

# EPIPOLAR LINE GENERATION FROM IKONOS IMAGERY BASED ON RATIONAL FUNCTION MODEL

Dan Zhao<sup>a</sup>, Xiuxiao Yuan<sup>a</sup>, Xin Liu<sup>a</sup>

<sup>a</sup>School of Remote Sensing and Information Engineering, Wuhan University,  
129 Luoyu Road, Wuhan 430079, China - zhaodanpppp@163.com

Commission IV, WG IV/9

**KEY WORDS:** Remote Sensing, IKONOS, Space Photogrammetry, Accuracy, Geography

## ABSTRACT:

High-resolution satellite imagery is expected to be a major source of 3D measurement of ground, especially automatic stereo plotting to generate DEM by stereo matching technology is highly required. Epipolarity is a very useful concept in processing stereo imagery, and it has been widely used in process images captured by frame cameras. Different from perspective image, every scan line of linear array scanner scene has different perspective centre and attitude; this has limited the use of epipolar theory in processing the linear array scanner scenes. The purpose of this paper is to develop a method which can be used to generate the approximate epipolar line of this kind of imagery. The paper firstly describes the epipolar geometry of linear array scanner scenes; then explains the rational function model (RFM) and proposes the use of RFM to generate the epipolar lines; finally, makes a accuracy assessment of the rational polynomial coefficients (RPCs) used in experiment and emphatically describes the procedure and experiment of the generation of epipolar lines and epipolar line pairs of IKONOS stereo imagery.

## 1. INTRODUCTION

### 1.1 General Instructions

High-resolution satellite imagery captured from linear array scanner is valued for their great potential in stereo plotting. The linear scanner with up to one-meter resolution from commercial satellites could deliver more benefits and provide a challenge to traditional topographic mapping based on aerial image. It is expected to be a major source of 3D measurement of ground. Due to the complicated imaging geometry of high-resolution satellite imagery, many traditional theories of photogrammetry can not be applied to this kind of imagery directly, so the appearing of high resolution satellite imagery provide new research contents to the space photography.

Epipolarity is an important concept in processing stereo images. A useful property of an epipolar line is that all corresponding image points lie on the corresponding epipolar line pairs. This property makes epipolar constraint as an important constraint condition for image matching. Many existing stereo matching algorithms use this constraint to confine search dimensions, reduce processing time and achieve reliable match estimates (Zhang et al., 1995; Kim, 2000). For the aerial and perspective image, epipolar geometry has been well founded and widely used. But due to the complex imaging geometry of linear array scanner scene, this geometry of such imagery is not easy to obtain and it has not been used as widely as it used in processing perspective images. To resolve this problem, many scholars have made a lot of efforts. Some have assumed that epipolar geometry would be the same for push-broom imagery as perspective image, some do not use this geometry at all. This paper is aims at developing a method which can be used to generate the epipolar lines of the linear array scanner scenes.

It is important to select an appropriate sensor model to be utilized before the study of the epipolar geometry. The rigorous

and generalized sensor models are the two broad categories of sensor model in use (McGlone, 1996). The representative of rigorous model is the collinearity equations. The generalized models include RFM, two-dimensional affine model, direct linear transformation (DLT). The sensor model used in this paper is RFM. It is a generalized model which can be generated from the physical sensor model and substituted for all sensor models and it is capable of achieving high approximation accuracy. However, RFM has a complex mathematical expression. Therefore, it is difficult to generate the exact epipolar curve for linear array scanner scenes use RFM. In this paper epipolar lines are generated by approximate method.

### 1.2 The Epipolar Geometry of Linear Push-Broom Scenes

For perspective image, there is only one perspective centre for the entire image, so the epipolar curve can be defined as the intersection between the epipolar plane and the image plane. The epipolar curve of any perspective images can be always represented as straight line, which is a well-known property of the epipolar geometry of perspective image. Conjugate epipolar pairs are exist for perspective image, the conjugate points must lie on the conjugate epipolar lines. Different from perspective image, every scan line of linear array scanner scene has different perspective centre and attitude. It is impossible to define an epipolar plane as perspective image, so it is not easy to give a rigorous definition for epipolarity of linear array scanner scenes.

There are two difficulties in deriving the epipolar geometry of linear push-broom scenes, the first difficulty is that a generally accepted mathematical method to describe the relationship between image and object space has not yet been established (McGlone, 1996), and the second difficulty comes from the complexity of such a geometric relationship and difficulty in expressing the relationship in mathematical forms.

Kim (2000) formulated exact epipolar curve for the linear array scanner scenes based on Orun and Natarajan's orientation model and derived some useful properties of the epipolar curve. The shape of such epipolar curve is not a straight line but hyperbola-like non-linear curves. It can be approximated by a straight line for a small range but not for the entire image. Conjugate epipolar pairs do not exist for this kind of imagery, but exist "locally".

The epipolarity equation derived by Kim is useful in analysing the property, but it is difficulty to apply this equation to the process of linear push-broom scenes due to the need of orientation information. Morgan (2004) derived the epipolarity of this kind of scenes based on two-dimensional affine model, and obtained a linear model which can be used to resample the entire scene. This paper proposes a method to generate the approximate epipolar lines based on RFM without deriving epipolarity equation.

## 2. DESCRIPTIONS OF RFM

RFM relates object point coordinates  $(X, Y, Z)$  to image pixel coordinates  $(l, s)$  or vice visa, as physical sensor models, but in the form of rational functions that are ratios of polynomials (Yong, 2005). The RFM is essentially a generic form of the rigorous collinearity equations. The RFM is divided into forward RFM and inverse RFM according to the relationship between object space and image space.

The forward RFM is a transformation from the coordinates in the object space to the row and column indices of pixel in the image space. The defined ratios have the form as follows:

$$\begin{aligned} l_n &= p_1(X_n, Y_n, Z_n) / p_2(X_n, Y_n, Z_n) \\ s_n &= p_3(X_n, Y_n, Z_n) / p_4(X_n, Y_n, Z_n) \end{aligned} \quad (1)$$

The inverse RFM is to transform the image coordinates to the object coordinates, which can be presented as follows:

$$\begin{aligned} X_n &= p_5(l_n, s_n, Z_n) / p_6(l_n, s_n, Z_n) \\ Y_n &= p_7(l_n, s_n, Z_n) / p_8(l_n, s_n, Z_n) \end{aligned} \quad (2)$$

where  $l_n, s_n$  are normalized image coordinates;  
 $X_n, Y_n, Z_n$  are normalized object space coordinates;  
 $P_i$  is a polynomial with the following form:

$$p(X, Y, Z) = \sum_{i=0}^N \sum_{j=0}^i \sum_{k=0}^j c_{ijk} \cdot X^{i-j} \cdot Y^{j-k} \cdot Z^k \quad (3)$$

Where  $N$  is the polynomial order,  $c_{ijk}$  called rational polynomial coefficients. In order to improve the numerical stability and minimize the introduction errors during computation, both image coordinates and object space coordinates are normalized in the range of  $[-1, +1]$  by applying offsetting and scaling factors(NIMA, 2000). When  $N$  is three, Eq. (3) becomes a three dimensional polynomial with 20 coefficients, which is the most

commonly used form by data and software vendors. The model used in this paper is in the form of three order polynomial.

## 3. EPIPOLAR LINE GENERATION

The epipolar geometry of linear array scanner scenes can be understood more easily with Figure 1. The  $p$  is a point on the left scene;  $S$  is the perspective centre of point  $p$ . The light ray pass through  $S$  and  $p$  hit the ground at point  $P$ . Every point on the light ray can be projected to the right scene, and the combination of these projected points can form a curve on the right scene, which is called the epipolar curve of point  $p$ .

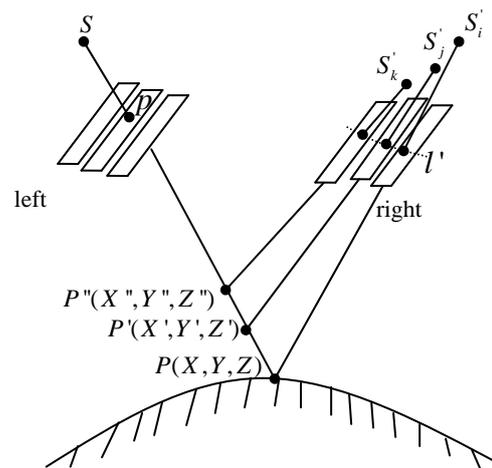


Figure 1. Epipolar geometry of linear array scanner scene

According to the definition of the epipolarity for linear array scanner scene described above and the property summarized by Kim (2000), a method to generate the approximate epipolar line of point  $p$  based on RFM can be developed.

The procedure of epipolar line generation is as follows. For a point  $p$  on the left scene give an elevation value  $Z$  and calculate the corresponding object space coordinate use the inverse RPCs of the left scene. Then project the object space point into the right scene use the forward RPCs of the right scene. Repeat this process for a quantity of times, each time the elevation of the object point should change with equal interval along the light ray connecting the perspective centre and image point on the left scene, a series of image point on the right scene can be obtained, fit them to be a line  $l'$ . The line  $l'$  should be the approximate epipolar line of point  $p$  and the conjugate point of  $p$  should be located on the line  $l'$  or near from it. During this process, the actual elevation of the object space point is not required. In the experiment section some details such as how to change the elevation and if the elevation range and the times elevation changed have influence on the accuracy of the epipolar line will be discussed.

## 4. EXPERIMENTS

### 4.1 Data Description

The dataset used in the experiment involves a stereo-pair captured by IKONOS-2 over the south of Australia with 1 meter resolution. The specifications of these scenes are listed in Table 1. Residential areas, water areas and mountains are

included in the area covered by the stereo-pair, and the elevation range of this area is about -200 to 1500 meters.

Scene	Left	Right
Focal length (m)	10	10
Date of capture	2003-02-22	2003-02-22
Time of capture	00:27 GMT	00:27 GMT
Rows and columns	12122 × 13148	12122 × 13148
Resolution (m)	1	1

Table 1. Specifications of the dataset

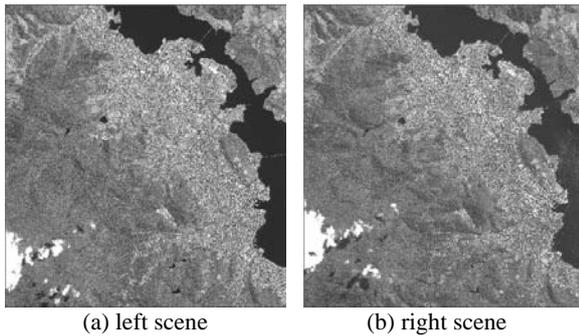


Figure 2. Overview of IKONOS scenes

#### 4.2 Experiment I

This experiment is performed to assess the accuracy of forward and inverse RPCs. The forward RPCs are provided by the imagery agent and the inverse RPCs are calculated by the authors. For the purpose of verification, 62 pairs independent check points are measured on stereo-pair by operator. The measurement accuracy of these points is 1/2 to 1/4 pixels.

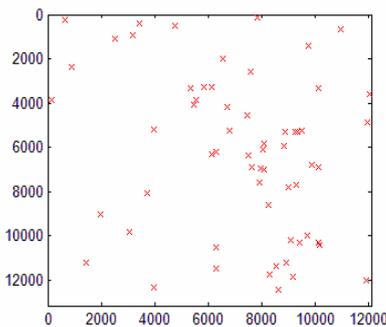


Figure 3. Distribution of check points on left scene

The experiment is carried out as follows, project the checkpoints on the left scene to ground use inverse RFM, and then the object space points are back-projected into the left scene to obtain scene points. Select the different value between the truth coordinates and the scene coordinates obtained by projection as accuracy standard. Tables 2 and 3 display the minimum error, maximum error and root mean square error (RMSE) of the forward and inverse RPCs. Tables 2 and 3 show the accuracy of forward and inverse RPCs for the same scene is better than 0.1 pixels, and there is no difference between the left and the right scene.

	Max	Min	RMSE
Column	0.081	-0.001	0.034
Row	-0.037	0.000	0.014

Table 2. Accuracy of the forward and inverse RPCs of the left scene (pixels)

	Max	Min	RMSE
Column	0.090	0.006	0.040
Row	0.057	0.000	0.019

Table 3. Accuracy of the forward and inverse RPCs of the right scene (pixels)

The accuracy is also assessed in another way. Start from the points on the left scene, project them to the right scene. Select the different value between the truth scene coordinates and the projected scene coordinates as the accuracy standard. Then do the opposite to assess the accuracy of the forward and inverse RPCs from the right scene to the left scene. As Tables 4 and 5 display, RMSE for the forward and inverse RPCs from the left to the right scene or from the right to the left scene is more than 1 pixel in row, which is not as well as the accuracy for the same scene.

	Max	Min	RMSE
Column	1.306	-0.063	0.606
Row	-2.480	-0.261	1.818

Table 4. Accuracy of the forward and inverse RPCs from left scene to right scene (pixels)

	Max	Min	RMSE
Column	-1.272	0.0138	0.585
Row	2.543	0.266	1.816

Table 5. Accuracy of the forward and inverse RPCs from right scene to left scene (pixels)

#### 4.3 Experiment II

Experiment II is performed to validate the method proposed and assess the accuracy of the approximate epipolar line generated. The distance from the conjugate point on the right scene to the epipolar line generated is presented as the accuracy standard.

The proposed method is performed with the change of elevation, so how to select the range of elevation and how many times elevation should be changed in the generation should be discussed at first. The experiment is carried out as follows. Use the conjugate points manual measured in experiment I, generate the right epipolar line of these points on left scene with different ranges of elevation and different times of elevation changed respectively.

With different elevation range respectively changes elevation for 1, 4, 6 times. Table 6 displays the minimum error, maximum error and root mean square error of the approximate epipolar

line generated. Nine groups of data are obtained. As Table 6 displays, the differences among these nine data groups are very small. The maximum error of the epipolar line is about 1 pixel, which can meet the need of the image matching. Comparing these data, it can be found that the increment of the time elevation changes has little influence on the accuracy within the

same elevation range. However, the range of elevation should be selected in a rational range. After the analysis, the authors come to the conclusion that the elevation range and the time elevation changes in the generation have little influence on the accuracy of the epipolar line.

Times Elevation Changed	Interval of Elevation (m)	Range of Elevation (m)	Min (pixel)	Max (pixel)	RMSE (pixel)
1	1000	-200-800	0.009	0.999	0.513
	2000	-500-1500	0.005	0.975	0.513
	3000	-1000-2000	0.006	0.963	0.512
4	300	-200-1000	0.010	1.015	0.514
	500	-500-1500	0.009	1.027	0.515
	1000	-1000-3000	0.022	1.292	0.558
6	300	-400-1400	0.009	1.026	0.515
	500	-500-2500	0.004	1.091	0.521
	1000	-1000-5000	0.027	1.475	0.614

Table 6. Accuracy of the approximate epipolar lines

In the actual generation of epipolar line, the true elevation range of the area the scenes covered can be used to generate the approximate epipolar line. Select a point  $p$  (6259.4, 4727.9) on the left scene, project  $p$  to the right scene respectively with the maximum elevation 1500 meters and the minimum elevation -200 meters, two points are obtained on the right scene, fit these two points to be the approximate epipolar line of the point  $p$  as Figure 4 shows.

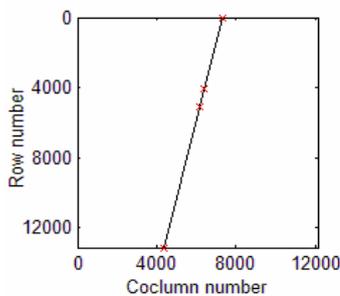
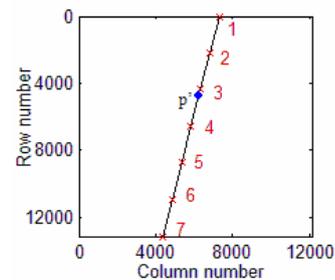


Figure 4. Approximate epipolar line of point  $p$

#### 4.4 Experiment III

Epipolar pairs are generated in this experiment. Start from the right epipolar line generated in experiment II, choose seven points evenly distributed on the right epipolar line, and generate the epipolar line of these points on the left scene, seven epipolar lines are obtained on left scene. Figure 5(a) shows the seven points selected on the right epipolar, and the blue dot is the conjugate point of  $p$ . Figures 5(b) and 5(c) display the epipolar lines generated. As Figure 5(b) shows seven epipolar lines generated on left scene coincide with each other.



(a) Distribution of selected points on the right epipolar line



(b) a piece of left scene (c) a piece of right scene

Figure 5. Approximate epipolar line pairs of point  $p$

The accuracy of these epipolar lines is assessed. As Figure 6 displays line 7 has the maximum error, which is 1 pixel; and line 4 has the minimum error, which is not more than 0.01 pixels. When the point selected on right epipolar line and  $p'$  are located closer, an epipolar line with higher precision can be obtained, and visa versa.

To highlight the difference among these epipolar lines, zoom in the scene within a minor range as Figure 7 shows. These lines are parallel to each other. The epipolar lines which locate closer are generated from the points locate closer. The second line and the third line are almost identical. For the same row coordinate, the maximum difference in column coordinate is 1.7 pixels and for the same column coordinate the maximum difference in row

coordinate is 7.3 pixels. Therefore, the epipolar pairs do not exist for the entire scene, but exist locally. This conclusion consists with the property of epipolarity derived by Kim (2000).

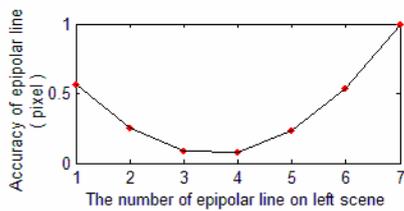


Figure 6. Accuracy of epipolar lines on the left scene

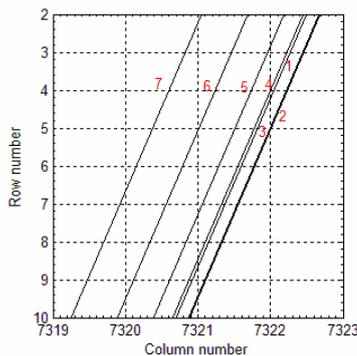


Figure 7. The difference among the seven epipolar lines

## 5. CONCLUSIONS

After analysing the epipolar geometry of linear array scanner scenes a method which can be used to generate the approximate epipolar line for linear array scanner scenes based on the forward and inverse transform of RFM was developed. The method proposed does not require DTMs or ground control points during the generation, but it is better to know the range of elevation of the area the scenes covered which can improve the accuracy of the epipolar line generated. A stereo pairs of IKONOS imagery are used to validate the feasibility of this method; the epipolar generated can achieve a precision better than 1 pixel which can meet the need of image matching. The existence of epipolar pairs are also discussed, the result of the experiment shows that the epipolar pairs do not exist in the range of entire scene, but exist in a small range.

The epipolar lines generated in this paper can be used in image matching, but the epipolar lines generated in this paper are approximated, the true correspondence point may not lie on the line but locate near from it. So the searching area for correspondence should be defined not just along the epipolar line but should be wider than it.

Recommendations for future work include performing more experiments using more data such as SPOT and QuickBird

imagery to test the generality of the method propose in this paper. Also, there is a need to do experiments of image matching use the approximate epipolar line generated based on RFM, and to compare it with other matching methods. We leave these issues as future research topics.

## ACKNOWLEDGEMENTS

Thanks for the supporting from the 973 Program of the People's Republic of China under Grant 2006CB701302 and the National Natural Science of China under Grant 407721001.

## REFERENCES

- Cho, W., Schenk, T., and Madani, M., 1992. Resampling digital imagery to epipolar geometry. *International Archives of Photogrammetry and Remote Sensing*, 29(B3), pp. 404-408.
- Sohn, H., Park, C., and Chang, H., 2005. Rational function model-based image matching for digital elevation models. *Photogrammetric Record*, 20(112), pp. 366-383.
- Hu, Y., and Tao, V., 2004. Understanding the rational function model: method and applications. [http://www.geoict.net/Resources/Publications/IAPRS2004\\_RF\\_M2394.pdf](http://www.geoict.net/Resources/Publications/IAPRS2004_RF_M2394.pdf) (accessed 16 Feb. 2008)
- Kim, T., 2000. A study on the epipolarity of linear pushbroom images. *Photogrammetric Engineering and Remote Sensing*, 62(8), pp. 961-966.
- Orun, A.B., and Natarajan, K., 1994. A modified bundle adjustment software for SPOT imagery and photography tradeoff. *Photogrammetric Engineering and Remote Sensing*, 60(12), pp. 1431-1437.
- Okamoto, A., and Fraser, C., 1998. An alternative approach of the triangulation of SPOT imagery. *International Archives of Photogrammetry and Remote Sensing*, 32(4), pp. 457-462
- Morgan, M., Kim, K., Jeong, S., and Habib, A., 2004. Indirect epipolar resampling of scens using parallel projection modelling of linear array scanners. <http://www.isprs.org/istanbul2004/comm3/papers/241.pdf> (accessed 16 Feb. 2008)
- NIMA, 2000. The compendium of controlled extensions for the national imagery transmission format (NITF). [http://geotiff.maptools.org/STDI-0002\\_v2.1.pdf](http://geotiff.maptools.org/STDI-0002_v2.1.pdf) (accessed 16 Feb. 2008)
- Zhang, Z., and Deriche, R., 1995. A robust technique for matching two uncalibrated images through the recovery of the unknown epipolar geometry. *Artificial intelligence*, 78(1-2), pp. 87-119.

