# THE THEORY AND APPLICATION OF STRUCTURED LIGHT PHOTOGRAMMETRY WITH KNOWN ANGLE 

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#### Abstract

: In computer vision field, structured-light is defined as the light with certain feature. Hence all of the light point, light line and light plane, which are projected by laser equipment and projector, can be called structured light. In many cases, the direction of those light from structured light resource are unknown in space. If the direction of the structured light can be obtained by the equipment from which it is transmitted, the surveying steps and the following data processing will be facilitated. According to what was mentioned above, two kinds of scanning mathematical models of structured light with known angle based on laser theodolite are introduced. When the horizontal circle is fixed and collimation axis rotates around the horizontal axis, a plumb line plane is formed with the scanning of transmitted laser beam. When the vertical circle is fixed and the alidade rotates around the vertical axis, a circular cone surface is formed with the scanning of transmitted laser beam. If one camera is used to shoot the structured light projected on the object, 3-D coordinate of point can be obtained after extracting structured light information on the image and calculating point of intersection of photography line and structured light scanning surface. Thereby, the detailed mathematical equations of calculating 3-D coordinates of structured light is deduced based on laser theodolite in structured light photogrammetry. Through the solution of the equations composed of collinearity condition equations and structured light scanning equation, the expressions of 3 D coordinates can be inferred simply and clearly. In order to testify the accuracy of the above theory, precision testing experiment and 3-D reconstruction of object experiment are conducted. In precision testing experiment, close range indoor 3D control field with high precision is regarded as the surveyed object whose precision is less than 0.1 mm . The known coordinates of control point is thought as true value and the calculated 3-D coordinates as measured value to calculate the precision of structured light photogrammetry. Under the case of plumb line surface scanning mode, twenty control points are chosen to calculate the precision. The mean square error of coordinates are $m_{X}= \pm 0.44 \mathrm{~mm}, m_{Y}= \pm 0.45 \mathrm{~mm}, m_{Z}= \pm 0.85 \mathrm{~mm}$ separately. Under the case of circular cone surface scanning mode, forty seven control points are chosen to calculate the precision. The mean square error of coordinates are $m_{X}= \pm 0.34 \mathrm{~mm}, m_{Y}= \pm 0.12 \mathrm{~mm}, m_{Z}= \pm 0.70 \mathrm{~mm}$ separately. In 3-D reconstruction of object experiment, a mannequin is considered as the reconstruction object and the experimental instruments are placed as what is done in the above experiments. The 3D model is reconstructed respectively according to plumb line surface scanning mode and circular cone surface scanning mode. The above experiments prove the accuracy of Photogrammetry theory of structured light based on laser theodolite.


## 1. INTRODUCTION

In computer vision field, structured-light is defined as the light with certain feature (Peng Ronghua etc., 2004). Hence all of the light point, light line and light plane, which are projected by laser equipment and projector, can be called structured light. In many cases, the direction of those light from structured light resource are unknown in space. Only through the system calibration, 3-D coordinates can be determined. When the binocular vision equipment is employed, the structured light is reputed as textures from the angle of photogrammetry in order to simplify the later step of image processing and image matching. If the direction of the structured light can be obtained by the equipment from which it is transmitted, the surveying steps and the following data processing will be facilitated. So in this paper, the theories of structured light scanning and the determination of corresponding object points 3-D coordinates are introduced.

Nowadays, in the surveying field the structured light producing apparatus with the largest volume are laser theodolite and total station instrument with laser. If the instrument is flattened, horizontal angle and vertical angle can be read through horizontal circle and vertical circle. Then, the space direction of laser beam, which can form structured light, can be determined. When the structured light is projected on the surveyed object, laser point target come into being. Then if the camera whose interior orientation elements and exterior orientation elements are known is used to shoot the laser point target, its 3-D coordinates can be obtained.

The laser beam of theodolite with two rotation freedoms corresponds with the collimation axis, and it can rotate around the horizontal axis and vertical axis. The laser spot can be adjusted by theodolite telescope. A laser theodolite is shown in Figure 1.


Figure 1. Theodolite with two rotation freedoms

## 2. SCANNING MATHEMATICAL MODELS OF STRUCTURED LIGHT WITH KNOWN ANGLE

There are two scanning modes based on laser theodolite. They are scanning mode of plumb line plane and scanning mode of circular cone surface (Feng wenhao, Li Xin, 1998).

### 2.1 The Scanning Mathematical Model of Plumb Line Plane

As shown in Figure 2, in the object space coordinate system $D$ $X Y Z$, laser theodolite is stationed at the point $L$, whose coordinate are $\left(X_{L}, Y_{L}, Z_{L}\right)$. Each coordinate axis of the defined laser theodolite coordinate system $L-X Y Z$ parallels that of the object space coordinate system $D-X Y Z$. When the horizontal angle $H$ is fixed and collimation axis rotates around the horizontal axis, a plumb line plane $S$ passing point $L$ is formed. Suppose $N$ is the point in the plumb line plane. Its coordinates in the coordinate system $D$ - $X Y Z$ are $(X, Y, Z)$ but $\left(X_{N}, Y_{N}, Z_{N}\right)$ in $L$ XYZ.


Figure 2. The sketch of scanning mode of plumb line plane
Obviously, in coordinate system $L-X Y Z$, the equation of the plumb line plane is:

$$
\begin{equation*}
S\left(X_{N}, Y_{N}, Z_{N}, H\right)=\operatorname{tg} H \cdot X_{N}-Z_{N}=0 \tag{1}
\end{equation*}
$$

In coordinate system $D-X Y Z$, the equation of the plumb line surface is:

$$
\begin{equation*}
S(X, Y, Z, H)=\operatorname{tg} H \cdot\left(X-X_{L}\right)-\left(Z-Z_{L}\right)=0 \tag{2}
\end{equation*}
$$

Special Case 1
When $H=0^{\circ}$, scanning plane equation is $Z=Z_{L}$.

Special Case 2
When $H=90^{\circ}$, scanning plane equation is $X=X_{L}$.

### 2.2 The Scanning Mathematical Model of Circular Cone Surface

In the object space coordinate system $D-X Y Z$, laser theodolite is set at the point $L$, whose coordinates are $\left(X_{L}, Y_{L}, Z_{L}\right)$. Each coordinate axis of the defined laser theodolite coordinate system $L-X Y Z$ parallels that of the object space coordinate system $D-X Y Z$. When zenith distance $V$ is put and collimation axis is rotated around vertical axis after the rotation of alidade, the circular cone surface passing $L$ is formed. Suppose $N$ is the point in the circular cone surface. Its coordinates in the coordinate system $D-X Y Z$ are $(X, Y, Z)$ but $\left(X_{N}, Y_{N}, Z_{N}\right)$ in $L$-XYZ.

In coordinate system $L-X Y Z$, the equation of the circular cone surface is:

$$
\begin{equation*}
X_{N}^{2}+Z_{N}^{2}-\operatorname{tg}^{2} V \cdot Y_{N}^{2}=0 \tag{3}
\end{equation*}
$$

The peak of the circular cone surface is the intersection point of the vertical axis and horizontal ones. In the object space coordinate system $D-X Y Z$, the equation is:

$$
\begin{equation*}
C(X, Y, Z, V)=\left(X-X_{L}\right)^{2}+\left(Z-Z_{L}\right)^{2}-\operatorname{tg}^{2} V \cdot\left(Y-Y_{L}\right)^{2}=0 \tag{4}
\end{equation*}
$$



Figure 3. The sketch of scanning mode of circular cone surface
Special Case 1
When $V=0^{\circ}$ or $V=180^{\circ}$, the circular cone surface is reduced to the plumb line passing point L , its equation is:

$$
\left\{\begin{array}{l}
X=X_{L} \\
Z=Z_{L}
\end{array}\right.
$$

Special Case 2
When $V=90^{\circ}$ or $V=270^{\circ}$, the circular cone surface is reduced to the horizontal plane, its equation is:

$$
Y=Y_{L}
$$

## 3. THE DETERMINATION OF 3-D COORDINATES OF STRUCTURED LIGHT

Because the location of the structured light scanning surface is known in space, only one camera whose elements of interior orientation and elements of exterior orientation are known is enough to be employed to shoot the structured light scanning surface, and then the 3-D coordinates of the intersection point between photography line and structured light scanning surface are calculated (Li Xin, 2006).

Suppose the laser point, which is formed after structured light is projected onto the surveyed object, is $P(X, Y, Z)$, the observation value of its corresponding image point $p$ is $(x, y)$, collinearity condition equations after the correction of system error is (Wang Zhizhuo, 1990):

$$
\left\{\begin{array}{l}
\frac{X-X_{S}}{Z-Z_{S}}=\frac{a_{1}\left(x-x_{0}+\Delta x\right)+a_{2}\left(y-y_{0}+\Delta y\right)-a_{3} f}{c_{1}\left(x-x_{0}+\Delta x\right)+c_{2}\left(y-y_{0}+\Delta y\right)-c_{3} f}  \tag{5}\\
\frac{Y-Y_{S}}{Z-Z_{S}}=\frac{b_{1}\left(x-x_{0}+\Delta x\right)+b_{2}\left(y-y_{0}+\Delta y\right)-b_{3} f}{c_{1}\left(x-x_{0}+\Delta x\right)+c_{2}\left(y-y_{0}+\Delta y\right)-c_{3} f}
\end{array}\right.
$$

where $f=$ principal distance
$x_{0}, y_{0}=$ coordinates of image principal point
$X_{S}, Y_{S}, Z_{S}=$ linear elements of exterior orientation
$a_{i}, b_{i}, c_{i}=$ direction cosine of orientation matrix $\Delta x, \Delta y=$ distortion error

Suppose $\bar{x}=x-x_{0}+\Delta x, \bar{y}=y-y_{0}+\Delta y$, and the equation (6) between image space coordinate system and image space auxiliary coordinate system is introduced.

$$
\left[\begin{array}{c}
X^{\prime}  \tag{6}\\
Y^{\prime} \\
Z^{\prime}
\end{array}\right]=\left[\begin{array}{lll}
a_{1} & a_{2} & a_{3} \\
b_{1} & b_{2} & b_{3} \\
c_{1} & c_{2} & c_{3}
\end{array}\right]\left[\begin{array}{c}
\bar{x} \\
\bar{y} \\
-f
\end{array}\right]
$$

Then equation (5) can be changed into:

$$
\left\{\begin{array}{l}
\frac{X-X_{S}}{Z-Z_{S}}=\frac{X^{\prime}}{Z^{\prime}}  \tag{7}\\
\frac{Y-Y_{S}}{Z-Z_{S}}=\frac{Y^{\prime}}{Z^{\prime}}
\end{array}\right.
$$

Where, $\left(X^{\prime}, Y^{\prime}, Z^{\prime}\right)$ are the coordinates of the image point in image space auxiliary coordinate system. If the elements of interior orientation and elements of exterior orientation and distortion coefficient of digital camera are computed through space resection, ( $X^{\prime}, Y^{\prime}, Z^{\prime}$ ) can be calculated through equation (6).

### 3.1 Calculation of 3-D Coordinates of Structured Light in Plumb Line Plane Scanning Mode

In the plumb line plane scanning mode, the calculation of the 3D coordinates of target points is equal to calculating the
intersection points of photograph line determined by the camera and the plumb line plane.

In the right-hand coordinate system $D-X Y Z$ shown in Figure 2, suppose the linear elements of exterior orientation of the camera are ( $X_{S}, Y_{S}, Z_{S}$ ). The linear equations consisting of equation (2) and equation (7) are:

$$
\left\{\begin{array}{l}
\frac{X-X_{S}}{Z-Z_{S}}=\frac{X^{\prime}}{Z^{\prime}}  \tag{8}\\
\frac{Y-Y_{S}}{Z-Z_{S}}=\frac{Y^{\prime}}{Z^{\prime}} \\
\operatorname{tg} H \cdot\left(X-X_{L}\right)-\left(Z-Z_{L}\right)=0
\end{array}\right.
$$

Then the 3-D coordinates can be expressed as following:

$$
\left\{\begin{array}{l}
X=\frac{X^{\prime}\left(\operatorname{tg} H \cdot X_{L}-\operatorname{tg} H \cdot X_{S}+Z_{S}-Z_{L}\right)}{\operatorname{tg} H \cdot X^{\prime}-Z^{\prime}}+X_{S}  \tag{9}\\
Y=\frac{Y^{\prime}\left(\operatorname{tg} H \cdot X_{L}-\operatorname{tg} H \cdot X_{S}+Z_{S}-Z_{L}\right)}{\operatorname{tg} H \cdot X^{\prime}-Z^{\prime}}+Y_{S} \\
Z=\frac{Z^{\prime}\left(\operatorname{tg} H \cdot X_{L}-\operatorname{tg} H \cdot X_{S}+Z_{S}-Z_{L}\right)}{\operatorname{tg} H \cdot X^{\prime}-Z^{\prime}}+Z_{S}
\end{array}\right.
$$

### 3.2 Calculation of 3-D Coordinates of Structured Light in Circular Cone Surface Scanning Mode

In the circular cone surface scanning mode, the calculation of the 3-D coordinates of target points is equal to calculating the intersection points of photograph line determined by the camera and the circular cone surface.
In the right-hand coordinate system $D-X Y Z$ shown in Figure 3, suppose the linear elements of exterior orientation of the camera are $\left(X_{S}, Y_{S}, Z_{S}\right)$, and in coordinate system $L-X Y Z$, the linear elements of exterior orientation are supposed as $\left(X_{s}^{\prime}, Y_{s}^{\prime}, Z_{s}^{\prime}\right)$. So we have:

$$
\left\{\begin{align*}
X_{S}^{\prime} & =X_{S}-X_{L}  \tag{10}\\
Y_{S}^{\prime} & =Y_{S}-Y_{L} \\
Z_{S}^{\prime} & =Z_{S}-Z_{L}
\end{align*}\right.
$$

In the right-hand coordinate system $L-X Y Z$, the equations consisting of equation (3) and equation (7) are:

$$
\left\{\begin{array}{l}
\frac{X_{N}-X_{S}^{\prime}}{Z_{N}-Z_{S}^{\prime}}=\frac{X^{\prime}}{Z^{\prime}}  \tag{11}\\
\frac{Y_{N}-Y_{S}^{\prime}}{Z_{N}-Z_{S}^{\prime}}=\frac{Y^{\prime}}{Z^{\prime}} \\
X_{N}^{2}+Z_{N}^{2}-\operatorname{tg}^{2} V \cdot Y_{N}^{2}=0
\end{array}\right.
$$

Thus, equation (11) can be changed into a single quadratic equation for $Z_{N}$ :

$$
\begin{align*}
{\left[\left(X^{\prime}\right)^{2}-\right.} & \left.\left(Y^{\prime}\right)^{2} \operatorname{tg}^{2} V+\left(Z^{\prime}\right)^{2}\right] Z_{N}^{2}  \tag{12}\\
& +2\left[X^{\prime} Z^{\prime} X_{S}^{\prime}-Y^{\prime} Z^{\prime} \operatorname{tg}^{2} V \cdot Y_{S}^{\prime}-\left(X^{\prime}\right)^{2} Z_{S}^{\prime}+\left(Y^{\prime}\right)^{2} \operatorname{tg}^{2} V \cdot Z_{S}^{\prime}\right] Z_{N} \\
& +\left[\left(Z^{\prime} X_{S}^{\prime}-X^{\prime} Z_{S}^{\prime}\right)^{2}-\left(Z^{\prime} Y_{S}^{\prime}-Y^{\prime} Z_{S}^{\prime}\right)^{2} \operatorname{tg}^{2} V\right]=0
\end{align*}
$$

Then the 3-D coordinates in the coordinate system $L-X Y Z$ can be expressed as following:

$$
\left\{\begin{align*}
X_{N} & =X^{\prime} \cdot W_{1}+X_{S}^{\prime}  \tag{13a}\\
Y_{N} & =Y^{\prime} \cdot W_{1}+Y_{S}^{\prime} \\
Z_{N} & =Z^{\prime} \cdot W_{1}+Z_{S}^{\prime}
\end{align*}\right.
$$

Where

$$
\begin{aligned}
W_{1}= & \frac{-X^{\prime} X_{S}^{\prime}+Y^{\prime} \operatorname{tg}^{2} V \cdot Y_{S}^{\prime}-Z^{\prime} Z_{S}^{\prime}}{\left(X^{\prime}\right)^{2}-\left(Y^{\prime}\right)^{2} \operatorname{tg}^{2} V+\left(Z^{\prime}\right)^{2}} \\
& +\frac{\sqrt{\left(X^{\prime} Y_{S}^{\prime}-Y^{\prime} X_{S}^{\prime}\right)^{2} \operatorname{tg}^{2} V-\left(X^{\prime} Z_{S}^{\prime}-Z^{\prime} X_{S}^{\prime}\right)^{2}+\left(Y^{\prime} Z_{S}^{\prime}-Z^{\prime} Y_{S}^{\prime}\right)^{2} \operatorname{tg}^{2} V}}{\left(X^{\prime}\right)^{2}-\left(Y^{\prime}\right)^{2} \operatorname{tg}^{2} V+\left(Z^{\prime}\right)^{2}}
\end{aligned}
$$

or

$$
\left\{\begin{align*}
X_{N} & =X^{\prime} \cdot W_{2}+X_{S}^{\prime}  \tag{13b}\\
Y_{N} & =Y^{\prime} \cdot W_{2}+Y_{S}^{\prime} \\
Z_{N} & =Z^{\prime} \cdot W_{2}+Z_{S}^{\prime}
\end{align*}\right.
$$

Where

$$
\begin{aligned}
W_{2}= & \frac{-X^{\prime} X_{S}^{\prime}+Y^{\prime} \operatorname{tg}^{2} V \cdot Y_{S}^{\prime}-Z^{\prime} Z_{S}^{\prime}}{\left(X^{\prime}\right)^{2}-\left(Y^{\prime}\right)^{2} \operatorname{tg}^{2} V+\left(Z^{\prime}\right)^{2}} \\
& -\frac{\sqrt{\left(X^{\prime} Y_{S}^{\prime}-Y^{\prime} X_{S}^{\prime}\right)^{2} \operatorname{tg}^{2} V-\left(X^{\prime} Z_{S}^{\prime}-Z^{\prime} X_{S}^{\prime}\right)^{2}+\left(Y^{\prime} Z_{S}^{\prime}-Z^{\prime} Y_{S}^{\prime}\right)^{2} \operatorname{tg}^{2} V}}{\left(X^{\prime}\right)^{2}-\left(Y^{\prime}\right)^{2} \operatorname{tg}^{2} V+\left(Z^{\prime}\right)^{2}}
\end{aligned}
$$

After changed into the coordinate system $D-X Y Z$, the $3-\mathrm{D}$ coordinates can be expressed as following:

$$
\left\{\begin{align*}
X_{N} & =X^{\prime} \cdot W_{1}+X_{S}^{\prime}+X_{L}  \tag{14a}\\
Y_{N} & =Y^{\prime} \cdot W_{1}+Y_{S}^{\prime}+Y_{L} \\
Z_{N} & =Z^{\prime} \cdot W_{1}+Z_{S}^{\prime}+Z_{L}
\end{align*}\right.
$$

or

$$
\left\{\begin{align*}
X_{N} & =X^{\prime} \cdot W_{2}+X_{S}^{\prime}+X_{L}  \tag{14b}\\
Y_{N} & =Y^{\prime} \cdot W_{2}+Y_{S}^{\prime}+Y_{L} \\
Z_{N} & =Z^{\prime} \cdot W_{2}+Z_{S}^{\prime}+Z_{L}
\end{align*}\right.
$$

## 4. TEST ABOUT STRUCTURED LIGHT WITH KNOWN ANGLE

The test on how to obtain the 3-D coordinates of the target point with structured light on known angle includes two experiments: they are precision checking experiment and 3-D reconstruction of object experiment, both of which are conducted in the indoor 3-D control field with high precision in School of Remote Sensing and Information Engineering of Wuhan University.

### 4.1 Precision Checking Experiment

Precision testing experiment is held under two cases, the plumb line plane scanning mode and the circular cone surface scanning mode. During the test, Laser theodolite aims at the control points in the control field with high precision and using the digital camera to shoot laser points in the dark environment facilitates the abstraction of coordinates of laser points later.

### 4.2 Precision test under the case of plumb line plane scanning mode

In the precision test under the case of plumb line surface scanning mode, the equipment used for transmitting the structured light is BOIF DJD2-1GJA electronic laser theodolite. The accuracy of angular measurement is $\pm 2^{\prime \prime}$. When the laser beam emits from the theodolite, it is coaxial with collimation axis. The digital camera used for shooting is FUJI-FinePix S1 Pro single lens reflex camera, which is equipped with SIGMA lens with 28 mm focal length, the CCD chip of which is 23.3 mm $\times 15.6 \mathrm{~mm}$ in size, the image resolution of which is 3040 pixel $\times$ 2016pixel (the size of each pixel is $7.7 \mu \mathrm{~m}$ ).

In the test, the laser theodolite is placed on the forced centering plate of the left observation post. Thus, the horizontal projection of the centre of the laser theodolite can be considered as the coordinate origin and its elevation can be calculated by observing the vertical angles of some control points in the control field. The control field can be shot after the digital camera is put in the proper place (the average photographic distance is 5.0 m . The interior orientation elements and exterior orientation elements of the camera can be obtained by using space resection.

Twenty control points are chosen in the control field. And the horizontal angle of these 20 control points can be obtained respectively after laser theodolite aims at them respectively. Their 3-D coordinates can be calculated according to equation (9). The difference between the coordinates of those computed 20 control points and true value of control points surveyed by the theodolite is displayed in Table 1.

| No. | $d X$ | $d Y$ | $d Z$ | No. | $d X$ | $d Y$ | $d Z$ |
| :---: | ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| 4308 | 0.085 | -0.345 | 0.651 | 4807 | 0.029 | -0.082 | 0.995 |
| 4307 | -0.309 | -0.009 | 0.803 | 4407 | 0.396 | 0.585 | -0.929 |
| 4306 | -0.426 | 1.165 | 0.830 | 4406 | 0.375 | -0.404 | -0.897 |
| 4305 | -0.620 | 0.246 | 0.748 | 4405 | 0.615 | 0.391 | -1.029 |
| 3810 | 0.355 | -0.367 | -0.801 | 3406 | 0.524 | 0.165 | -1.173 |
| 3809 | 0.121 | 0.555 | -0.746 | 3407 | 0.478 | -0.280 | -0.977 |
| 3808 | -0.027 | -0.484 | -0.796 | 3408 | 0.709 | 0.643 | -0.768 |
| 3807 | 0.586 | 0.421 | -0.625 | 2810 | -0.906 | 0.035 | 0.601 |
| 3806 | 0.264 | 0.176 | -0.579 | 2809 | -0.038 | 0.209 | -0.833 |
| 4806 | 0.060 | -0.378 | 0.954 | 2808 | 0.267 | -0.454 | -0.966 |

Table 1. Coordinates Difference under Plumb Line Plane Scanning Mode

The mean square error of coordinates shown in Table 1 are $m_{X}= \pm 0.44 \mathrm{~mm}, m_{Y}= \pm 0.45 \mathrm{~mm}, m_{Z}= \pm 0.85 \mathrm{~mm}$ separately.

## 5. PRECISION TEST UNDER THE CASE OF CIRCULAR CONE SURFACE SCANNING MODE

In the precision test under the case of circular cone surface scanning mode, the equipment used for transmitting the structured light is SOKKIム SET1130R3 total station. The accuracy of angular measurement is $\pm 1^{\prime \prime}$. When the laser beam emits from the total station, it is coaxial with collimation axis. The digital camera used for shooting is Kodak DCS Pro SLR/n, which is equipped with Nikon lens with 28 mm focal length, the CMOS chip of which is $36 \mathrm{~mm} \times 24 \mathrm{~mm}$ in size, the image resolution of which is 4500 pixel $\times 3000$ pixel(the size of each pixel is $8 \mu \mathrm{~m}$ ).

Following the procedure of the experiment in section 4.1.1, the concerned coefficients of total station and digital camera can be determined. What is different, is the employment of circular cone surface scanning mode. Forty seven control points are chosen in the control field. The difference between the coordinates of those computed 47 control points and true value of control points surveyed by the theodolite is displayed in Table 2.

| No. | $d X$ | $d Y$ | $d Z$ | No. | $d X$ | $d Y$ | $d Z$ |
| :---: | ---: | ---: | ---: | :---: | ---: | ---: | ---: |
| 2203 | 0.051 | 0.104 | -0.404 | 4310 | -0.726 | -0.022 | 0.923 |
| 2204 | 0.257 | 0.099 | -0.609 | 2807 | 0.506 | 0.130 | -0.773 |
| 2205 | 0.011 | -0.066 | 0.092 | 2808 | 0.456 | 0.152 | -1.099 |
| 2206 | -0.165 | -0.100 | -0.240 | 2809 | 0.551 | 0.007 | -1.135 |
| 2207 | -0.083 | -0.149 | -0.030 | 2810 | -0.028 | -0.162 | -0.256 |
| 2304 | 0.598 | 0.479 | -0.671 | 2811 | -0.076 | -0.204 | -0.292 |
| 2305 | 0.438 | 0.069 | 0.043 | 3805 | 0.310 | 0.194 | -0.329 |
| 2306 | 0.435 | -0.026 | -0.100 | 3806 | 0.285 | 0.196 | -0.752 |
| 2307 | 0.031 | -0.075 | 0.277 | 3807 | 0.467 | 0.143 | -1.192 |
| 2308 | -0.428 | 0.057 | 1.356 | 3808 | 0.226 | 0.077 | -1.068 |
| 2309 | 0.028 | 0.037 | 0.952 | 3810 | -0.527 | 0.058 | 0.501 |
| 2310 | -0.074 | -0.096 | -0.398 | 3811 | -0.233 | 0.063 | -0.249 |
| 3304 | 0.287 | 0.191 | -0.528 | 4805 | -0.213 | 0.206 | 0.688 |
| 3305 | 0.329 | 0.135 | -1.022 | 4806 | -0.082 | 0.061 | 1.098 |
| 3307 | 0.217 | -0.026 | -0.479 | 3405 | 0.390 | -0.008 | -0.434 |
| 3308 | 0.216 | 0.082 | -0.817 | 3406 | 0.243 | -0.048 | -0.234 |
| 3309 | 0.057 | 0.022 | -0.446 | 3407 | -0.166 | 0.008 | 0.366 |
| 3310 | -0.197 | 0.007 | -0.077 | 3408 | -0.147 | -0.077 | -0.131 |
| 3311 | -0.262 | 0.031 | 0.352 | 3409 | -0.594 | -0.142 | 0.480 |
| 3312 | -0.130 | -0.098 | -0.475 | 4406 | 0.391 | 0.071 | -1.022 |
| 4306 | 0.027 | -0.034 | -1.337 | 4407 | 0.010 | -0.133 | -0.504 |
| 4307 | 0.171 | 0.015 | -1.329 | 4408 | -0.538 | -0.083 | 0.082 |
| 4308 | 0.254 | 0.148 | -0.908 | 4409 | -0.865 | 0.029 | 0.151 |
| 4309 | -0.300 | -0.025 | -0.511 |  |  |  |  |

Table 2. Coordinates Difference under Circular Cone Surface Scanning Mode

The mean square error of coordinates shown in Table 2 are $m_{X}= \pm 0.34 \mathrm{~mm}, m_{Y}= \pm 0.12 \mathrm{~mm}, m_{Z}= \pm 0.70 \mathrm{~mm}$ separately.

### 5.1 Test of 3-D Reconstruction of Object

In the test of 3-D reconstruction of object, plumb line plane scanning mode and circular cone surface scanning mode are applied into scanning the mannequin made of toughened glass, the size of which is, height 95 cm , width 40 cm , thickness 30 cm . Thirteen common tie points made up of steel nail are distributed in the head, shoulder, waist and the plank in the lower part of the mannequin's body as shown in figure 5 . The mannequin is set before the control field when the structured light is used to scan and camera is used to shoot.


Figure 4. Reconstructed object: a mannequin

### 5.1.1 Reconstruction of Object under the Case of Plumb Line Plane Scanning Mode

The equipment which is the same to the equipment in section 4.1.1 is used. The distance between the digital camera and the mannequin is about 2.4 m . Long time exposure mode is employed to get the liner structured light. Only one scanning line is recorded on each image. The digital camera stays motionless during the whole test. In order to reconstruct the mannequin, the rotation of the mannequin is accomplished manually because it should face the theodolite and the digital camera in different sides. Common tie points are used to realize the connection of target points in different sides. Linear structured light scanning image is shown in Figure 6.


Figure5. Images of plumb line plane scanning mode
In the test, 90 images of structured light are shot and 87046 target points are calculated. The target points of mannequin in different sides in the form of scanning line are shown in figure 6.

From the result of the 3-D reconstruction, the loss of model evidence of medial leg is brought about on account of scanning dead angle. As a whole, the reconstructed model is close to the real object.


Figure 6. The target points of mannequin in different sides in the form of scanning line

After points resampled, TIN of mannequin is reconstructed. The contrasts between the 3-D reconstruction models and the real images in different sides are shown in Figure 7.


Figure 7. Contrasts between the 3-D reconstruction model and the real image in different sides

### 5.1.2 Reconstruction of Object under the Case of Circular Cone Surface Scanning Mode

The equipment which is the same to the equipment in section 4.1.2 is used in this test. The whole test procedure is also the same. What is different is the usage of circular cone surface scanning mode. In the test, 249 images of structured light are shot and 115302 target points are calculated. The images of liner structured light are shown in Figure 8. The target points of mannequin in different sides in the form of scanning line are shown in Figure 9.


Figure8. Images of circular cone surface scanning mode


Figure 9. The target points of mannequin in different sides in the form of scanning line

After points resampled, TIN of mannequin is reconstructed. The rendered 3-D mannequin reconstruction models in different sides are shown in Figure 10.


Figure10. 3-D reconstruction model in different sides
From the consequence of the test, the loss of model evidence of medial leg is brought about still because of the scanning dead angle. Nevertheless, as a whole, the reconstructed model is close to the real object. The model of pleura reflects the profile of mannequin vividly under the case of this mode as it does under the case of plumb line surface scanning mode.

## 6. CONCLUSION

In the paper, two kinds of scanning mathematical models of structured light with known angle based on laser theodolite are introduced, how to obtain images of structured light on the basis of singular digital camera and how to determine the equation of 3-D coordinates of structured light points are given. The correctness of various equations is demonstrated in tests.

When the theory of structured light photogrammetry with known angle is applied, the acquisition of 3-D coordinates of structured light points only depends on calculating the intersection point of photography beam and the scanning surface of structured light. That avoids image matching and suits the surveying of objects without textile. Additionally, it is fit for surveying the objects in the dim or without average light around. The scanning mathematical models of structured light mentioned in the paper can be used as the design prototype of some laser scanning system. If some advanced hardware is equipped, the automation of scanning surveying can be realized.

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