THE PRECISION ANALYSIS OF 3D RECONSTRUCTION BASED ON GENERALIZED STEREOPAIR

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ABSTRACT:
According to the idea of realizing the 3D reconstruction using different images obtained from different sensors, this paper focuses on the different-source high resolution remotely sensed imagery covering the same regions, constructs the generalized stereopairs, and investigates the space intersection mathematics models based on RFM. Then quickly extracts the ground 3D information, and realizes the 3D reconstruction of target objects. After that, this paper carries on the precision analysis towards the reconstructed three dimensional models, and compares the relative precision of these 3D models reconstructed utilizing three different methods, and then validates that the generalized stereopair can realize the 3D reconstruction of ground objects, and the precision can absolutely satisfy the requirement of production.

1. INTRODUCE
Along with the development of satellite remote sensing (ab. as RS) technology, the high resolution RS imagery play a more and more important role in military, economy and government decision-making, etc. It becomes a hot researching point to realize targets 3D reconstruction using RS imagery. For example, in the military applications, it is a urgent need to obtain the medium and small scale terrain data and 3D model reconstruction of the whole world key military targets using the satellite RS imagery; In the development of city navigation system, it urgently needs to construct the 3D reconstruction model, and provide the data support for government decision-making in the forecast of disaster scope, the disaster effect evaluation, and the crowd dispersal.

In photogrammetry and RS, the 3D reconstruction is familiarly obtained by stereopairs, and we can get many types of RS images for now. However, we still suffer from the following two critical reasons:

(1) Many regions may have no stereopairs;
(2) Even have, but very expensive.

Investigating the reasons, there are some factors to cause them (Weixi Wang, 2007a): (1) Polity (information blockage to non-allies or regions); (2) Price (the prince of stereo RS imagery is usually more expensive than single image several times); (3) Data lacking (many special regions or targets usually have no stereopairs of same satellite); (4) Security (the high resolution stereopairs of some special regions are not provided).

Fortunately, more and more imagery could cover the same area, which are of different satellite sensors or aerial cameras, these images therefore have different space resolutions and different colors.

Focusing on the above two problems, and according to the idea of realizing the 3D reconstruction using different images obtained from different sensors, this paper proposes a method to construct a generalized stereopair taking the overlap high resolution images from different sensors or cameras as the left and right images, and establishes new rigorous space intersection model based on Rational Function Model (ab. as RFM), Then quickly extracts the ground 3D information, and realizes the 3D reconstruction of target objects. After that, this paper carries on the precision analysis towards the reconstructed three dimensional models, and compares the relative precision of these 3D models reconstructed utilizing three different methods, and then validates that the generalized stereopairs can realize the 3D reconstruction of ground objects, and the precision can absolutely satisfy the requirement of production.

2. CONSTRUCTION OF GENERALIZED STEREOPAIR BASED ON RFM

2.1 Principle
The orientation model of image decides which kind of models should be used in space intersection. The common orientation models have the following types: Polynomial Model (PM), Collinearity Equation Model (CEM), Affine Model (AM), Direct Linear Transformation (DLT), and Rational Function Model (RFM). Among them, RFM has already become the main orientation model for high resolution imagery. Comparing with other models, the RFM has following advantages (Weixi Wang, 2007b; Wang Weixi, 2007c): (1) independence from sensors or cameras; (2) suit for any kind of coordinate system, and free form the coordinate system; (3) can imitate rigorous models of any existing sensors or cameras. According to the above advantages, this paper takes the RFM as the basis of generalized...
stereopair space intersection and makes the principle deduce and analysis.
In the construction of generalized stereopair, the different-source images have the different sensor models and imaging techniques. In order to unify all these different sensor models under the identical kind of model, therefore takes the RFM as the image orientation model for each single image to construct generalized stereopair. And all these kind of space intersection models constructed by this method are all based on the RFM model.

2.2 Generalized Stereopair Space Intersection Model Based on RFM

When the orientation model of left and right image to construct the generalized stereopair are all RFM, then the corresponding space intersection mathematical model can be described as the RFM+RFM model, and its mathematical formula is:

$$
\begin{align*}
\begin{cases}
    r_l &= P_{rl}(X_l, Y_l, H_l) \\
    c_l &= P_{cl}(X_l, Y_l, H_l) \\
    r_r &= P_{rr}(X_r, Y_r, H_r) \\
    c_r &= P_{cr}(X_r, Y_r, H_r)
\end{cases}
\end{align*}
$$

(1)

Because the normalization parameters of left and right image are different, the corresponding normalized space coordinates \((X_l, Y_l, H_l)\) are also inconsistent in expressing, which causes it cannot directly solve the equation group by the least square method and get \((X, Y, H)\). In order to unify the form of unknown values as solving the equation group, it needs to import the unnormalized original object space coordinates \((X_u, Y_u, H_u)\) into the equation group to calculate (Gene Dial, 2002), but not the normalized coordinates \((X, Y, H)\).

Linearize the above formula 4, and imports original ground 3D coordinates \((X_u, Y_u, H_u)\), then can get following formula:

$$
\begin{align*}
\begin{cases}
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u + \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u + \frac{1}{H_u} \frac{\partial}{\partial H} H_u + r(X_u, Y_u, H_u) = 0 \\
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u + \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u + \frac{1}{H_u} \frac{\partial}{\partial H} H_u + c(X_u, Y_u, H_u) = 0 \\
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u + \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u + \frac{1}{H_u} \frac{\partial}{\partial H} H_u + r(X_u, Y_u, H_u) = 0 \\
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u + \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u + \frac{1}{H_u} \frac{\partial}{\partial H} H_u + c(X_u, Y_u, H_u) = 0
\end{cases}
\end{align*}
$$

(2)

And corresponding error equations are:

$$
\begin{align*}
\begin{bmatrix}
    V_{r_l} \\
    V_{c_l} \\
    V_{r_r} \\
    V_{c_r}
\end{bmatrix}
= \begin{bmatrix}
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u & \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u & \frac{1}{H_u} \frac{\partial}{\partial H} H_u \\
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u & \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u & \frac{1}{H_u} \frac{\partial}{\partial H} H_u \\
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u & \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u & \frac{1}{H_u} \frac{\partial}{\partial H} H_u \\
    \frac{1}{X_u} \frac{\partial}{\partial X} X_u & \frac{1}{Y_u} \frac{\partial}{\partial Y} Y_u & \frac{1}{H_u} \frac{\partial}{\partial H} H_u
\end{bmatrix}
\begin{bmatrix}
    \Delta X_u \\
    \Delta Y_u \\
    \Delta H_u
\end{bmatrix}
- \begin{bmatrix}
    r_l - r_r' \\
    c_l - c_r' \\
    r_r - r_r' \\
    c_r - c_r'
\end{bmatrix}
\end{align*}
$$

(3)

In these formulas:

\(X_u, Y_u, H_u\) refers to the scale factors in normalization parameters for left image; \(r_l, c_l\) refers to the measured image coordinates of left image after normalization; \(r_l', c_l'\) refers to the calculated image coordinates of left image after normalization using initial values; \(r_r, c_r\) refers to the measured image coordinates of right image after normalization; \(r_r', c_r'\) refers to the calculated image coordinates of right image after normalization using initial values;

In the solution of ground 3D coordinates it needs to carry on on the iterative computation, and the initial values of iteration can directly use the normalization parameters of ground coordinates: \(\text{LongOffset}, \text{LatOffset}, \text{HeightOffset}\).

This selection method of initial values not only may avoid complex formula computation in commonly iteration initial values calculation, moreover because these three normalization parameters are the known data, they may be conveniently and quickly imported into the iterative computation, and satisfy the request of fast localization to the ground objects.

Having the iteration initial values, it can import them into the space intersection model based on RFM, which is constructed in before, and carry on iteration during solving the error equation group. As soon as the different value of neighboring two coordinates is smaller than the set limit error, then finish the iteration, and calculate the optimal solution of original object space 3D coordinates \((X_u, Y_u, H_u)\) (C. Vincent Tao, 2004; Arie Croitoru, 2004).

3. 3D RECONSTRUCTION OF GROUND OBJECTS

The reconstruction of ground objects mainly includes two parts: firstly it needs to calculate the 3D coordinates of each point on ground objects, secondly it needs to calculate the length, width, height, and areas of each surface of these objects according to the 3D coordinates.

3.1 Calculation of 3D Coordinates of Ground Points

It needs to establish the normal equation utilizing the error equation group (formula 2 and 3), and then calculate the 3D coordinates of each ground point.

3.2 3D Reconstruction of Target Objects

According to the above calculated 3D coordinates of each point, this paper calculates the length, width, height, and areas of each surface of these objects. In the reconstruction, because of the errors occurring in the calculation of point 3D coordinates, it may cause the reconstructed target objects are short of spatial geometrical features. Therefore, towards the same target object, it needs to do the spatial geometrical restriction in the 3D reconstruction. It means to do the parallel, perpendicularity, and evolution consistency. Taking the regular object as an example, the computation method is as following:

Parallel:
The purpose of parallel is to guarantee that the opposite sides of the object are parallel.
If line $P_1P_j$ is parallel to the line $P_kP_l$, then
\[(X_i - X_j)(Y_l - Y_j) - (Y_i - Y_j)(X_l - X_j) = 0 \tag{4}\]

If the coordinates have errors, then the condition equation is:
\[(X_i + \Delta x_l - X_j - \Delta x_j)(Y_l + \Delta y_l - Y_j - \Delta y_j) - (Y_i + \Delta y_i - Y_j - \Delta y_j)(X_l + \Delta x_l - X_j - \Delta x_j) = 0 \tag{5}\]

After the linearization, and get:
\[(Y_i - Y_j)W_{l1} + (X_i - X_j)W_{l2} + (Y_i - Y_j)W_{l3} + (X_i - X_j)W_{l4} + (Y_i - Y_j)W_{l5} + (X_i - X_j)W_{l6} = 0 \tag{6}\]
and $W_{l1} = (X_i - X_j)(Y_j - Y_i) - (Y_i - Y_j)(X_j - X_i)(Y_j - Y_i)$

Perpendicularity:
The purpose of perpendicularity is to guarantee that the adjoining sides in the same plane of the object are perpendicular.
If line $P_1P_j$ is perpendicular to the line $P_kP_l$, then
\[(X_i - X_j)(X_l - X_j) + (Y_i - Y_j)(Y_l - Y_j) = 0 \tag{7}\]

If the coordinates have errors, then the condition equation is:
\[(X_i + \Delta x_l - X_j - \Delta x_j)(X_l + \Delta x_l - X_j - \Delta x_j) + (Y_i + \Delta y_i - Y_j - \Delta y_j)(Y_l + \Delta y_i - Y_j - \Delta y_j) = 0 \tag{8}\]

After the linearization, and get:
\[(X_i - X_j)(X_l - X_j)W_{l1} + (Y_i - Y_j)(Y_l - Y_j)W_{l2} + (X_i - X_j)(Y_l - Y_j)W_{l3} + (Y_i - Y_j)(X_l - X_j)W_{l4} = 0 \tag{9}\]
and $W_{l2} = (X_i - X_j)(X_l - X_j)(Y_l - Y_j) + (Y_i - Y_j)(Y_l - Y_j)(X_l - X_j)$

Elevation consistency:
The purpose of elevation consistency is to guarantee that the elevation values of each corner in the object roof are equal. If there are $n$ corners in the object roof, and the calculated elevation values are $Z_1, Z_2, Z_3, \ldots, Z_n$, then it can take the average value of these elevation values as the adjustment value, and the formula is:
\[Z = \frac{1}{n}\sum_{i=1}^{n} Z_i \tag{10}\]

4. EXPERIMENTS

The experiment data include: one pair of QuickBird panchromatic imagery and part of SPOT5 color image of the same region, some 1:1000 topographic maps of this region, 5 GPS control points. These two RS images overlay part of same regions. Among them, the pair of QuickBird panchromatic imagery are the whole images, and these sizes are both 22880 x 17492 pixels, space resolution is 0.5m, the RPC and the normalization parameters of the both images are known; The SPOT5 color image is a small part of the whole image, and the size is 3104 x 4632 pixels, space resolution is 0.6m, the RPC and the normalization parameters of this image are unknown. Among these different-source high resolution remotely sensed imagery, one of the QuickBird panchromatic imagery stereopair and the SPOT5 colorful image are used to realize the reconstruction of generalized stereopair; the pair of QuickBird panchromatic imagery are used to realize the reconstruction of traditional stereopair; and one of the QuickBird panchromatic imagery stereopair is used to realize the reconstruction of single image.

In experiments, this paper selects ten ground objects, and realizes the reconstruction of these ten objects utilizing generalized stereopair, traditional stereopair, and single image three different reconstruction methods, and then carries on the comparison and analysis about the relative precisions of these reconstructed object models obtained from different reconstruction methods, for example, the relative precision of length, width, and height of these models.

4.1 3D Reconstruction Based on Generalized Stereopair

Because the SPOT5 image has no RPC, it needs to solve the RPC of it using some ground control points (GCP). After that, this paper will construct the generalized stereopair using QuickBird and SPOT5 images based on RFM, and then realize the reconstruction based on generalized stereopair.

Before solves the RPC, in order to strengthen the stability of normal equation coefficient matrix, and avoids having the rank defect matrix, it needs to firstly carry on normalization processing to the GCP ground 3D coordinates and the corresponding image coordinates, and causes the coordinates normalized between -1.0 and +1.0. The method of normalization processing is stated as following formulas:

\[\text{normalization of the image coordinates:}\]

If the size of image is $2m \times 2n$, and the pixel coordinates of image center is $(m,n)$, then the corresponding normalization parameters of image coordinates can be calculated by following formula:

\[\text{SampOffset = } m, \text{ LineOffset } = n, \text{ SampScale } = m, \text{ LineScale } = n\]

\[\text{normalization of the ground 3D coordinates:}\]

\[\text{LongOffset} = \sum_{n=1}^{n} \frac{\text{Longitude}}{n} - \frac{\text{Depth}}{n}, \text{ LatOffset} = \sum_{n=1}^{n} \frac{\text{Latitude}}{n} - \frac{\text{Height}}{n}\]

\[\text{LongScale} = \max(\text{Longitude} - \text{LongOffset}, \text{Longitude} - \text{LongOffset}), \text{ LatScale} = \max(\text{Latitude} - \text{LatOffset}, \text{Latitude} - \text{LatOffset})\]

\[\text{HeightScale} = \max(\text{Height} - \text{HeightOffset}, \text{Height} - \text{HeightOffset})\]

(12)

According to above formula, the 10 normalization parameters of image coordinates can be calculated, then the ground coordinates $(X, Y, H)$ and corresponding image coordinates $(r, c)$ after normalization can be calculate by these normalization parameters. Finally, the RPC also can be solved by these coordinates after normalization under the least square method.

Having the RPCs of QuickBird and SPOT5 images, this paper establishes the space intersection model according to the formulas (2) and (3), calculates the 3D coordinates of ground objects using image coordinates, and then realizes the reconstruction of these objects.

The results of 3D reconstruction based on generalized stereopair are as following Table 1. show:
4.2 3D Reconstruction Based on Traditional Stereopair

Using the homogenous QuickBird panchromatic imagery stereopair, and according to the RPCs of them and few GCPs, this paper realizes the 3D reconstruction of the same ten ground objects based on formulas (2) and (3). And The results of 3D reconstruction based on traditional stereopair are as following Table 2. show:

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
<th>Model 9</th>
<th>Model 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>50.14</td>
<td>40.72</td>
<td>41.09</td>
<td>40.91</td>
<td>71.36</td>
<td>110.35</td>
<td>45.87</td>
<td>48.69</td>
<td>41.22</td>
</tr>
</tbody>
</table>

Table 2. Results of 3D reconstruction based on traditional stereopair (Unit: meter)

4.3 3D Reconstruction Based on Single Image

Using one of the QuickBird panchromatic imagery stereopair, and according to the RFM and corner measuring method, this paper realizes the 3D reconstruction of the same ten ground objects. And The results of 3D reconstruction based on single image are as following Table 3. show:

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
<th>Model 9</th>
<th>Model 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>50.23</td>
<td>40.89</td>
<td>40.85</td>
<td>41.10</td>
<td>71.46</td>
<td>110.60</td>
<td>45.99</td>
<td>48.78</td>
<td>41.40</td>
</tr>
<tr>
<td>Height</td>
<td>18.69</td>
<td>19.50</td>
<td>19.37</td>
<td>21.34</td>
<td>18.35</td>
<td>56.15</td>
<td>18.91</td>
<td>21.94</td>
<td>21.54</td>
</tr>
</tbody>
</table>

Table 3. Results of 3D reconstruction based on single image (Unit: meter)

4.4 Precision Comparison and Analysis

The evaluation method of relative precision for reconstructed models:

For the same ground object A, this paper adopts three different reconstruction methods, and obtains three different reconstruction values \( a_1, a_2, a_3 \), let \( \bar{a} = (a_1 + a_2 + a_3)/3 \), then separately calculates \( (a_1 - \bar{a}), (a_2 - \bar{a}), (a_3 - \bar{a}) \), and variance: \( D = \sqrt{(a_1 - \bar{a})^2 + (a_2 - \bar{a})^2 + (a_3 - \bar{a})^2}/3\) - \(1\). If \( D \) is smaller than the threshold value \( \delta \), which had been assigned, then can consider that the \( \bar{a} \) is close to the actual value of ground object, and can calculates the relative precisions of these reconstructed models using \( (a_1 - \bar{a})/\bar{a}, (a_2 - \bar{a})/\bar{a}, (a_3 - \bar{a})/\bar{a} \), and then evaluates the precision and reliability of these different 3D reconstruction methods.

The average values and variances of 3D reconstruction results are as following Table 4. show, and the relative precisions of three different 3D reconstruction methods are as following Table 5. show:

<table>
<thead>
<tr>
<th>Model</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
<th>Model 9</th>
<th>Model 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. Length</td>
<td>50.16</td>
<td>40.74</td>
<td>41.03</td>
<td>40.97</td>
<td>71.38</td>
<td>110.40</td>
<td>45.89</td>
<td>48.70</td>
<td>41.25</td>
</tr>
</tbody>
</table>

Table 4. Average values and variances of 3D reconstruction results
### Table 5. Relative precisions of three different 3D reconstruction methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
<th>Model 6</th>
<th>Model 7</th>
<th>Model 8</th>
<th>Model 9</th>
<th>Model 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generalized Stereopair</td>
<td>0.0134</td>
<td>0.0034</td>
<td>0.0031</td>
<td>0.0027</td>
<td>0.0042</td>
<td>0.0065</td>
<td>0.0060</td>
<td>0.0078</td>
<td>0.0062</td>
<td>0.0081</td>
</tr>
<tr>
<td>Traditional Stereopair</td>
<td>0.0053</td>
<td>0.0024</td>
<td>0.0014</td>
<td>0.0026</td>
<td>0.0015</td>
<td>0.0046</td>
<td>0.0030</td>
<td>0.0058</td>
<td>0.0033</td>
<td>0.002</td>
</tr>
<tr>
<td>Single Image</td>
<td>0.0187</td>
<td>0.0058</td>
<td>0.0045</td>
<td>0.0053</td>
<td>0.0057</td>
<td>0.0111</td>
<td>0.0080</td>
<td>0.0136</td>
<td>0.0095</td>
<td>0.0101</td>
</tr>
</tbody>
</table>

5. CONCLUSION

(1) Based on the RFM, and taking the overlap high resolution images from different sensors or cameras as the left and right images, this paper constructs the generalized stereopair, and establishes the space intersection model of generalized stereopair. The results of 3D reconstruction validates that the generalized stereopairs can realize the reconstruction of ground objects.

(2) According to the relative precisions of three different 3D reconstruction methods (as Table 5. shows), it can be known that the relative precision of 3D reconstruction objects obtained from generalized stereopair is higher than the relative precision of single image, and almost equal to the relative precision of traditional stereopair.

REFERENCES


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