PRECISE DETERMINATION OF FISHEYE LENS RESOLUTION

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Theme Session C2

KEY WORDS: Close-Range, Resolution, Optical Systems, Accuracy, Camera, Non-Metric, Fisheye Lens

ABSTRACT:

Using fisheye lens in close range photogrammetry gives great possibilities in acquiring photogrammetric data of places, where access to the object is very difficult. But such lens have very specific optical and geometrical properties resulting from great value of radial distortion. In my approach before calibration, Siemens star test was used to determination of fisheye lens resolving power. I have proposed a resolution examination method that is founded on determination of Cassini ovals and computation of some coefficient. This coefficient modifies resolution calculations. Efficiency of my method was estimated by its comparison with classical way and statistical analysis. Proposed by me procedure of lens resolutions investigations can be used not only in case of fisheye lens, but also in case of another wide-angle lenses. What have to be mentioned is fact, that the smaller focal length of lens, the more accurate is the method. Paper presents brief description of method and results of investigations.

1. INTRODUCTION

Very important are: lens calibration and appropriately chosen test. Depending on gradient of resolution variations, the proper number of points (X, Y, Z) of the test, and their density (depending on radial radius) should be matched. Another possibility of using result of researches is creating map of fisheye lens resolution and determination of some difficult areas in the photo. In these areas (even after calibration) it is impossible using directly the content of the photo, because of its degradation. For these regions proper interpolation method of spectral response value for new complementary pixels has to be found. Such a process enables the full usage converted photos of fisheye lens camera.

That is why investigations connected with lens resolution will have to be done. Additionally, selection of suitable photographing parameters is also depended on lens resolution. Conducted investigations of using fisheye lens, caused necessity of creating a map of fisheye lens resolution. One of possibilities of optical systems quality determination is evaluation of their resolution ability. It consists of determining the smallest structure that can be distinguishable in the image. Making investigations with fisheye lens, and taking into consideration its distortions, such a test has to be done in different way than in case of classical lenses. Strip test will not be proper in such investigations. In that case, test of Siemens is needed. In one photo, there is 192 identical Siemens stars, arranged in columns and rows in equal intervals. Error of such location is 0.01 mm.

Current methods of optical systems quality estimation use image contrast in connection with information of transcribed details. These details are carried by the system and expressed by spatial frequencies (1/mm). The transfer function is the most universal tool of optical system quality assessment.

The angular resolution criteria of Rayleigh are the basic measure of lens resolution. However, the quantity expressed by the Rayleigh criteria is the limiting resolution. More complete understanding of the system is expressed by the Optical Transfer Function (OTF), of which the limiting resolution is one point. The Optical Transfer Function describes the spatial (angular) variation as a function of spatial (angular) frequency.

On the other side, I would like to propose different approach to determination of resolution, using Siemens star. All Siemens stars and their unrecognizable areas in the photo (broadening rings) derived from fisheye lens camera form not circles(ellipse), but Cassini ovals (a>b). Theoretically in fully symmetric lens, Bernoulli's lemniscate (a=b) should appear on the diagonal of the image, but with assumption, that centers of some Siemens stars pass through this diagonal. Remaining stars, especially the main axis of Cassini oval, depending on quarter of the image, form with horizontal straight line characteristic angle. Because our image comes from fisheye lens, this are not straight lines, but arcs, so the angle includes between two arcs. Difference between value of theoretic angle (between straight line), and real angle (between straight lines) enables also determination of influence of distortion in particular point, on decrease of resolution. In my researches I have proofed, that according to proposed by me formula, it is possible to create resolution map of fisheye lens camera with much higher accuracy than in classic approach, what I proofed in my researches. What is more, probable lens errors being results of improper lens connection can be found. Resolution map is creating as a numeric model of lens resolution decrease. To this aim, best model is GRID. It enables average of noise, that can appear in our investigations. Very important criteria of such a model is its resolution, which should be in the range between 3-8% of difference between maximal and minimal value of lens resolution. Lower limit is a result of precision of used method, and using smaller values do not have any sense. Upper limit comes from possibility of presentation of the result.

My researches I have made using Nikon lens with 10.5 mm focal length, mounted in digital camera Kodak DCS 14n pro (14 mln pixels). Measurement was made in Image Analyst and Microstation software. The only problem during measurements is chromatic aberration effect.

2. THEORY

Before lens calibration, resolution should be examined. It is conditioned by necessity of test and calibration model choice. Because of the fact, that we cannot describe different distortions by single scalar value, we have to use point spread function PSF. This function is a real function of two spatial variables . It determinates intensity distribution in output plane of the system that is depended on point activation on input.

Unfortunately, though function plays important role in image forming process, there is great problems with its measurement. For lens with a diameter function PSF can be presented as a function of radius r (Castleman, 1996) :

$$PSF(r) = \left[2 \frac{J_1\left(\pi \frac{r}{r_1}\right)}{\left(\pi \frac{r}{r_1}\right)} \right]^2 \tag{1}$$

where:

$J_1(x)$- is a first degree of Bessel function
r_1 - scale parameter
r - radial radius

Whereas, line spread function LSF, describing intensity variations across line in output plane, and related with point broadening is easy to measure.

Line broadening function can be obtain, registering of luminance signal by displacing detector (narrow slit) along dispersion spot.

If direction of this shift determines x axis, and slit is parallel to y axis, it corresponds to relation between PSF and LSF (2).
With those broadening functions correlates subsequent function, this is an answer to edge spread function ESF.

$$LSF(x) = \int_{-\infty}^{+\infty} PSF(x, y)dy \tag{2}$$

The way of receiving differs from LSF determination. The main difference is a detector form – now it is a half-plane, not a slit. In can be described by formula (3).

$$ESF(x) = \int_{-\infty}^{x} dx \int_{-\infty}^{+\infty} PSF(x, y)dy \tag{3}$$

Broadening functions can be used not only in quality assessment of image processing systems, but also can serve in determination of resolution or modulation transfer function - MTF. MTF function is an answer of optical system versus spatial frequency.It is contrast dependency on spatial frequency in relation to contrast for low frequencies. MTF can be express by relation of signal modulation factor in output to signal modulation factor on input (depended on spatial frequencies f_x, f_y) and present as Fourier transform module of function PSF.

$$MTF(f_x, f_y) = \left| \Im\left[PSF(x, y)\right] \right| \tag{4}$$

Because phase shift Φ is also dependent on f_x, f_y, and can be expressed by formula (5), where function $f(f_x, f_y)$ determinate changes in signal phase during its transfer. Such a function is called Phase transfer function PTF.

$$\phi(f_X, f_Y) = \arctan\left[\frac{F_s(f_X, f_Y)}{F_c(f_X, f_Y)}\right] = Arg\{\Im[PSF(x, y)]\} \tag{5}$$

where:

$$F_s(f_X, f_Y) = \iint_{-\infty} PSF(x, y)\sin[2\pi(f_X x + f_Y y)]dxdy \tag{6}$$

$$F_c(f_X, f_Y) = \iint_{-\infty} PSF(x, y)\cos[2\pi(f_X x + f_Y y)]dxdy \tag{7}$$

On the basis of formula (4) and (5) we can state, that MTF and PTF are module and argument of the same Fourier transform of PSF function.

$$\Im[PSF(x, y)] = MTF(f_X, f_Y)\exp[jPTF(f_X, f_Y)] \tag{8}$$

Because PSF presents point answer of system, equation (8) is an expression for spatial transfer function $S_P(f_X, f_Y)$ of the image processing system, that is named Optical Transfer Function - OTF. From formulas (9) and (10) arise, that normalization of PSF function means normalization of OTF function to unity when spatial frequency values equal zero (Wożniak,1996).

$$OTF(f_X, f_Y) = \Im[PSF(x, y)] \tag{9}$$

$$MTF(f_X, f_Y) = |OTF(f_X, f_Y)| \tag{10}$$

The possibility of OTF normalization is very important, because it enables conversion of absolute signal values into relative signal values, that are contrast and image quality measure. “Wave” deforms passing optical system (lens) or being registered by photosensitive element. Wave is flattened, so contrast is worse. The denser waves, the bigger flatness and the higher contrast decrease.

Lens construction, quality of glass, type of photographic film emulsion, type and technology of CCD matrix production –all these elements and many other, influence on image sharpness.

3. METHOD DESCRIPTION

By my own researches, I reached a conclusion, that Siemens star test is best in resolution of fish eye lens. I have planned test with 192 Siemens stars, in order that all frame was fulfilled with stars, as it is visible in the figure 1.

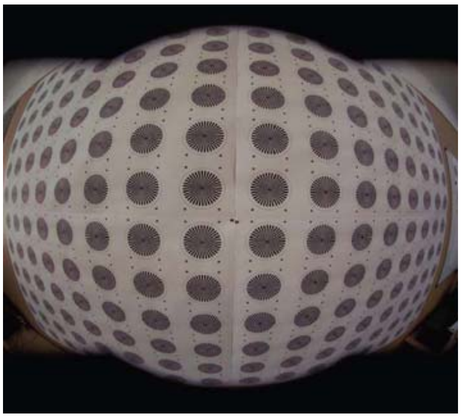

Figure 1. Photograph of Siemens star test, took with digital camera Kodak DCS 14nPro of 10.5mm lens.

In case of fisheye lens we can notice that broadening ring is not an ellipse (like in case of lenses with focal length bigger than 20 mm). This perturbation, presented in the figures 2 and 3 results from big geometrical aberrations, that means from distortion.

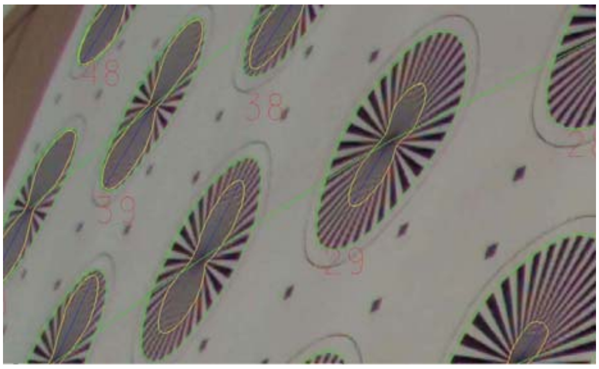

Figure 2. Fragment of photo of Siemens star test took with digital camera Kodak DCS 14nPro (10.5 mm) with broadening rings

In case of focal length 10.5 mm, after 3 measurements series, we can say (with confidence level 0.91), that shape of broadening ring complies with both Cassini oval and equation (11) in case a>b.

These researches were made using 182 Siemens stars (test has 192 stars). Discrepancies in Siemens stars numbers follows from vignetting shield in 10.5 mm lens.

$$(x^2+y^2)^2 = 2b^2(x^2-y^2)+a^4-b^4 \tag{11}$$

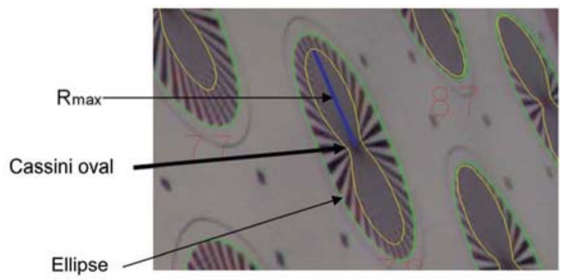

Figure 3. Broadening ring in shape of Cassini oval, digital camera Kodak DCS 14nPro (10.5mm lens)

In classical approach, using Siemens star test, resolution is calculated according to formula (19). Nevertheless, such method do not take into consideration deformations of broadening ring caused by distortion (important in this type of lens).

Therefore, in my opinion, there have to be taken into consideration a fitting factor, proposed by me. Assuming that ellipse area is:

$$P_{elp} = \pi \cdot a \cdot b \tag{12}$$

Cassini oval area

$$P_{OwC} = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} r^2 d\theta \tag{13}$$

Where:

$$r^2 = a^2 \cdot \cos 2\theta \pm \sqrt{a^4 \cdot \cos^2 2\theta - (a^4 - b^4)}$$

If not influence of distortion, shape of broadening ring wouldn't be Cassini oval, but ellipse. Thus, we can assume that this ellipse would have the same shape as ellipse formed from Siemens star. In that case, their flattening would be equal.

$$e'_1 = e'_2 \tag{15}$$

$$\sqrt{1-\frac{b_1^2}{a_1^2}} = \sqrt{1-\frac{b_2^2}{a_2^2}} \tag{16}$$

After reductions and excluding non-physical solutions we obtain:

$$b_2 = \frac{a_2 \cdot b_1}{a_1} \tag{17}$$

thus, we could determine the value of semi-minor axis of broadening ring if not distortion influence. Having such data it is possible to define a coefficient enabling corrections in calculations of fisheye lens resolution.

Coefficient is defined as a ratio of factual broadening area to hypothetical broadening area if not distortion influence.

$$\frac{P_{OwC}}{P_{Elipsy}} = W \tag{18}$$

To formula (19) describing resolution (from Siemens star test), we insert coefficient W (computed according to formula (18)) and we replace d by $2r_{max}$, what is a double broadening radius (such approach facilitates measurements).

$$R = \frac{L_{psek}}{\pi \cdot d} \tag{19}$$

where:

L_{psek} – is a number of sector pairs in Siemens star, D – is a diameter of broadening ring

As a consequence we obtain modified formula of resolving power:

$$W \cdot \frac{L_{psek}}{\pi \cdot 2r_{max}} = R_W \tag{20}$$

4. RESULTS AND CONCLUSION

The part of results in table 1 is representative for all test. Visually, best of all is to compare both methods in the lens resolution map, presented in the figure 4.

Star number	R [L/mm]	Rw [L/mm]	Star number	R [L/mm]	Rw [L/mm]
1	45,56	37,10	49	48,03	47,31
10	37,83	28,69	58	44,30	43,87
19	26,31	16,88	67	23,46	13,09
28	18,63	8,43	76	16,54	8,19
37	15,60	8,78	85	13,31	7,50
46	12,11	7,32	94	12,39	6,22
145	45,61	44,03	97	47,81	38,44
154	35,10	27,19	106	38,58	29,72
163	28,85	18,45	115	27,25	16,97
172	18,24	9,56	124	20,46	12,50
181	15,18	9,22	133	14,88	8,26
190	13,50	9,11	142	12,92	8,86

Table1. Results of resolution in classical and my own approach in four circumradial directions.

It is worth to notice, that there is a place in the lens, where distortion influence is negligible (coefficient close to zero). Such place in classical cameras correspond to principal point of best symmetry (PPS).

	R [L/mm]	Rw [L/mm]
Mean value	19,78	12,18
Median	16,05	9,18
Standard deviation	8,88	8,19
Min	10,48	3,18
Max	48,03	47,31
Variance	78,92	67,03
Correlation coefficient	0,95	

Tab. 2 Statistical comparison of results. Resolution can be present also as resolution map (figure 4).

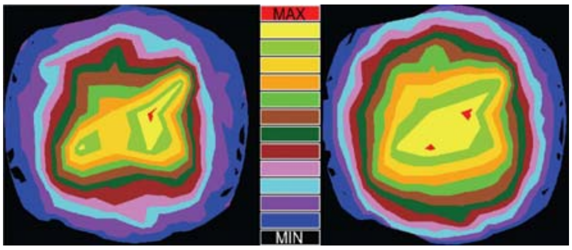

Figure 4 Comparison of resolution computed according to classical approach (right image) and according to discussed method (left image).

5.CONCLUSION

Computation of fisheye lens resolution according to proposed method enables elimination of geometric aberration influence on resolution (especially distortion). When absolute values are noticeable, distortion has a great influence in case fisheye lens cameras. Additionally, resolution map can be useful in designing calibration tests. Dependency of Cassini oval area and ellipse area of broadening can be used in lens distortion determination.

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