# ROBUST ALGORITHM FOR FITTING SPHERE TO 3D POINT CLOUDS IN TERRESTRIAL LASER SCANNING


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**ABSTRACT:**

Sphere target is one of the most important tools in terrestrial laser scanning. Different point clouds obtained form different views can be transformed into the same coordinate system frame by using it, where the determination of sphere centre is a key. Because the traditional estimation method---least-squares method does not have the ability to resist outliers, and results in poor estimated parameter, a new method to determine sphere centre is presented. The method is based on M-estimation, and the estimated parameters are not affected by outliers of point clouds. Experiments using simulation data and real data are conducted. Results show that the proposed method can overcome the effect of outliers, and improve the accuracy of sphere centre coordinates. Compared with least square estimation, it has the advantage of robust.


## 1. INTRODUCTION

Terrestrial laser scanning is a new uprising technology. Using the technology, a mass of 3D coordinates from objects surface can be obtain rapidly, called point clouds. Because the technology has many advantages, such as speediness, high efficiency, and high accuracy etc., its has attracted many attentions from all kinds of areas, such as archaeology & cultural heritage documentation,industry detection, topography, Mining, architecture & Facade Measurement, monitoring & civil engineering, city modeling and so on.

Spherical targets are the important auxiliary tools in 3D terrestrial laser scanning system. They can be used to transform different point clouds into a common coordinate system, i.e registration of point clouds, or transform point clouds to certain local coordinate system(Lai J-Y, UengW-D etc.,1999). They are also used in calibration of laser scanning(W.Boehler, M.Bordas Vicent,2003, THorsten Schulz etc., 2004). In all of tasks mentioned above, the key problem is to determine a fitted sphere to point clouds of sphere, that is, to determine the radius and location of the sphere from point clouds. The positioning accuracy of sphere target will have a great effect on the following registration accuracy and quality of object modeled.

The positioning method of sphere target is often based on least-square method. We known that LS methods have no ability to resist gross error or outliers, and make the positioning of sphere trustless. So we propose a new method based on M-estimation to determine the sphere position.

## 2. METHOD BASED ON LS

We can describe the question as follows: given a collection of data points (xi, yi, zi) of sphere obtained from point clouds by laser scanning, determine a fitted sphere that represent the sphere best. When fitting spheres, two different tasks may be encountered, one is to fitting a sphere with its radius “free” to be determined, and the other is to fitting a sphere with a specified “fixed” radius. In this paper, in consideration of radius of standard sphere targets is often provided by instrument manufacturers, we fit sphere with a fixed radius. Sphere equation is

$$\sqrt{\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 + \left(z_i - z_0\right)^2} = R \tag{1}$$

Where
$(x_i, y_i, z_i)$ is coordinate of the ith data point of sphere target recorded by laser scanner
$(x_0, y_0, z_0)$ is coordinate of the sphere center , to be determined
R is the radius of the sphere, provided by manufacturers of laser scanners

On consideration of the errors, the fitting problem can be converted to an optimization problem: to minimize the distances between data points and fitted sphere, therefore, the objective function is as follows:

$$\sum_i \left(\sqrt{\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 + \left(z_i - z_0\right)^2} - R\right)^2 = \min \tag{2}$$

he least-squares method is often used to solve the problem(C.M. Shakarji. 1998, Yuriy Reshetyuk, etc. 2005). Firstly, we linearize the equation (1)

$$\begin{aligned} v_i &= -\sqrt{\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 + \left(z_i - z_0\right)^2} + R \\ &= \frac{x_i - x_0}{d_i} dx_0 + \frac{y_i - y_0}{d_i} dy_0 + \frac{z_i - z_0}{d_i} dz_0 - \left(R_i^0 - R\right) \end{aligned} \tag{3}$$

Where

$$d_i = \sqrt{\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 + \left(z_i - z_0\right)^2}$$

$$R_i^0 = \sqrt{\left(x_i - x_0^0\right)^2 + \left(y_i - y_0^0\right)^2 + \left(z_i - z_0^0\right)^2}$$

the error equation is

$$v = bX - l$$

if we have n points , then total error equation represented by matrix form is

$$\underset{n\times 1}{V} = \underset{n\times 3}{B}\,\underset{3\times 1}{X} - \underset{n\times 1}{L}$$

Where

$$B = \begin{pmatrix} \frac{x_1 - x_0}{d_1} & \frac{y_1 - y_0}{d_1} & \frac{z_1 - z_0}{d_1} \\ \vdots & \vdots & \vdots \\ \frac{x_n - x_0}{d_n} & \frac{y_n - y_0}{d_n} & \frac{z_n - z_0}{d_n} \end{pmatrix} \qquad X = \begin{pmatrix} dx_0 \\ dy_0 \\ dz_0 \end{pmatrix} \qquad L = \begin{pmatrix} R_1^0 - R \\ \vdots \\ R_n^0 - R \end{pmatrix}$$

using indirect adjustment, we got

$$X = \left(B^T B\right)^{-1} B^T L \tag{4}$$

iterative processes are needed.

It is well known that least-squares method is based on condition that the observation data have no gross error or outliers. Outliers, in general, are defined as observations that appear to be inconsistence with the remainder of the data sets (M. Kern, T. Preimesberger etc., 2005). Numerous studies have been conducted, which clearly show that least squares estimators are vulnerable to the outliers, Sometimes, even when the data contains only one bad datum, least-squares estimates may be completely perturbed(Zhengyou Zhang,1995) . In practice, as far as sphere data from point clouds obtained by TLS are concerned, many factors, such as object occlusion, ambiance, unfavorable reflection behaviour at the margin of the sphere and segmentation of sphere data from point clouds etc., result in existence of a large number of outliers. So the performance and accuracy of the least squares approach degrade, and the estimated parameter values may be unreliable. Therefore a robust method should be developed to fitting sphere so as to identify and remove outliers to weaken their effect on parameter estimates, and improve the accuracy of estimated parameters.

## 3. ROBUST METHOD BASED ON M-ESTIMATION

An algorithm based on M-estimators (Zhengyou Zhang,1995, John Fox ) is presented. Let $r_i$ be the residual of the $i^{th}$ data point, i.e. the distance of $i^{th}$ datum to fitted sphere

$$r_i = d_i - R = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} - R$$

The standard least squares method tries to minimize $\sum_i r_i^2$ ,

but the M-estimators try to reduce the effect of outliers by replacing the squared residuals ri2 by another function of the residuals, i.e.

$$\min \sum_i \rho(r_i) \tag{5}$$

Where $\rho$ is a symmetric, positive-definite function with a unique minimum at zero.

Let **b**= $[b_1,b_2,\ldots,b_n]^T$ be the parameter vector to be estimated. Put partial derivatives of (5) to the vector **b** and let derivatives be zeros. At the same time, let $\psi(\bullet) = \rho'(\bullet)$, we can obtain the following equations:

$$\sum_i \psi(r_i) \cdot \frac{\partial r_i}{\partial b_j} = 0 \qquad j = 1, 2, \cdots n \tag{6}$$

Define a weight function

$$\omega(r_i) = \frac{\psi(r_i)}{r_i}$$

Then equation (6) becomes

$$\sum_i \omega(r_i) \cdot r_i \cdot \frac{\partial r_i}{\partial b_j} = 0 \qquad j = 1, 2, \cdots n \tag{7}$$

it equals to the solution of the following iterated re-weighted least-squares problem

$$\min \sum_i \omega(r_i) \cdot r_i^{\,2} \tag{8}$$

There are many kinds of weight function, in the paper, we choose one proposed by professor Zhou Jiangwen(Huang Youcai, 1990)

$$\omega_i = \begin{cases} 1 & |r_i| \le 1.5\sigma \\ \frac{2.5\sigma}{|r_i|} & 1.5\sigma < |r_i| \le 2.5\sigma \\ 0 & 2.5\sigma < |r_i| \end{cases} \tag{9}$$

where $\sigma$ is the standard deviation of residuals. Meanings of the weight function is: when the residual of data is large than 2.5 times standard deviation, the data are regarded as outliers, so the corresponding weight is zero, which indicates that the data has no function is parameter estimation, or else , different weight is given to each data point.

From equation (9), we know that the weights depend upon the residuals, and however, the residuals depend upon the estimated coefficients, and the estimated coefficients depend upon the weights. So, an iterative solution is therefore required.

The algorithm based on M-estimators is as follows:

1) Select initial estimates $\mathbf{b}^0$;
2) At each iteration k, calculate residuals $r_i^{(k-1)}$, and the

corresponding standard deviation $\sigma^{(k-1)}$, k is an iterative number;

3) give associated weights $\omega\left(r_i^{(k-1)}\right)$ to each data point, according to equation (9);
4) Solve for new weighted-least-squares estimates

$$X^{(k)} = \left(B'\omega^{(k-1)}B\right)^{-1} B'\omega^{(k-1)}L$$

Steps 2 to 4 are repeated until the estimated coefficients converge.

## 4. NUMERIC EXPERIMENTS

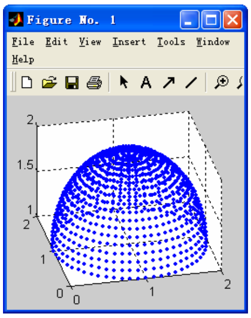
In order to verify the liability and robustness of the algorithm, we use simulation data and real point clouds data to check it. Software Matlab (Pu Jun, Ji Jiangfeng etc.) are used in computation.

### 4.1 Simulation Data Experiment

In simulation test, supposing the sphere to be fitted is as follows

$$(x-1)^2 + (y-1)^2 + (z-1)^2 = 1$$

we draw more than one thousand point from a unit sphere; among which seven are made to be outliers purposely.



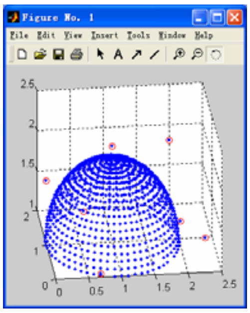




Figure 1 Figure 2
Figure 1 Simulated point clouds data
Figure 2 Simulated data with outliers

Table 1 lists the results respectively.

| Parameters / methods | $x_0$ | $y_0$ | $z_0$ |
|---|---|---|---|
| Predefined parameters | 1 | 1 | 1 |
| LS method | 1.002029 | 1.001783 | 1.000482 |
| Robust method | 1.000091 | 1.000001 | 0.999963 |

Table 1 Parameters of simulated model and its estimated values

From the tables above, we can see clearly that the robust method is superior to the least-squares method.

### 4.2 Real Point Cloud Data Experiment

Figure 3 is real point cloud data of sphere target obtained by Faro Large scanner.
From figure 3, we can see that there are large numbers of outliers, so we use matlab to initially delete some obvious outliers, the result is seen figure 4.

The radius of faro sphere is 72.5mm. Table 2 list results of different methods. Compared with result of faro scene software, we can see that method based on M-estimation is more accurate than LS method. This further demonstrates accuracy and reliability of the method propose.

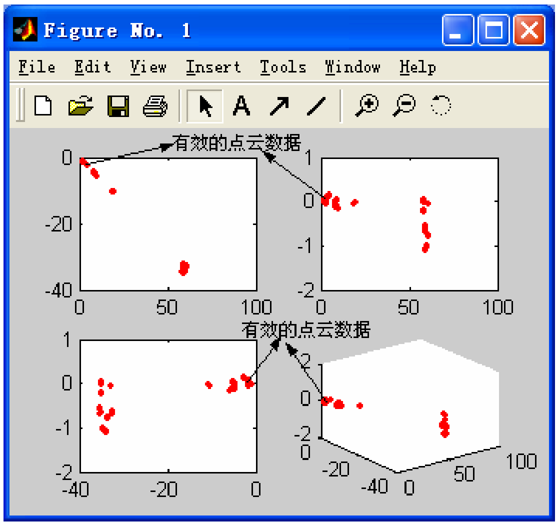



Figure 3 spheres target data from real point cloud
(top left represents projective graph on xy plane; top right represents on xz plane; low left represents on yz plane; low right represents graph on 3D coordinate system )

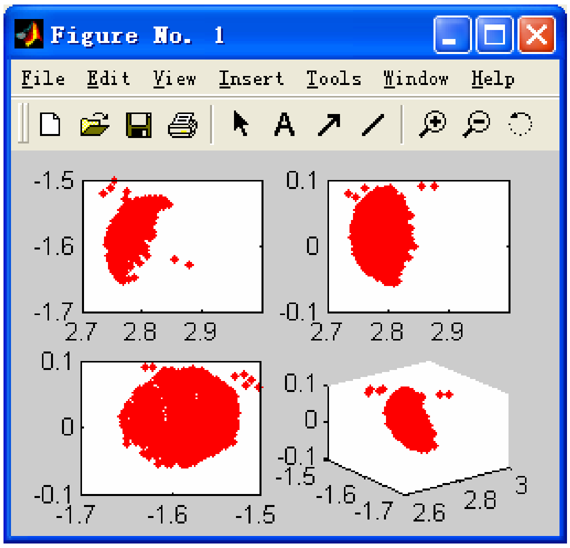



Figure 4 Spheres target data from real point cloud after deleting outliers roughly
(top left represents projective graph on xy plane; top right represents on xz plane; low left represents on yz plane; low right represents graph on 3D coordinate system, apparent outliers roughly)

| parameters / methods | $x_0$ | $y_0$ | $z_0$ |
|---|---|---|---|
| Results using commercial software | 2.818048 | -1.601085 | 0.018759 |
| LS method | 2.823747 | -1.604789 | 0.018325 |
| Robust method | 2.819342 | -1.602488 | 0.017821 |

Table 2 Estimated values from real point clouds data

## 5. CONCLUSION

There are two steps to position location of sphere target, first segment sphere data from point clouds, and then fitting sphere. On the assumption that segmentation has been completed, we proposed a robust method based on M-estimation to fitting sphere target and determine the center coordinate of sphere. Experiments using simulation data and real data are shown that the proposed method can overcome the effect of outliers, and improve the positioning accuracy of sphere centre coordinates. Compared with least square estimation, it has the advantage of robust.

On further research, we will focus on how to segment sphere data from point clouds automatically.

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