# 3D DETERMINATION OF VERY DENSE PARTICLE VELOCITY FIELDS BY TOMOGRAPHIC RECONSTRUCTION FROM FOUR CAMERA VIEWS AND VOXEL SPACE TRACKING

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**ABSTRACT:**

The paper presents some improvements to a novel approach for the determination of 3-D flow velocity fields, which is based on 3-D particle tracking in a tomographic reconstruction of an observation volume seeded with tracer particles. The basic idea of the approach is using multiple camera views for a full tomographic reconstruction of the object space, which is represented by a 3-D voxel structure with a resolution adapted to the camera resolution. Based on the images of four or more convergent cameras with their orientation known from a prior calibration procedure, a complete 3-D light intensity distribution in the observation volume can be reconstructed by a projective transformation of each camera image contents into each depth layer of the object space and a consecutive minimum search. 3-D velocity field information can then be obtained by volume-based tracking in time-resolved voxel space representations. This procedure represents a rather elegant way of completely avoiding detection and matching ambiguities, thus allowing for a significant increase of the spatial resolution of 3-D particle tracking. The paper will show the basic concept of tomographic reconstruction and tracking in 3D-PTV and show some first results from processing synthetic data sets. The computational effort, accuracy and spatial resolution potential of the technique will be compared to conventional 3-D particle tracking velocimetry.

## 1. Introduction

Flow measurement techniques are used in many science and engineering tasks, where quantitative flow velocity information in liquid or gas flows has to be determined. Flow measurement techniques determine velocity vectors or velocity vector fields in a defined observation volume of a natural flow, an engine or a laboratory facility. The techniques can be classified into different categories:

Non-contact or contact measurement techniques.

- Observation volume: Point-wise (0-D), profile-wise (1-D), sheet-wise (2-D) and full-field (3-D) measurement.
- Velocity information: Determination of absolute velocity only or determination of one, two or all three components of the velocity vector.
- Determination of instantaneous velocity vectors or longer particle trajectories.
- Necessity or needlessness of particle seeding.

See e.g. Nietsche and Brunn, (2006) or Raffel et al. (2007) for a general overview of flow measurement techniques. In fluid mechanics there is a special interest for non-contact time-resolved fully 3C3D (= determination of 3 components of velocity in 3-D space) measurement techniques. Most methods base on seeding particles, which visualise the flow and can be recorded by an imaging system. 3-D particle tracking velocimetry (3D-PTV, section 2) is an established method to determine 3-D trajectories of a large number of particles in a flow from multiple camera particle image sequences. It is based on the detection of discrete particles in the images, the establishment of multi-image correspondences, the determination of 3-D particle coordinates and a subsequent discrete particle tracking in 3-D object space. The spatial resolution potential of 3D-PTV is limited by ambiguities occurring in the processes of particle detection and multi-image correspondence establishment (Maas et al., 1993).

Recently, Elsinga et al. (2005) have proposed an alternative approach to 3-D particle tracking, which is based on a tomographic reconstruction of the observation volume (section 3). In section 4, we show some improvements to the tomographic reconstruction approach. Section 5 and 6 shows some first particle tracking results from the tomographic approach.

## 2. 3D-PTV

In terms of the categorization made in section 1, 3-D particle tracking velocimetry can be characterized as a non-contact seeding-based full-field 3-D flow measurement technique delivering 3-D trajectories for a large number of particles in a flow. It is based on seeding a flow with neutrally buoyant tracer particles, which are imaged by a stereo camera system. Discrete particle image coordinates in the images (Figure 1) are determined by image analysis techniques.

The core of a 3-D PTV data processing schedule is the spatio-temporal matching process (Figure 2). In the 3D-PTV approach shown by Maas et al., (1993), multi-image correspondences between these particle image positions are established using multi-ocular epipolar line information. Based on these particle image correspondences, particle 3-D coordinates can be determined by spatial intersection. In liquid flow measurement applications, where particles in a liquid are observed by outside cameras through a plane glass interface, multimedia photogrammetry techniques (e.g. Maas, 1995) are employed to handle the spatial intersection of twice-broken beams. In a last processing step, 3-D particle trajectories are determined by applying tracking techniques to discrete 3-D particle constellations.

Netzsch and Jähne (2005) reverse the processing step order in spatio-temporal matching by first tracking particles in 2-D image space and then establishing stereo correspondences between image space trajectories. A step towards integrated spatio-temporal matching is shown by Willneff and Grün (2002), who combine the two approaches by a back-projection search of missing trajectory links.

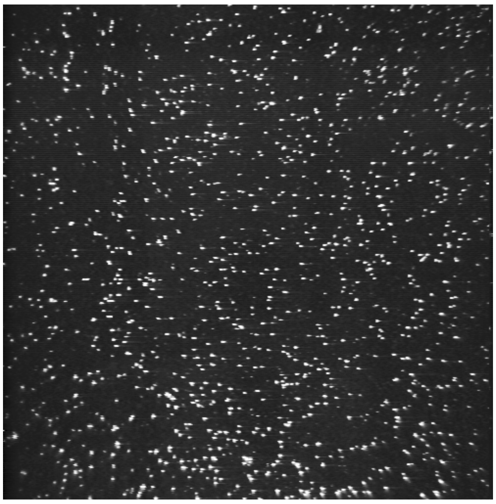

Figure 1. PTV image with ~ 1000 particles

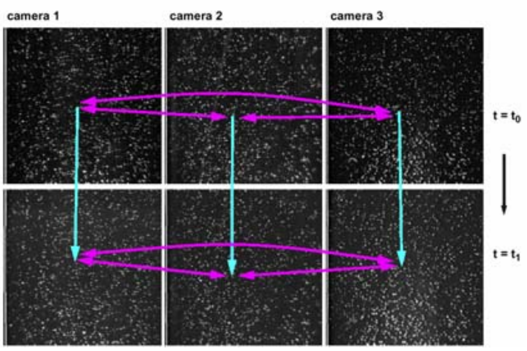


Figure 2. 3D-PTV spatio-temporal matching process

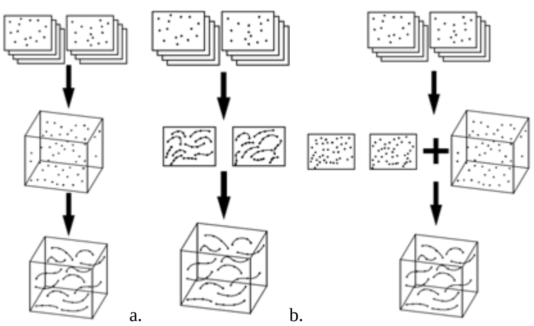


Figure 3. 3-D PTV with 3-D priority (a.), tracking priority (b.) and integrated spatio-temporal matching (c.) (Willneff, 2003)

## 3. Tomo-PIV

Recently, a tomography-based 3-D particle tracking approach has been introduced by Elsinga et al. (2005). In analogy to the established 2-D PIV (particle image velocimetry) technique (Adrian, 1986), the technique is called tomo-PIV. PIV is based on a double pulse exposure of particles in a flow and the determination of flow velocity vector fields by area-based image correlation techniques. The advantage of PIV, besides a rather simple implementation, can be seen in the insensitivity to high seeding densities. While high seeding densities will cause ambiguities in the detection of discrete particles in the images in PTV, the area-based matching tracking approach in PIV does not rely on discrete particles detected in the images. A major drawback of standard PIV is in the fact, that it determines only two components of the velocity vector in a thin layer in the observation volume (2D2C technique). Extensions like scanning-PIV (Brücker, 1995; Hoyer et al., 2005) or multiple-plane stereo-PIV (Kähler and Kompenhans, 2000) can partially solve these limitations, while holographic PIV (Hinsch, 2002) requires a large instrumental effort. These drawbacks of standard PIV are solved by tomo-PIV.

Tomo-PIV generates a tomographic reconstruction of a 3-D particle constellation from a limited number of camera views in an approach similar to shape-from-silhouette (e.g. Matusik et al., 2000). As the particles are moving, the camera views (typically four) have to be captured simultaneously by synchronized cameras. A 3-D observation space reconstruction can for instance be performed by the MART (multiplicative algebraic reconstruction technique) algorithm (Herman and Lent 1976). The basic idea of the technique is to represent the observation volume by a 3-D voxel structure with a resolution adapted to the camera resolution by the following procedure:

Every pixel of the first image is projected into the voxel space through the projection center of its camera. Every voxel, which is hit by the projected ray, gets a greyvalue obtained by interpolation from the originating pixel (Figure 4a).

Then, every pixel of the second image is projected into the voxel space. In every voxel, which is hit by the projected ray, the existing voxel greyvalue (obtained from the first image) is multiplied by the greyvalue of the originating pixel (Figure 4b). Likewise, the content of all other camera views is projected into the voxel space (Figure 4c).

As a result, the voxel space will contain multiplicatively accumulated image intensity information of the instantaneous particle constellation. It is obvious, that only voxels at valid particle positions will show very high values (as it has high values in all factors of the greyvalue multiplication), while all remaining voxels will show rather low values. Repeating the procedure for each time step, a time-resolved 3-D voxel space representation is obtained. In this voxel data sequence, 3-D flow velocity vectors can be obtained by 3-D cross correlation or similar techniques.

Elsinga et al. (2005) show, that four camera views will usually be sufficient for the reconstruction. The requirements to the geometric camera configuration are identical with those for conventional 3D-PTV (Maas et al., 1993). The advantage of tomo-PIV over 3D-PTV is in the fact that it avoids the ambiguity-prone processing steps of discrete particle detection and establishment of multi-view correspondences, thus allowing for a higher seeding density and delivering denser flow velocity field information.

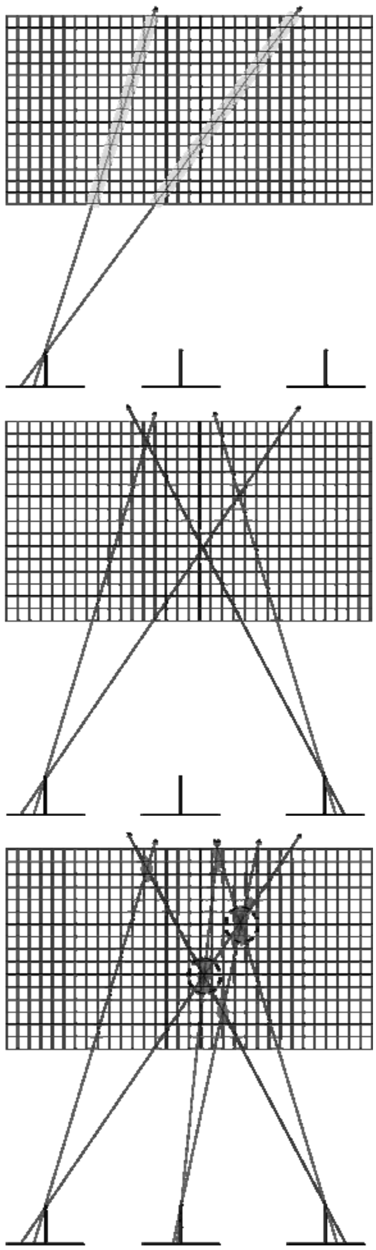


Figure 4. Tomographic reconstruction principle

In the following, we will show several enhancements to tomo-PTV to improve the speed of the tomographic reconstruction process and to optimize the tracking.

## 4. Tomographic reconstruction

The pixel-wise projection method as described in section 3 is straightforward, but computationally rather inefficient and time consuming. The core of our new tomographic reconstruction technique, which is presented in detail in (Putze, 2008), is a multiple projective transformation based approach. The object space voxel structure is initialized by setting the value of every voxel to 255. The reconstruction of the object space light intensity field is performed by transforming the content of each camera image into each depth layer of the voxel space. Using homogeneous coordinates, a simple and fast computation can be performed. The relationship between the image coordinates x' and the voxel coordinates of a depth layer Di in object space is:

$$x' = H_i \cdot D_i$$

Hi contains the 8 parameters of a projective transformation. For a layer D0, the elements of H0 can be determined from the parameters of the exterior and interior orientation of the camera (projection center X0,Y0,Z0, 3x3 rotation matrix r, camera constant c):

$$H_0 = \begin{bmatrix} -c \cdot r_{11} & -c \cdot r_{21} & \mathrm{c} \cdot (r_{11} \cdot X_0 + r_{21} \cdot Y_0 + r_{31} \cdot Z_0) \\ -c \cdot r_{12} & -c \cdot r_{22} & \mathrm{c} \cdot (r_{12} \cdot X_0 + r_{22} \cdot Y_0 + r_{32} \cdot Z_0) \\ r_{13} & r_{23} & -r_{13} \cdot X_0 - r_{23} \cdot Y_0 - r_{33} \cdot Z_0 \end{bmatrix}$$

The transformation matrices Hi of all further depth layers can be determined by adding an increment hi to H0. Due to the parallelism of the depth layers, the determination hi of is simplified:

$$h_i = \begin{bmatrix} 0 & 0 & -c \cdot r_{31} \cdot Z_i \\ 0 & 0 & -c \cdot r_{32} \cdot Z_i \\ 0 & 0 & r_{33} \cdot Z_i \end{bmatrix}$$

$$H_i = H_0 + h_i$$

In homogeneous coordinates, it is sufficient to go through the transformation for the corner pixels of a layer. All other greyvalues can be obtained by a bilinear interpolation.

The layer-wise rectification procedure is repeated for each camera view. Obviously, each object space voxel will obtain different greyvalues from different views. The voxel space particle reconstruction is based on a very simple rule: A voxel belonging to a valid particle must have a high greyvalue in every image. This rule has been realized by a multiplication of the greyvalues from each projection in the implementation of Elsinga et al. (2005). This way, only those voxels, which get a high greyvalue from every view, will 'survive'. The rule can be implemented even more efficiently by a minimum operator, where the greyvalue of a voxel GV is the minimum of its greyvalues in all views gvj:

$$GV = \min \{gv_j\}, \quad gv_j \in \{0...255\}$$

A 3-D particle constellation can then easily be obtained by a thresholding in voxel space.

A background image obtained from spatio-temporal histogram analysis is subtracted from each image beforehand to eliminate the effect of background reflections. Multimedia geometry (i.e. the handling of a broken optical path when observing particles in liquids through a glass interface, see e.g. Maas 1995) can be incorporated into the rectification process by ray tracing from the camera through the air-glass interface to the glass-water interface and consecutive linear linear depth layer mapping using a matric hi with direction vector components obtained from the ray tracing process using Snell's Law.

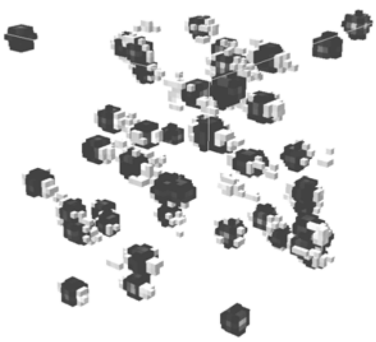

Figure 5. Tomographic reconstruction from synthetic data (Putze, 2008)

Figure 5 shows a reconstruction of a synthetic 50x50x50 voxel volume. The dark voxels show the hulls of particles in the synthetic data, the grey voxels show artefacts from the reconstruction process. These artefacts usually have a greyvalue in the order of only 1 ... 5 in 8 bit data. They can easily be discarded in a thresholding process and have negligible influence to the results of tracking. Putze (2008) shows on the basis of simulated data, that the effect of these artefacts on the particle motion vector is < 0.02 voxel.

## 5. Voxel space tracking

While conventional 3D-PTV produces a large number of discrete particles with metric 3-D object space coordinates, tomo-PIV produces a 3-D voxel space containing the particles. This facilitates the implementation of volume-based particle tracking techniques without the necessity of detecting individual particles. Techniques for tracking discrete particle constellations in 3-D PTV have been presented by Papantoniou and Dracos (1989). These procedures may produce unsolvable ambiguities at high seeding densities. Generally, PIV with its area-based matching between consecutive images is less sensitive to ambiguities and can handle higher seeding densities. Likewise, tomo-PIV allows using volume-based matching techniques for tracking in consecutive voxel datasets, thus enabling 3-D tracking at significantly higher seeding densities than conventional 3-D PTV. These volume-based matching techniques could for instance be 3-D cross correlation like in Elsinga et al. (2005) or 3-D least squares matching (Maas et al., 1994).

3-D cross correlation depicts a rather simple technique to determine the 3-D displacement vectors between cuboids of the dimension (2K+1)x(2L+1)x(2M+1) in the voxel space of two consecutive epochs A,B:

$$\rho = \frac{\sum_{k=-K}^{K}\sum_{l=-L}^{L}\sum_{m=-M}^{M}(A_{klm}-\overline{A})\cdot(B_{klm}-\overline{B})}{\sqrt{\sum_{k=-K}^{K}\sum_{l=-L}^{L}\sum_{m=-M}^{M}(A_{klm}-\overline{A})^2\cdot\sum_{k=-K}^{K}\sum_{l=-L}^{L}\sum_{m=-M}^{M}(B_{klm}-\overline{B})^2}}$$

Subvoxel precision can be obtained by fitting a Gaussian function into the cross correlation coefficient field. Cross correlation offers the advantage of a simple implementation. Elsinga et al. (2005) implemented it in a hierarchical manner (multigrid correlation) to improve the convergence behaviour. It is, however, limited to the determination of three cuboid shift parameters. Cuboids with significant deformations will not be tracked well. This may be partially compensated by iterative window deformation techniques, which have been presented for 2D-PIV (Scarano, 2002), but at a much larger computational effort.

As an alternative, 3-D least-squares-tracking (3-D LST) is a volume-based tracking technique, which is adaptive to cuboid deformation and rotation. In analogy to 2-D least-squares-matching (LSM), 3-D LST minimizes the sum of the squares of voxel value differences by determining the coefficients of a 3-D affine transformation (Maas et al., 1994). In addition to the three displacement vector components, the 12 parameter of the 3D affine transformation in 3-D LST contain scale, rotation and shear information. This allows for a higher precision in case of velocity gradients in the interrogation volume. Moreover, these parameters enable to determine a deformation tensor for each interrogation cube. The result of 3-D LST applied to sequences of tomographically reconstructed voxel structures is a dense 3D velocity vector field with additional shear tensor information.

## 6. Validation with simulated data

Simulated data were used for a first validation of the 3D cross correlation tracking approach. The dataset consists of a 100x100x100 voxel volume with 300 randomly distributed particles. The average distance between neighbouring particles is 10.6 voxels (center-center). Two experiments were performed, both with 300 particles in two time instances. In the first experiment, a linear translation of (1.3/-1.3/0.0) voxels was applied to the particles. In the second experiment, a shear of about 4° was applied (vx = vy = 0.07z ). The particle positions were projected into the images of four synthetic cameras, where particle images were generated using the point spread function. These synthetic images were then used to reconstruct a voxel representation of the object space applying the reconstruction method as described in section 4.

The resulting voxel data were processed by 3D cross correlation using cuboids of 15x15x15 voxels on a 3D grid with 5 voxel spacing. The cuboids contained 2 ... 8 particles. The resulting motion vector fields are shown in Figure 6(shift) and Figure 7(shear). As expected, the 3D cross correlation performed very well in the presence of a pure translation in the particle motion field. The rms deviation of the reconstructed motion vectors from the simulated flow is 0.17/0.16/0.16 voxel in the three vector components, with slightly larger errors occurring in the corners of the dataset.

The results become much worse, when a moderate shear of 0.07 is introduced: The rms deviation of the reconstructed motion vectors increases to 0.48/0.45/0.83 voxels. Moreover, the presence of shear effects in the cuboid may lead to larger errors in case of sparse and asymmetric particle density within the cuboid. This can be checked automatically with the option of omitting cuboids with a poor particle distribution. An adaptive cuboid growth technique is not a good solution here, as the shear-induced errors will grow with increasing cuboid size.

While 3D cross correlation is a simple and fast method for tracking in voxel data, its sensitivity to non-translational movements is known and is explained by the purely translational movement of the correlation cuboid over the search area. A possible solution for this limitation could be the application of a volume deformation technique as suggested for 2D-PIV by Scarano (2002) or by 3D-LST (Maas et al., 1994 – cmp. section 5).

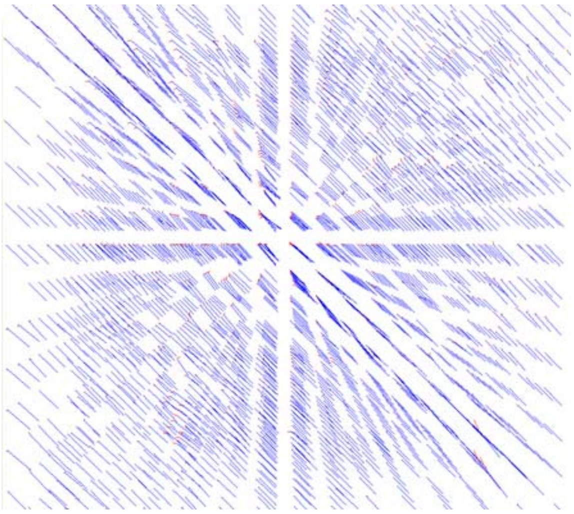

Figure 6. Vectors from tracking (blue) and difference to the simulated data (red) of a flow with a translation

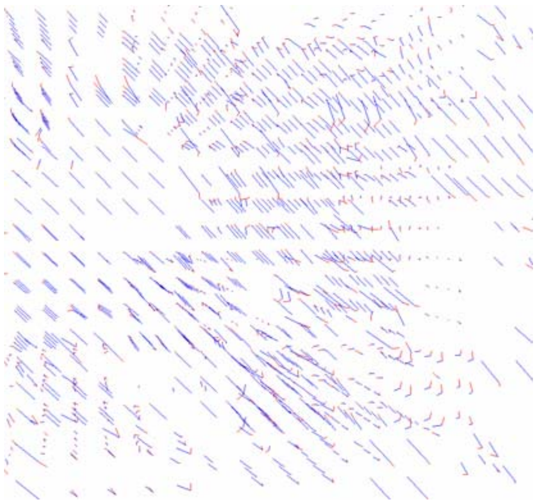

Figure 7. Vectors from tracking (blue) and difference to the simulated data (red) of a flow with a shear

## 7. Conclusion

A 3-D tomographic object space reconstruction forms a viable alternative to conventional photogrammetric 3-D coordinate determinations in applications of 3-D particle tracking velocimetry. Four views of an observation volume seeded with tracer particles are sufficient to reconstruct a voxel space representation of a particle constellation. This procedure avoids the ambiguity error prone processing steps of detecting discrete particles in camera images and establishing multi-view correspondences in a conventional photogrammetric data processing chain. Moreover, the voxel space representation forms a basis for the application of volume-based 3-D tracking techniques such as cuboid cross correlation or 3-D least squares tracking, which are again less ambiguity error prone. As a result, a significantly higher particle seeding density can be processed, allowing for a higher resolution in the quantitative description of flow fields.

The computational efficiency of the tomographic reconstruction can be increased significantly by a sequential projective transformation approach in homogeneous coordinates. Particle tracking in a voxel space representation can easily be performed by cuboid cross correlation with subvoxel interpolarion or – more complex, but adaptive to cuboid deformations – by 3-D least squares tracking.

The tomographic reconstruction and cuboid tracking has been implemented and tested with simulated data. Future work will concentrate on tests with real data, the verification of the obtainable gain in seeding density, the interpretation of the 3D-LST transformation parameters and the implementation in a multimedia photogrammetry environment.


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