COMBINED KNOWLEDGE PROPAGATION FOR FACADE RECONSTRUCTION

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ABSTRACT:

Frequently, algorithms for 3D facade reconstruction extract high resolution building geometry like windows, doors and protrusions from terrestrial LiDAR and image data. However, such a bottom-up modelling of facade structures is only feasible if the observed data meets considerable requirements on the amount of detail and coverage. For this reason, within our work, the explicit reconstruction of facades is enhanced by the integration of rules. The rules are derived automatically from already reconstructed facades, which serve as knowledge base for further processing. As an example, dominant or repetitive features and regularities as well as their hierarchical relationship are detected from the modelled facade elements. The rules together with the 3D representations of the modelled facade elements constitute a formal grammar. It holds all the information which is necessary to reconstruct facades in the style of the given building. In our approach, they are used for both the verification of the facade model generated during the data driven reconstruction process and the generation of synthetic facades for which no observed sensor data is available.

1. INTRODUCTION

In the age of Google Earth and Microsoft Virtual City, 3D urban models are of growing interest. Aiming at a wide availability of virtual city models, applications like digital globes require reconstruction tools with a high degree of automation. Such tools are usually based on the interpretation of measured data. Alternatively, especially for visualization applications, suitable rules can be used in order to automatically generate synthetic buildings. Mostly generated from aerial data, the vast majority of existing city models consists of coarse building models such as block models (LOD-1) or geometry models (LOD-2). Such representations that feature building models with detailed roof structures and planar facades are for example sufficient for simulations or visualizations at small or medium scale. However, new developments in the areas of computer graphics, virtual reality, the entertainment industry or navigation systems push the demand for more complex and realistic models (LOD-3). For this purpose, the building facades have to be enriched by further information usually related to terrestrial image or LiDAR data. One appropriate approach might be using texturing methods that project colour or depth information onto the planar facade. Another possibility is explicit geometric modelling of building facades, which is the focus of our work.

In principle, object reconstruction is feasible either knowledge based in a top-down fashion or in a bottom-up manner, which is more data driven. Knowledge based techniques usually apply formal grammars that ensure the plausibility and the topological correctness of the reconstructed object elements. A famous example for formal grammars is given by Lindenmayer-systems (L-systems), which can be used to model the growth processes of plants. Since L-systems allow for the procedural modelling of complex objects, they serve as a basis for the development of further grammars appropriate for the modelling of architecture. For instance, (Müller et al, 2006) produce detailed building shells without any sensor data by means of a shape grammar. The context-sensitive shape rules basically implement splits along the main axes of the facades. Other approaches follow the trend towards appearance based and generative modelling which combine sensor data with a priori knowledge in the form of grammars or libraries. For example, (Alegre & Dallaert, 2004), (Brenner & Ripperda, 2006) and (Ripperda & Brenner, 2007) apply formal grammars on images or depth data in order to automatically extract the hierarchical structure of the objects on the facades. Systems which derive procedural rules from given images or models as proposed in (Bekins & Aliaga, 2005), (Müller et al, 2007) and (Van Gool et al, 2007) still resort to semi-automatic methods. Generally, the variety of facade structures to be generated is restricted to the knowledge base inherent in the grammar rules or libraries. In this respect, data driven approaches are more flexible. The facade structure is not subject to predefined rules since geometrical features are directly extracted and modelled from the measured data. As a consequence, such approaches are relatively sensitive to erroneous or incomplete data.

To overcome these difficulties, we pursue an approach which runs fully automatically and includes both bottom-up and top-down propagation of knowledge. The goal is to extract rules from observed facade geometries, which are - due to limitations during data acquisition - mostly available only for parts of a building. These rules then can be applied to generate facade structure for the remaining parts of the building. Our algorithm starts with the extraction and modelling of facade geometries using terrestrial LiDAR and image data in a bottom-up fashion as it was suggested in our previous work (Becker & Haala, 2007). After this interpretation step, the resulting reconstructed facade serves as knowledge base for further processing. Dominant or repetitive features and regularities as well as their

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hierarchical relationship are detected from the modelled facade elements. At the same time, production rules are automatically inferred. The rules together with the 3D representations of the modelled facade elements constitute a formal grammar which we will call *facade grammar*. It contains all the information which is necessary to reconstruct facades in the style of the respective building. We take advantage of this in two ways. Top-down predictions are activated and used for the verification and robustification of the reconstruction result that has already been derived from the observed measurements during the bottom-up modelling. Moreover, the facade grammar can be applied to synthesize facades for which no sensor data is available.

The paper is organized as follows: Section 2 briefly presents the data driven reconstruction process which has been the subject of our previous work. In section 3, the basic definitions and notations that will be used for our facade grammar are introduced. While the inference of the facade grammar is described in section 4, section 5 concerns the grammar application. Results and conclusions are given in the remaining sections.

2. DATA DRIVEN FACADE RECONSTRUCTION

Our approach for data driven facade reconstruction aims at refining an existing coarse building model by adding 3D geometries to the planar facades (Becker & Haala, 2007). For this purpose, terrestrial LiDAR data as well as facade imagery is applied. In the first part of our algorithm, windows, doors and protrusions are modelled from the LiDAR data by searching for holes in the point cloud. In a second step, these structures are refined by integrating further 3D information derived from images of high resolution. The modelling process applies a 3D object representation by cell decomposition, which can be used efficiently for building reconstruction at different scales. For the exemplary dataset "Alte Kanzlei, Stuttgart", Figure 1 depicts the coarse building model with the measured LiDAR points (left) (Schuhmacher & Böhm, 2005) and the refined facade after the reconstruction process (right). The obtained 3D facade model can now be used to infer the facade grammar.

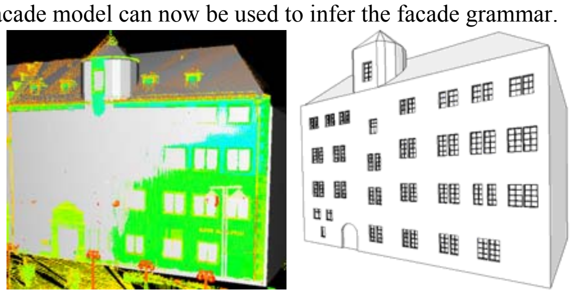


Figure 1. Alte Kanzlei, Stuttgart: LiDAR points aligned with coarse 3D building model (left) and refined facade model (right)

3. FACADE GRAMMAR

In our application, a formal grammar will be used for the generation of facade structure where no sensor data is available. In principle, formal grammars classified into different hierarchies by (Chomsky, 1965) are applied to generate words of a language and to determine whether a word is in the language (Appelrath & Ludewig, 1991). They provide a vocabulary V and a set of production or replacement rules P. The vocabulary comprises symbols of various types. The symbols are called *non-terminals* if they can be replaced by other symbols, and *terminals* otherwise. The sets of non-terminals and terminals are usually denoted by N and T. The non-terminal symbol which defines the starting point for all replacements is the *axiom* ω. A formal grammar can be written as a four-tuple $\Gamma(N,T,P,\omega)$. The grammar's properties mainly depend on the definition of its production rules. They can be, for example, deterministic or stochastic, parametric and context-sensitive. A common notation for productions which we will refer to in the following sections is given by

$$id: \quad lc < pred > rc : cond \quad \rightarrow \quad succ : prob$$

The production identified by the label *id* specifies the substitution of the predecessor *pred* for the successor *succ*. Since the predecessor considers its left and right context, *lc* and *rc*, the rule gets context-sensitive. If the condition *cond* evaluates to true, the replacement is carried out with the probability *prob*. Based on this definitions and notations we develop a facade grammar $\Gamma^{facade}(N,T,P,\omega)$ which allows us to synthesize new facades of various extents and shapes. The axiom ω: *F(polygon)* refers to the new facade to be modelled and, thus, holds information on the facade polygon. The sets of terminals and non-terminals, T and N, as well as the production rules P are automatically inferred from the reconstructed facade obtained by the data driven reconstruction process (section 2).

4. KNOWLEDGE INFERENCE

The result of the data driven facade reconstruction serves as knowledge base for further modelling. It provides information on both the basic facade elements, which constitute the vocabulary of the architectural composition, and their interrelationship. The coaction of vocabulary and its system of relationship, its syntax, characterizes the architectural style of buildings (Mitchell, 1990). In an architect's sense the reference to a specific style is indispensable when attempting to design. The same holds true for the procedural modelling of buildings and building facades.

In the following sections, we propose an approach for the automatic inference of a facade grammar in the architectural style of the observed building facade. Three main phases can be distinguished. Firstly, the vocabulary is set up by recognizing elementary facade objects, i.e. window and wall elements, which will then represent the terminals (section 4.1). Secondly, structural and hierarchical relations between these elements are analysed and used for a compact facade description in form of a character string (section 4.2). Based on the detected structures and hierarchies, production rules are inferred in a final step (section 4.3).

4.1 Searching for Terminals

In order to yield a meaningful set of terminals for the facade grammar, the building facade is broken down into some set of elementary parts, which are regarded as indivisible and therefore serve as terminals. For this purpose, the facade is segmented into floors and each floor is further divided into *tiles*. Tiles are created by splitting the floors along the vertical delimiters of *geometries*. A geometry describes a basic object on the facade that has been generated during the data driven reconstruction process (section 2). It represents either an indentation like a window or a protrusion like a balcony or an oriel. Geometrically, it can be characterized by a set of solids.

By default, this set of solids includes a solid which has the size of the corresponding indentation or protrusion object. Additional solids define the object's design in detail. For example, in case of a window with crossbars these additional solids would be the crossbar solids. The formal definitions for a tile and a geometry are as follows:

$$tile = (geometry, space)$$

$$geometry = \{solid_1, solid_2, ..., solid_n\}$$

$$space = \begin{cases} space_{horizontal} & if \quad geometry = \varnothing \\ space_{vertical} & if \quad geometry \neq \varnothing \end{cases}$$

Each tile has got two attributes: a *geometry* and a *space*. If the geometry is a non-empty set of solids, the tile describes a region on the facade that includes an indentation or a protrusion. The attribute space defines the vertical distance between the floor plane and the geometry. The width of the tile is given by the bounding box of the corresponding solids. If the geometry is an empty set of solids, the tile represents a blank wall element. In this case, the space describes the horizontal extent of the tile. According to this definition, two main types of tiles can be distinguished: *wall tiles* and *geometry tiles*. In the remaining sections of the paper, wall tiles will be denoted by the symbols W for non-terminals and w_i for terminals. Geometry tiles will be referred to as G and g_i in case of non-terminals and terminals, respectively. Figure 2 depicts a facade floor which is split into tiles. Wall tiles are marked in light grey, geometry tiles in dark grey.

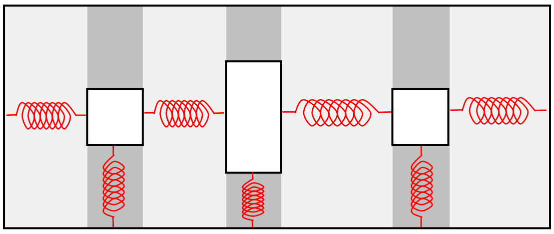


Figure 2. Floor with wall tiles (light grey), geometry tiles (dark grey) and relative spaces (springs)

Based on this concept, the facade can be encoded by a series of tiles. In order to ensure the adaptability to facades with various widths and floor heights, we implement a kind of spring-model. For this purpose, we consider both absolute values, which do not scale, and relative values, which do scale. A geometry and, thus, its associated solids consist of absolute values. That means that the size of indentations and protrusions is constant and does not change when being applied to new facades. By contrast, the vertical spaces of geometry tiles as well as the horizontal spaces of wall tiles are relative values relating to the floor height and the facade width. In Figure 2 relative spaces are illustrated by springs.

In the following, the generation of tiles will be described in detail. Section 4.1.1 deals with the segmentation of the facade into floors and tiles. Within a clustering process described in section 4.1.2, these tiles are discriminated and sorted into classes according to their observed similarities and differences.

4.1.1 Spatial Partitioning:

The facade is segmented into floors by applying horizontal partition planes. For the determination of the partition planes a horizontal plane is shifted across the building from bottom to top. See for example Figure 1 (right) where the geometries are represented by the windows and the door. Areas where there is no intersection with geometries are marked as potential regions for a floor plane. The first floor plane is defined by the ground level. Further floor planes are inserted in the middle of such potential regions if their distance to the previous floor plane specifies a reasonable architectural floor height. We assume a minimal floor height of 3m for all our data sets.

Additionally, each floor has to be divided into tiles. This is done by vertical splits along the left and right borders of all geometries lying in the floor. As a result, we obtain an alternating sequence of wall tiles and geometry tiles:

floor = wall, geometry, wall, geometry, wall, ... , wall

We define this type of series as *topologically correct* arrangement of tiles. Topologically correct sequences of tiles are the basis for the determination of their interrelationship which will be addressed in section 4.2.

4.1.2 Clustering Tiles:

The partitioning steps discussed in section 4.1.1 result in a set of tiles. These tiles are clustered within a classification process for two reasons. Firstly, positional and geometrical inaccuracies of the reconstructed facade can be adjusted. Secondly, the aggregation of similar tiles prevents a huge amount of terminals and alleviates the search for structural interrelations.

According to the differentiation in wall tiles and geometry tiles, instances of these two main types are clustered separately. While the clustering of wall tiles only considers the space attribute, the classification of geometry tiles regards both, the space and the geometry. In this case, depth images are derived from all geometries. This is done by laying a regular 2D grid over the facade plane. According to the sampling theorem, the grid size of the depth image should be less than half the size of the smallest geometry solid to be recognized. From each grid point that lies within the region of a geometry a ray, perpendicular to the facade plane, is projected onto the solids of the geometry. A ray tracing algorithm returns the distance between the first intersection point and the facade plane. This distance is interpreted as depth value and converted into a grey value pixel of the depth image. The similarity of depth images from different geometries is evaluated by calculating the correlation coefficient. If the correlation coefficient for two depth images is above a specific threshold value, the corresponding geometries are assumed to have similar solid configurations. The clustering process results in a set of tiles $\{w_1, w_2, ... , g_1, g_2, ...\}$ that are used as terminals within our facade grammar. Based on these terminals the facade design can be encoded by a sequence of discrete symbols which will be the basis for detecting interrelationships between the terminals (section 4.2).

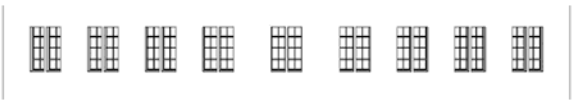
4.2 Interrelationship between Terminals

Having distinguished elementary parts of the facade, as discussed in section 4.1, we now aim at giving further structure to the perceived basic tiles by grouping them into higher-order structures. This is done fully automatically by identifying hierarchical structures in sequences of discrete symbols. Existing algorithms like SEQUITUR proposed by (Nevill-Manning & Witten, 1997) are designed to infer structure from naturally occurring sequences such as for example language, where no knowledge about the meaning behind the single symbols can be assumed. By contrast, topologically correct terminal strings that we are working on carry information about the alternation of symbols as well as the geometrical extents of the corresponding tiles. Taking advantage of this kind of a priori knowledge, we develop an algorithm which is best suited to the facade modelling problem.

We aim at both structural inference and compression for sequences of discrete symbols. The structural inference, which is performed incrementally, reveals hierarchical interrelationships between the symbols in terms of rewrite rules. These rules identify phrases that occur more than once in the string. Thus, redundancy due to repetition can be detected and eliminated. Our algorithm can be stated concisely in the form of three constraints: (c1) The smallest unit of adjacent symbols to be examined is triplets of the form (g,w,g). (c2) No triplet of adjacent symbols appears more than once in the rules. (c3) The order in which triplets are examined depends on the width of their wall tile element. Triplets with small wall tiles are processed first.

Constraint 1 ensures that a structure represents a meaningful part of the facade. First and last elements are assigned to be geometry tiles. Constraint 2 prevents redundancy. Constraint 3 is motivated by the law of proximity which is one of the "Gestalt laws" (Arnheim, 1974). It says that elements close to each other tend to be grouped into a unit. This is also a principle of generalization where close elements are aggregated when the level of detail is reduced. Following this idea, our algorithm extracts structures by means of a fine-to-coarse search.

As a result, we obtain structures of the form Si→[gl|Sj],wm,[gn|Sk] with (i≠j,k), where the symbol Si is a non-terminal denoting the extracted structure "i". Structures are always triplets with a wall tile in the middle and a geometry tile or a structure at the beginning and the end. As an example, Figure 3a shows a modelled floor of the data set "Prinzenbau, Stuttgart". While Figure 3b depicts the corresponding tile string in its original version, the compressed string and the extracted structures are given in Figure 3c. The hierarchical relations between the facade elements can be stored in a parse tree illustrated in Figure 3d.

a) 

b) floor 1 → $w_1\ g_1\ w_3\ g_1\ w_1\ g_1\ w_3\ g_1\ w_1\ g_1\ w_3\ g_1\ w_1\ g_1\ w_3\ g_1\ w_2\ g_1\ w_3\ g_1\ w_2\ g_1\ w_3\ g_1\ w_1\ g_1\ w_3\ g_1\ w_1\ g_1\ w_3\ g_1\ w_1\ g_1\ w_3\ g_1\ w_1$

c) floor 1 → $w_1\ S_3\ w_2\ S_1\ w_2\ S_3\ w_1$

$S_1 \rightarrow g_1\ w_3\ g_1$

$S_2 \rightarrow S_1\ w_1\ S_1$

$S_3 \rightarrow S_2\ w_1\ S_2$

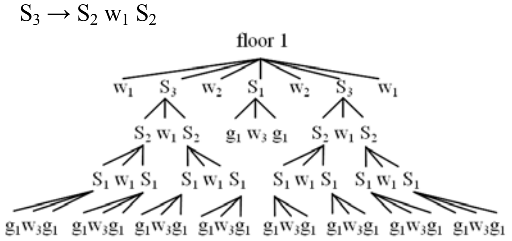
d) 

Figure 3. Modelled floor of the building "Prinzenbau, Stuttgart" (a), corresponding tile string (b), compressed tile string and extracted structures (c), parse tree (d)

Based on the parse tree the terminals g_i, w_i and the structures S_i can be assigned a hierarchy value. This value corresponds to the highest level that the respective terminal or structure occurs at. Levels are numbered from the bottom level to the root. For instance, the terminals and structures in the example of Figure 3d get the following hierarchy values:

	w_1	w_2	w_3	g_1	S_1	S_2	S_3
hierarchy	4	4	3	3	4	3	4

Table 1. Hierarchy values of tiles and structures

4.3 Inference of production rules

Based on the sets of terminals $T=\{w_1, w_2, \ldots, g_1, g_2, \ldots\}$ and non-terminals $N=\{W, G, \ldots, S_1, S_2, \ldots\}$, which have been set up in previous sections, the production rules for our facade grammar can be inferred. The different types of rules to be derived are listed in section 4.3.1. A detailed description of the rules and their properties follows in section 4.3.2.

4.3.1 Rule inference:

Following types of production rules are obtained during the inference process:

p_1: F → W+

p_2: W : cond → W G W
cond = width(W) ≥ width(W G W)

p_3: G : cond → S_i : P($\mathbf{x}$|p_3)
cond = width(G) ≥ width(S_i)

p_4: G : cond → g_i : P($\mathbf{x}$|p_4)
cond = width(G) ≥ width(g_i)

p_5: lc < W > rc : cond → w_i : P($\mathbf{x}$|p_5)
cond = width(W) ≥ width(w_i) &&
hierarchy(lc) ≤ left_hierarchy (w_i) &&
hierarchy(rc) ≤ right_hierarchy (w_i)

The production rules p_1 and p_2 stem from the partitioning steps in section 4.1.1. P_1 corresponds to the horizontal partitioning of the facade into a set of floors. According to a repeat split as applied by (Müller et al, 2006) the facade will be divided into as many floors as there is space. At this stage, each floor element is represented by a wall tile. The vertical partitioning into tiles is reflected in rule p_2. A wall tile, which in the first instance can stand for a whole floor, is replaced by the sequence *wall tile, geometry tile, wall tile*. Each structure found in section 4.2 gives rise to a particular production rule in the form of p_3. This rule type states the substitution of a geometry tile for a structure S_i. In addition, all terminal symbols generate production rules denoted by p_4 and p_5 in the case of geometry terminals g_i and wall terminals w_i, respectively.

As it can be seen from the notation of the rules, our facade grammar is parametric since attributes and conditions are involved. It contains probability information and some rules are context-sensitive. In the following, these properties are discussed in detail.

4.3.2 Rule Properties:

Each terminal is parameterized by a set of attributes which can be propagated to the non-terminals. This set of attributes consists of a width value, which refers to the horizontal extent of the corresponding tile, and a hierarchy value as introduced in section 4.2. While this set is complete for geometry terminals, wall terminals additionally know about the maximum hierarchies of their left and right context. The formal notation for parameterized terminals is gi (width, hierarchy) and wi (width, hierarchy, left_hierarchy, right_hierarchy).

Attributes are used within rule conditions. In p2, p3 and p4 these conditions ensure that there is enough space on the facade for the replacement. P5 achieves context sensitivity by integrating hierarchy attributes in its condition. A wall tile can only be inserted if its width is not too big for the replacement and its left and right hierarchies do not undershoot the hierarchies of the left and the right context. This constraint is made in order to prevent the generation of novel structures during the production process.

Production rules pi (i=3,4,5) are associated with a conditional probability P(x|pi). It constitutes the probability of the position x given the occurrence of the rule pi. Since x describes a two-dimensional position on the facade, P(x|pi) can be interpreted as a 2D probability histogram laid over the facade.

5. KNOWLEDGE PROPAGATION

Our facade grammar implies information on the architectural configuration of the observed facade concerning its basic facade elements and their interrelationships. This knowledge is used in two ways. On the one hand, the facade model generated during the data driven reconstruction process is verified and made more robust against inaccuracies and false reconstructions due to imperfect data. Section 5.1 will deal with this verification process. On the other hand, the knowledge is the basis for synthesizing new, unobserved facades. The applied production process will be addressed in section 5.2.

5.1 Verification

The result of the data driven reconstruction process, which is the basis for knowledge inference, may contain false facade structures and therefore be partly incorrect. As mentioned in section 2, the reconstruction process identifies facade elements such as windows by searching for holes in the laser data. Thus, two types of reconstruction errors may occur: (1) False or too large windows due to holes in the laser data resulting from occlusions during data acquisition, (2) no or too small windows in real window areas because of no or only little holes in the laser data due to closed shutters or grilles.

While errors of type 1 can be avoided by scanning the facade from different viewpoints, errors of type 2 are eliminated during an iterative image based verification approach. For this purpose, an orthophoto of the facade is generated. Within this orthophoto each geometry tile refers to a specific image region which is used as mask for the following image correlation process. Based on the detected floors and tiles, hypotheses about possible positions of each geometry tile are generated and projected onto the orthophoto. A position is accepted if the correlation value between the proposed image region and the respective image mask exceeds a given threshold. The geometry can be inserted; existing geometries that intersect with the new one are deleted. Afterwards, the resulting improved facade model is used to update the set of terminals and production rules. The series of verification and knowledge inference can be carried out iteratively. The process stops when the facade model does not change anymore.

Figure 4 Figure 4 depicts the orthophoto of the data set "Prinzenbau, Stuttgart" as well as parts of the reconstruction results before and after verification. The grilles of the arched windows in the ground floor cause reconstruction errors of type 2 (Figure 4b). Only the window to the right of the door could be reconstructed correctly. Its corresponding image mask and the hypothesized and accepted positions are marked by a yellow rectangle and white crosses, respectively (Figure 4a).

a)
b)
c)

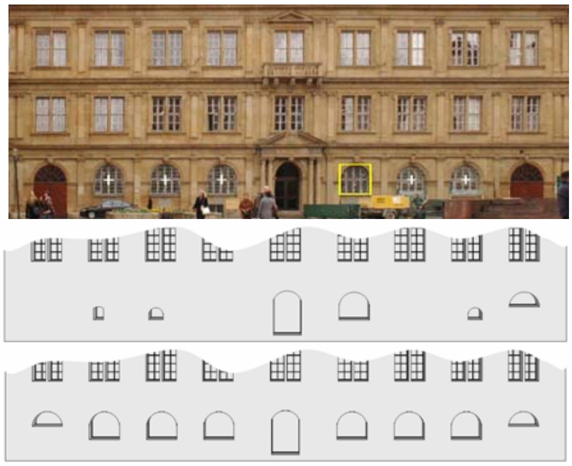


Figure 4. Orthophoto of the data set "Prinzenbau, Stuttgart" (a), 3D model before (b) and after verification (c)

5.2 Production Process

The production process starts with an arbitrary facade, called the axiom, and proceeds as follows: (1) Select a non-terminal in the current string, (2) choose a production rule with this non-terminal as predecessor, (3) replace the non-terminal with the rule's successor, (4) terminate the production process if all non-terminals are substituted, otherwise continue with step (1). Steps 1 and 2 will be described in sections 5.2.1 and 5.2.2 in detail.

5.2.1 Non-Terminal Selection:

The selection process of non-terminals is exemplarily illustrated in Figure 5 For clearness, we assume a facade with only one floor. In each step, the non-terminal selected for the next substitution is marked in red.

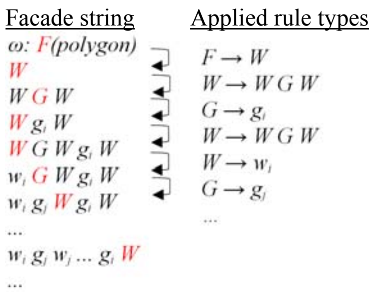


Figure 5. Non-terminal selection

As long as the facade string consists of only one symbol, the non-terminal selection is trivial. In the third line, substitution starts with the non-terminal G in the middle of the string. According to this replacement, the chosen geometry tile g_i will be placed about in the middle of the facade floor. The following replacements are taken from the left to the right of the string. When there is only one non-terminal left on the right end of the string (see the last line in Figure 5), the left part of the facade floor is completely filled with a sequence of wall and geometry tiles. At this stage, symmetry can be enforced by substituting the remaining non-terminal W by a mirrored version of the left terminal string. If no symmetry is required, the replacement can be continued as described before.

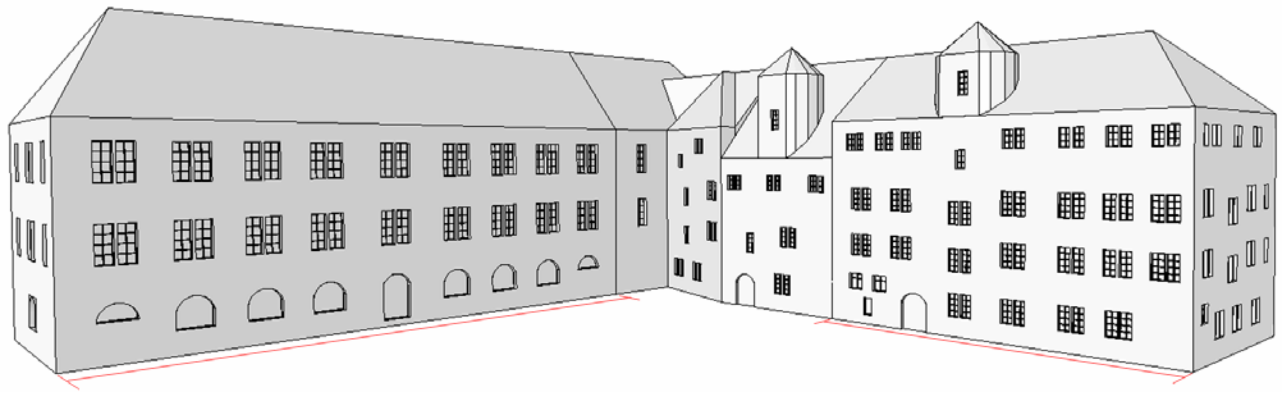


Figure 6. 3D facade models for the buildings “Prinzenbau” (left part) and “Alte Kanzlei” (right part) at Schillerplatz, Stuttgart

5.2.2 Rule Selection:

When more than one production rule is possible for replacement, we choose the rule with the highest probability value. Following the basic idea of the naïve Bayes classifier, we are searching for the maximum probability $P(p_i|\boldsymbol{x})$ which is defined as follows:

$$P(p_i \mid \boldsymbol{x}) = \frac{P(\boldsymbol{x} \mid p_i) \cdot P(p_i)}{P(\boldsymbol{x})}$$

$P(p_i|\boldsymbol{x})$ denotes the required posterior probability of the rule p_i for a given position $\boldsymbol{x}$ on the facade. $P(\boldsymbol{x}|p_i)$, which has already been derived during the rule inference in section 4.3, is the conditional probability of position $\boldsymbol{x}$ when rule p_i happens to be true. The probability for the occurrence of p_i is given by $P(p_i)$. It is a prior probability that we replace with the hierarchy of p_i. Thus, rules with higher-order structures are preferred during the selection process. $P(\boldsymbol{x})$ represents the marginal probability of $\boldsymbol{x}$. Since the denominator is constant for all p_i, only the nominator, which in our case can be written as $P(\boldsymbol{x}|p_i) \cdot hierarchy(p_i)$, has to be evaluated.

6. RESULTS

Figure 6 shows the facade models for the buildings “Alte Kanzlei” and “Prinzenbau” at Schillerplatz, Stuttgart. The parts of the buildings that have been modelled during the data driven reconstruction process are marked by red lines on the ground. All remaining facades are synthesized based on the grammars inferred from the marked facades.

7. CONCLUSIONS

We proposed an automatic approach for the explicit geometric modelling of 3D building facades. Grammar rules are extracted from observed 3D facade geometries and applied for the generation of synthetic facade structures for unobserved building parts. Due to the presented combination of bottom-up and top-down knowledge propagation, the algorithm is highly flexible towards various cases: If a new facade to be modelled is covered by inaccurate, noisy or incomplete sensor data, grammar rules can be used for the verification, improvement and completion of facade structures. In case of facades that have not been observed at all, the grammar allows for the prediction of structural information in the style of the respective building. Moreover, knowledge propagation is not restricted to the facades of one single building. Based on a small set of facade grammars derived from just a few observed buildings, facade reconstruction is also possible for whole districts featuring uniform architectural styles.

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