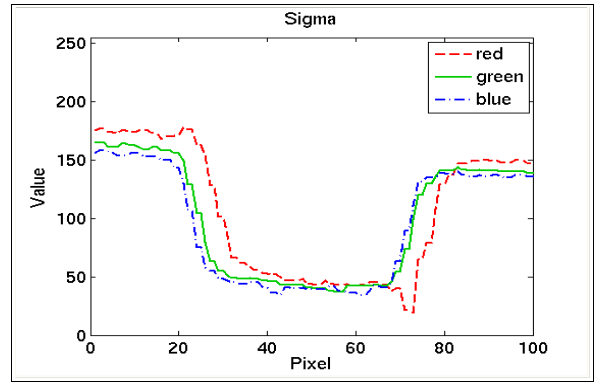


Figure 6. Colour profiles of the Sigma lens.

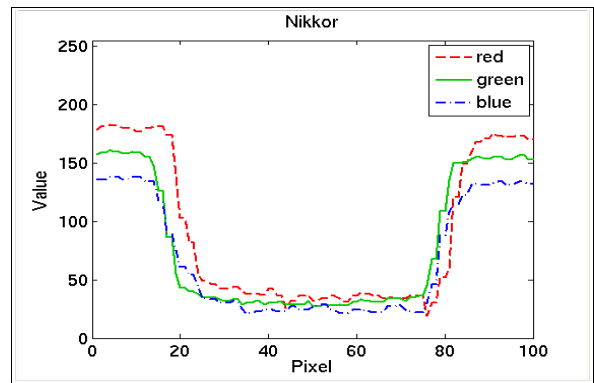


Figure 7. Colour profiles of the Nikkor lens.

most simple model included shifts ( $\Delta x$  and  $\Delta y$ ) and a scale ( $s$ ),

$$x_{corr} = x_{obs} s + \Delta x; \quad y_{corr} = y_{obs} s + \Delta y \quad (2)$$

The affine model consisted of six parameters ,

$$x_{corr} = a_0 x_{obs} + a_1 y_{obs} + a_2; \quad y_{corr} = a_3 x_{obs} + a_4 y_{obs} + a_5 \quad (3)$$

and the projective model included eight parameters,

$$x_{corr} = \frac{a_0 x_{obs} + a_1 y_{obs} + a_2}{a_6 x_{obs} + a_7 y_{obs} + 1}; \quad y_{corr} = \frac{a_3 x_{obs} + a_4 y_{obs} + a_5}{a_6 x_{obs} + a_7 y_{obs} + 1} \quad (4)$$

The most complicated model was a polynomial with 16 parameters,

$$\begin{aligned} x_{corr} &= a_0 x_{obs} + a_1 y_{obs} + a_2 x_{obs} y_{obs} + a_3 x_{obs}^2 \\ &+ a_4 y_{obs}^2 + a_5 x_{obs}^2 y_{obs} + a_6 x_{obs} y_{obs}^2 + a_7 \\ y_{corr} &= a_8 x_{obs} + a_9 y_{obs} + a_{10} x_{obs} y_{obs} + a_{11} x_{obs}^2 \\ &+ a_{12} y_{obs}^2 + a_{13} x_{obs}^2 y_{obs} + a_{14} x_{obs} y_{obs}^2 + a_{15} \end{aligned} \quad (5)$$

Contrary to (Kaufmann et al., 2006) the model parameters were determined by registering the whole colour channel at once to the reference channel. The model parameters were adjusted using Gauss-Newton method so that the sum of squared pixel value differences was minimized. This iterative calculation doesn't need point correspondences but it does need initial values for the unknown parameters. In this case they are easy to specify due to the similarity of the different colour channels. The registering process is automatic and uses only the original images as input.

The registering process outputs the model parameters with their

standard errors and the transformed colour channels. Because the applied models have different parameters their standard errors can not be compared directly. Evaluation based on significance levels is also troublesome because there aren't any observations (coordinates of corresponding points) which could be compared to the solved model. So the only way to compare the results is to use the registered colour channels. Both numerical and visual comparisons were carried out. Before comparison the different channels were normalized to have equal mean and standard deviation. This operation compensates the general brightness and contrast differences but not the discrepancies in vignetting. The quality of the registering was evaluated by subtracting different channels and by studying the result.

The performed tests revealed that shifting and scaling of the colour channels is adequate to compensate the chromatic aberration also in the case of fisheye lenses. The results were not improved by applying the more complicated models. Figures 8 and 9 introduce the deformation of red and blue channels relative to the green channel when the shifting and scaling model was applied to the image taken with Sigma lens. Figures indicate that the red channel must be reduced and blue channel expanded. This conclusion concurs with the observations based on the colour profiles in Figure 6. The shift parameter was less significant compared to the scale parameter.

Figures for affine, projective and polynomial models were so similar to the Figures 8 and 9 that they were left out of this paper. The summary of the deformations in x- and y-direction for Sigma lens is presented in Figures 10 and 11. None of the applied models differs clearly from the others.

Figures 12-15 present equivalent plots for the Nikkor lens. As can be seen in Figures 12 and 13 both red and blue channel have to be reduced in order to match with the green one. Also this conclusion is supported by the colour profiles seen in Figure 7. Images taken with Nikkor lens have slightly less aberration than those taken with Sigma lens.

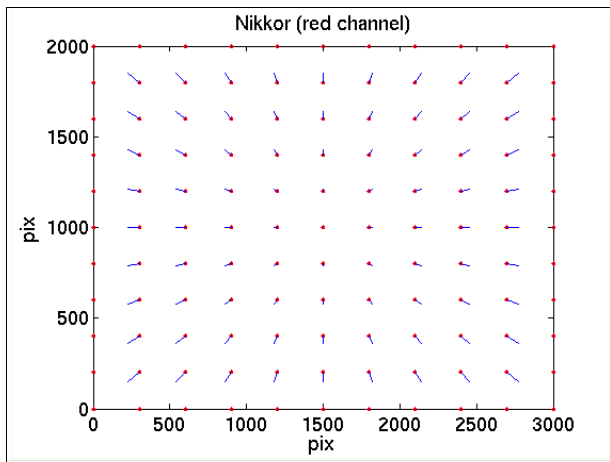


Figure 8. Deformation (x50) of red channel (Sigma lens). The applied model included shift and scale parameters. Note that the imaging circle of the Sigma lens didn't reach the sensor corners.

After transforming the colour channels with determined parameters they were subtracted from each others. The numerical evaluation of the differences of certain channels is presented in Table 16. The means and standard deviations of differences are almost identical and therefore less informative. A more descriptive way to judge the results is to compare image details and see if there are some discrepancies. Figure 17 shows a small crop from the original and corrected image. In this case

the affine model was applied. The improvement is clear, coloured borders have disappeared. Figure 18 illustrates one set of colour profiles corrected with all different models. The differences are almost impossible to distinguish by eye. Figures 19 and 20 represent the equivalent data for the Nikkor lens.

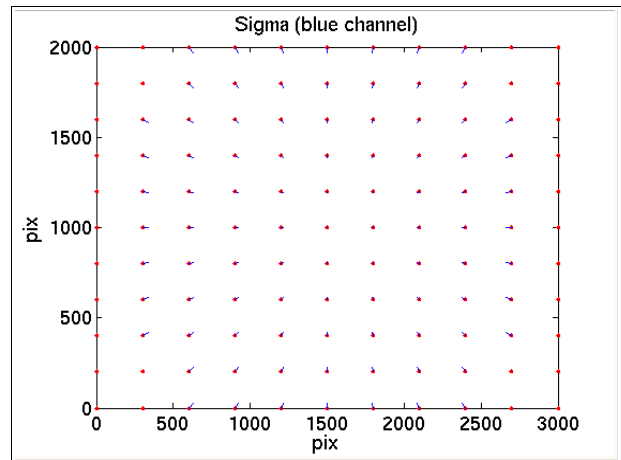


Figure 9. Like in Figure 8. but for blue channel.

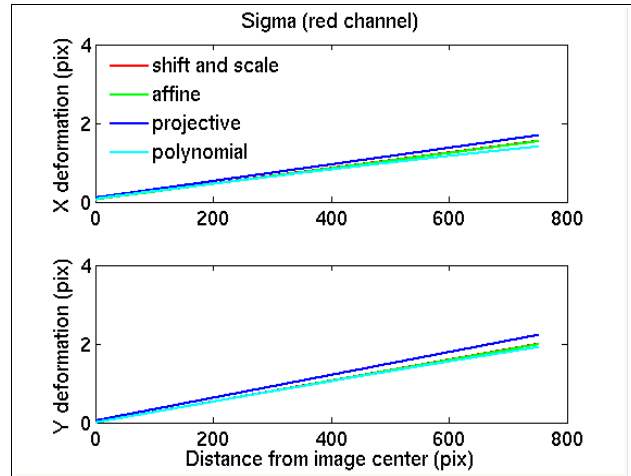


Figure 10. Deformations of the red channel in x- and y-direction (note: due to the similarity the graphs are overlapping).

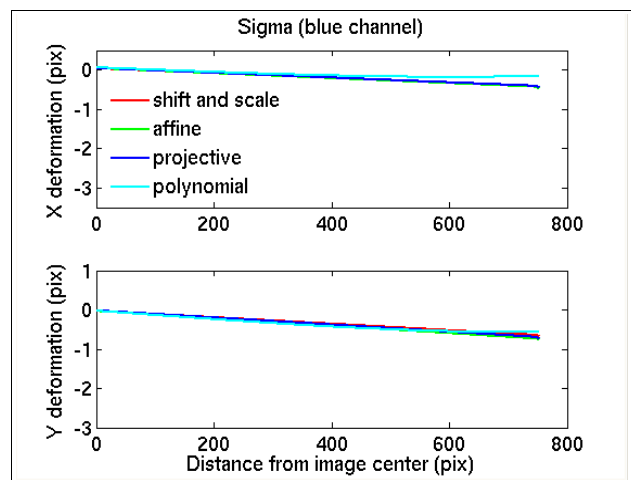


Figure 11. Deformations of the blue channel in x- and y-direction (Sigma lens).

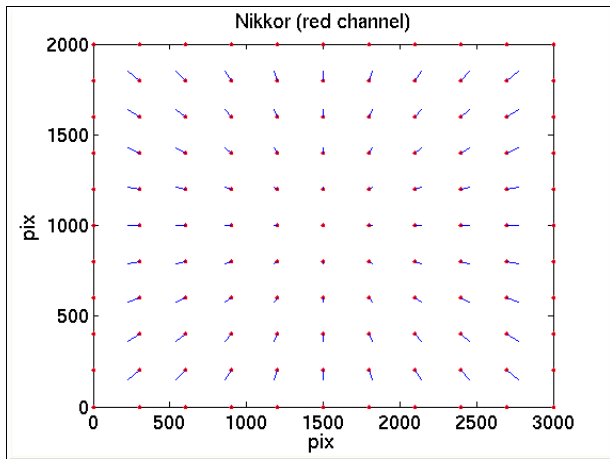


Figure 12. Deformation (x50) of red channel (Nikkor lens). The fitted model included shift and scale parameters.

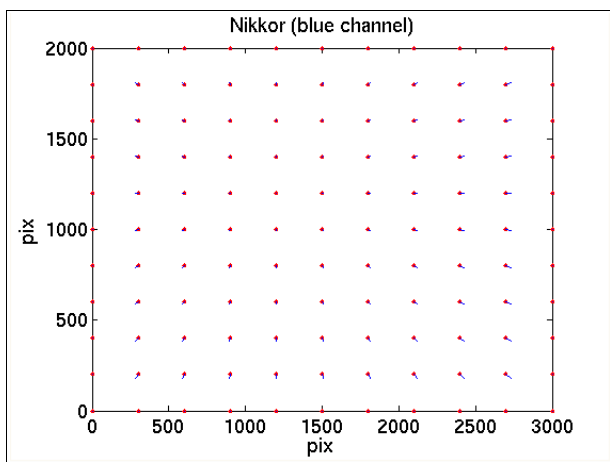


Figure 13. Deformation (x50) of blue channel (Nikkor lens). The fitted model included shift and scale parameters.

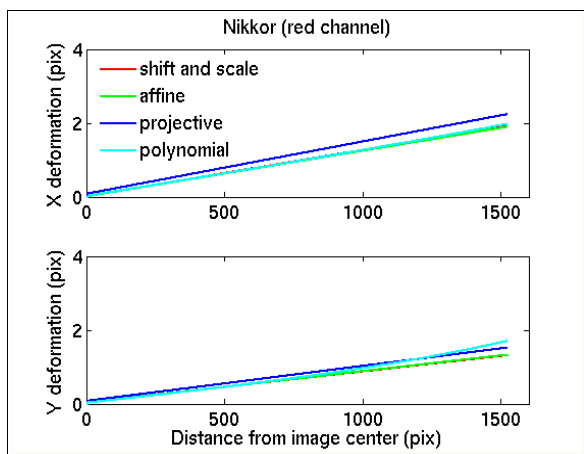


Figure 14. Deformations of the red channel in x- and y-direction (Nikkor lens).

#### 4. APERTURE AND FOCUS

This section describes the effect of aperture and focus settings on the chromatic aberration. Image sets with major aperture numbers and focus settings were collected with both lenses. The image sets were corrected using shift and scale model and the

results compared.

An initial examination of the colour profiles exposed that stopping down the lens will move the green and red profiles towards the image border (see Figure 21) while the blue profile stays almost fixed. Because the green channel was chosen to reference it would be expected that the aberration of the blue channel will change more according to the aperture. Figure 22 shows the maximum aberrations on the image border for the different aperture settings. As expected the aberration of blue channel seems to have a clear trend. Instead the aberration of the red channel is nearly constant.

The colour profiles obtained with maximum and minimum apertures of Nikkor lens are presented in Figure 23. The red channel seems to be shifted more towards the image border when using the smallest aperture. The blue channel instead has moved less than the green one. The results shown in Figure 24 confirm previous observations. Both red and blue channel's aberration will change according to the aperture.

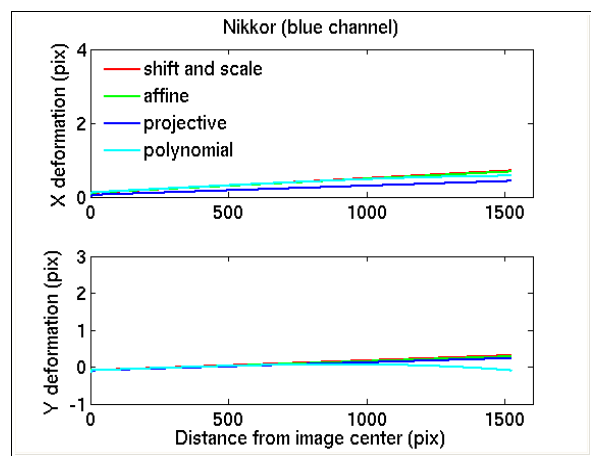


Figure 15. Deformations of the blue channel in x- and y-direction (Nikkor lens).

|                 | Sigma (blue-green) |      | Nikkor (blue-green) |      |
|-----------------|--------------------|------|---------------------|------|
|                 | mean               | std  | mean                | std  |
| original        | 6.19               | 6.63 | 5.88                | 5.4  |
| shift and scale | 5.59               | 5.42 | 5.59                | 6.53 |
| affine          | 5.58               | 5.43 | 5.59                | 6.52 |
| projective      | 5.58               | 5.43 | 5.61                | 6.57 |
| polynomial      | 5.57               | 5.39 | 5.59                | 6.52 |

Table 16. The difference means and standard deviations of subtracted channels. Only minor deviations appear.

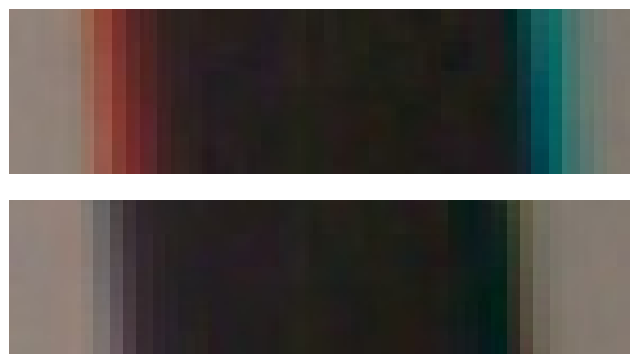


Figure 17. The crop on the top is from the original Sigma image and on the bottom from the image corrected with affine model.

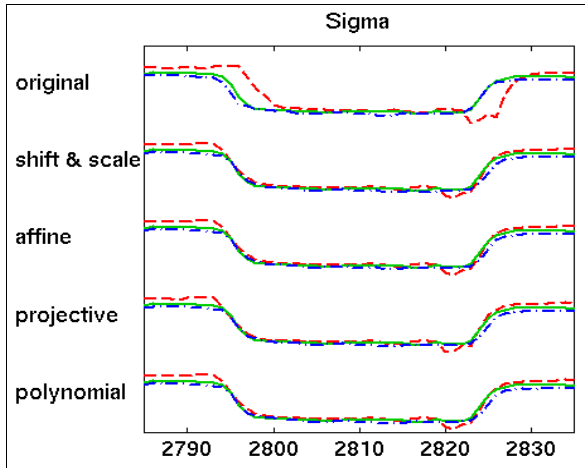


Figure 18. Comparison of colour profiles obtained with different correction models (Sigma lens).

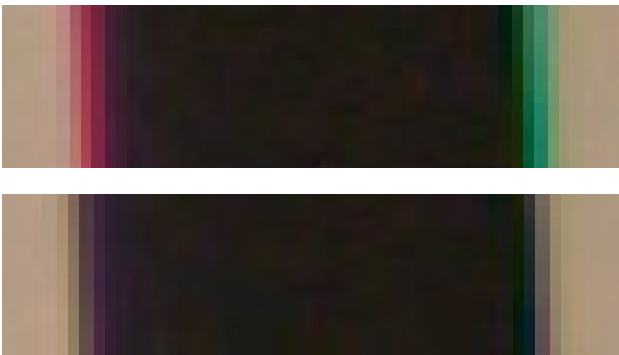


Figure 19. The crop on the top is from the original Nikkor image and on the bottom from the image corrected with affine model. The difference is obvious.

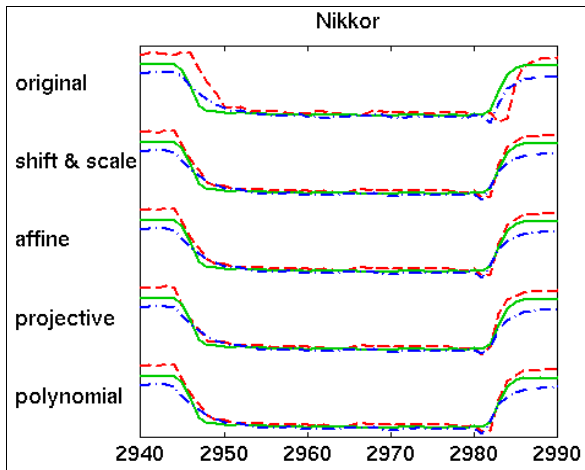


Figure 20. Comparison of colour profiles obtained with different correction models (Nikkor lens).

Changing the focus modifies the images so much that the colour profiles taken with different focus settings can't be compared directly. The image area will be slightly different and colour profiles will be moved dozens of pixels. Another problem is the blurring of the image if the camera hasn't been moved. But of course the different channels can be registered and the results analysed. Figures 25 and 26 present the results of the registration process. The aberration of the Sigma lens seems to be almost independent of focus setting. Instead the aberration of

Nikkor lens varies according to the focus. The aberration of the red channel slightly increases when the focus distance grows. The aberration of the blue channel reduces remarkably while focusing to the infinity. The change in aberration of the blue channel is almost two pixels.

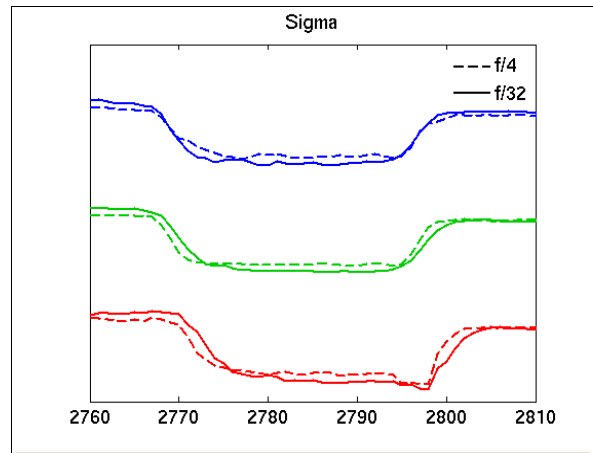


Figure 21. Colour profiles obtained with maximum and minimum aperture (Sigma lens).

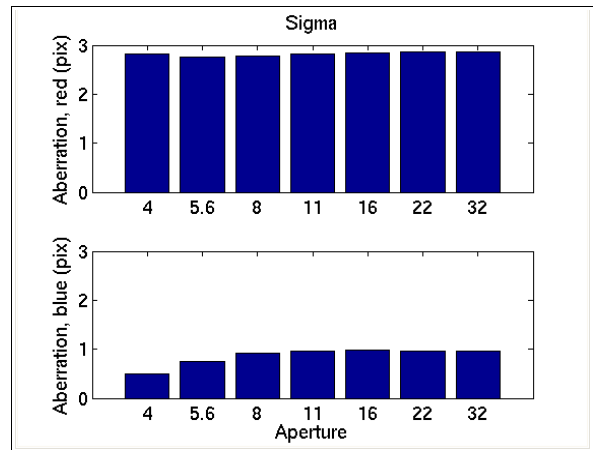


Figure 22. Statistics of the maximum aberration on the image border with different apertures (Sigma lens).

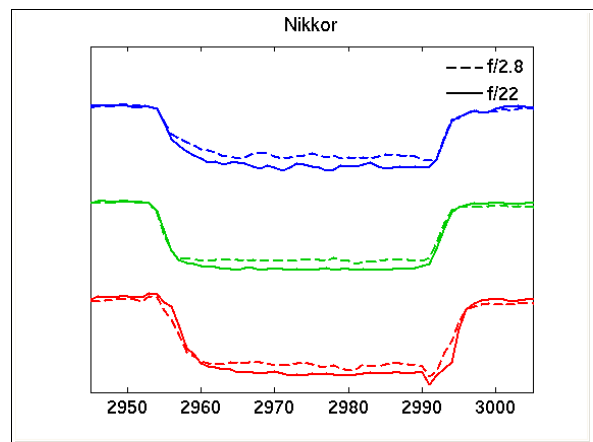


Figure 23. Colour profiles obtained with maximum and minimum apertures (Nikkor lens).

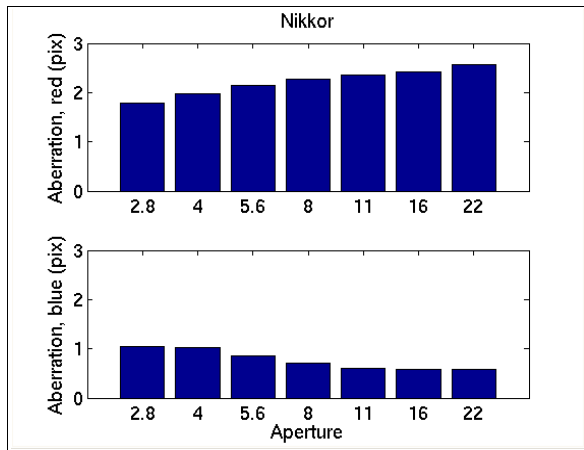


Figure 24. Statistics of the maximum aberration on the image border with different apertures (Nikkor lens).

### 5 CONCLUSIONS AND DISCUSSION

This paper presented a study on the chromatic aberration of two fisheye lenses. The tested Sigma and Nikkor fisheye lenses have notably chromatic aberration on the image borders.

Examination of four different models revealed that shifting and scaling is adequate to compensate most of the chromatic aberration of tested lenses. The three other models, affine, projective and polynomial, didn't improve the result.

The maximum chromatic aberration of the tested Sigma lens with aperture number  $f/22$  was 2.9 pixels for the red and 0.8 pixels for the blue channel. Corresponding values for Nikkor lens were 2.6 and 0.5.

According to the performed tests aperture has effect on the magnitude of the aberration. In the case of the Sigma lens the red channel didn't react to the aperture change but the aberration of the blue channels increased along with the aperture number. The difference between the smallest and biggest aberration was approximately 0.5 pixels at the image border. Tests with Nikkor lens showed that the aberration of the red channel increased when the lens was stopped down. Instead the aberration of the blue channel had an opposite behaviour. The maximum change for the red channel was around 0.8 pixels and for the blue channel 0.6 pixels at the image border.

Also the change of focus have an effect on the chromatic aberration. The maximum aberration difference with Nikkor lens was around two pixels, while the Sigma lens was almost immune to the focus changes.

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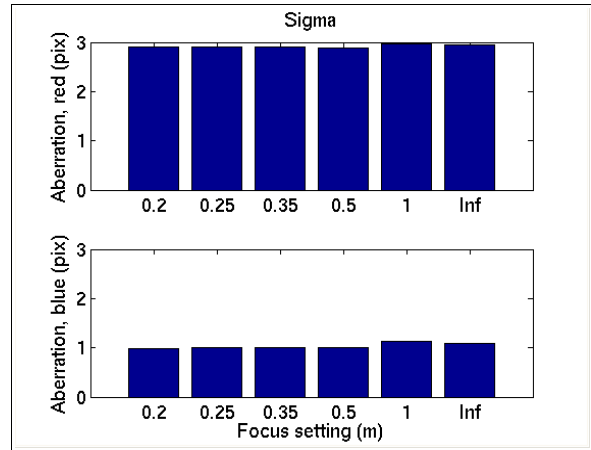


Figure 25. Colour aberration on the image border with different focus settings (Sigma lens).

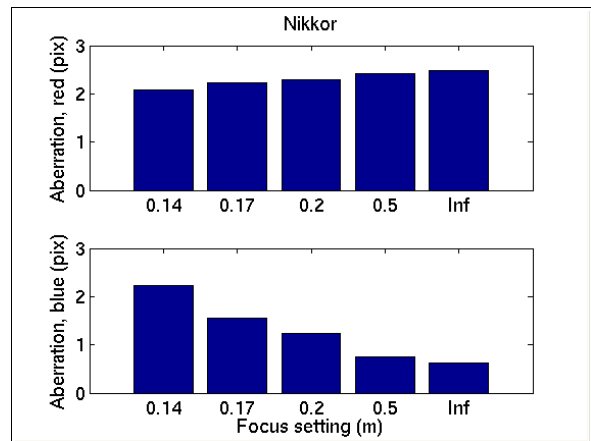


Figure 26. Colour aberration on the image border with different focus settings (Nikkor lens).