

NEW METHOD IN MAP PROJECTION FOR INDIRECT COEFFICIENTS

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ABSTRACT

This paper is opening a new methodology in conformal transverse Projection for Gauss or "Mercator", and Conformal Russell stereographic projection.

We know that Map Projection is using new method " harmonic multinomial equations by Laplace equations, if we want to find the coefficients for direct method it will be very easy, while indirect method coefficients it will be very difficult because known only C_{10} coefficients, but now we can apply new law for coefficients indirect method unlimited C_j where $j=1,2,4,\dots,n$.

INTRODUCTION

The Map Projections divided to topographic projection and geodetic projection; where topographic projection use for normal accuracy less one meter in field surveying, while the geodetic projection in modern measurements with global positioning system GPS and development program in technology of computer GIS become necessary to construct high precision projection.

Traditional geodetic projection has difficulties in construct of mathematical equations for indirect method "transportation rectangular coordinates to geographic coordinates "method Kruger ", while it can be solved only 5 boundaries from series equations [4].

Since computer technology has begin to use a new methodology in geodetic projections method harmonic multinomial equations by Laplace equations [2]; as we know in map projections isometric system coordinate, are used condition solves harmonic multinomial equations, but after 8 boundaries, it will be difficult to solve by equations.

This problem was solved by Professor Uladzimir Padshyvalau, where he can solved any boundaries by general equation [1], also he solved indirect coefficients for 8 boundaries. We know that method indirect coefficients have 10 boundaries [3], using for zone 16-18 grades.

We developed a general series methodology for indirect coefficients of cartographic projections to get unlimited coefficients.

GENERAL HARMONIC MULTINOMIAL EQUATIONS BY LAPLACE EQUATIONS

Application these equations in present have more deployment in construct map projection, because it is very easy dealing with computer technology [1] by the following:

$$\begin{aligned}x &= x_0 + \sum_{j=1}^n C_j P_j \\y &= y_0 + \sum_{j=1}^n C_j Q_j\end{aligned}\tag{1}$$

Where x_0, y_0 = initials coordinates systems for zone projection

C_j = coefficients expansion of projection by direct method

P_j, Q_j = elements of harmonic multinomial equations apply to Laplace equations

An initial coordinates systems for zone projection; where we can get it from meridian arc, and parallels ellipsoid.

$$\begin{aligned}P_j &= P_{j-1}P_1 - Q_{j-1}Q_1, & P_0 &= 1 \\Q_j &= P_{j-1}Q_1 + Q_{j-1}P_1, & Q_0 &= 0\end{aligned}\tag{2}$$

Where P_j = different values between latitudes

Q_j = different values between longitudes

Where the different values between latitudes calculated by q (isometric latitude), and difference between longitudes begin from L_0 (center meridian) to given meridian; isometric latitude value can be got from following equation

$$q = \ln \sqrt{\left(\frac{1 + \sin B}{1 - \sin B}\right) \left(\frac{1 - e \sin B}{1 + e \sin B}\right)^e} \quad (3)$$

The last equations use for direct method, while equations (4,5,6) using for indirect method following them[1]

$$q = q_0 + \sum_{j=1}^n C_j P_j \quad (4)$$

$$L = L_0 + \sum_{j=1}^n C_j Q_j$$

$$\begin{aligned} P_j &= P_{j-1} P_1 - Q_{j-1} Q_{j-1}, & P_0 &= 1 \\ Q_j &= P_{j-1} Q_1 + Q_{j-1} P_1, & Q_0 &= 0 \end{aligned} \quad (5)$$

For geographic latitude using iteration value by following equation

$$B = 2 \arctan \left[\sqrt{\left(\frac{1 + e \sin B}{1 - e \sin B}\right)^e} * \exp(q) \right] - \frac{\pi}{2} \quad (6)$$

EXPANSION COEFFICIENTS FOR INDIRECT METHOD

We know these coefficients very difficult, and using traditional method by adding series into other series; this method needs long time to any coefficients after $n=8$ and error possibility 50 %, while error possibility after $n=10$ by 100%.

And now we can represent new equation for unlimited coefficients with two controls, first control by adding digital of coefficients, and second control by recurrent function; The form equation to series mathematic.

From this equation we have got coefficients even $n=C_{12}=12$ and more, by these coefficients (C_{12}) we can make project in one zone by 20 grades different for longitudes by precision equal 0.001m or 0.00003", these results the same has of GPS.

Test and application indirect coefficients

Here we will study two positions; first in North Africa and second in Europe with Gauss projection or Mercator projection.

Libya- Characteristic of projection: (general latitude) $B_0=26^{\circ}30'00.0000''N$, (center meridian) $L_0=17^{\circ}15'00.0000''E$, wide of (zone) 20°

Data four points geographic coordinates system in edges and mediums zone.

- 1- $\varphi=33^{\circ}30'00.0000''N$ $\lambda=27^{\circ}15'00.0000''E$
- 2- $\varphi=19^{\circ}00'00.0000''N$ $\lambda=27^{\circ}15'00.0000''E$
- 3- $\varphi=33^{\circ}30'00.0000''N$ $\lambda=22^{\circ}15'00.0000''E$
- 4- $\varphi=19^{\circ}00'00.0000''N$ $\lambda=22^{\circ}15'00.0000''E$

These geographic coordinates can be transformed into rectangular coordinates system, and from rectangular coordinates into geographic coordinates, results in table-1.

Table 1(Transformation geographic coordinates in rectangular coordinates and inverse)

| Transformation geographic coordinates in rectangular coordinates | | | | |
|--|-----------------|-----------------|---------------|---------------|
| | φ | λ | X | y |
| 1 | 33°30'00.0000"N | 27°15'00.0000"E | 3753391.51695 | 931090.05563 |
| 2 | 19°00'00.0000"N | 27°15'00.0000"E | 2131959.38724 | 1057198.23312 |
| 3 | 33°30'00.0000"N | 22°15'00.0000"E | 3719480.43007 | 46852.55635 |
| 4 | 19°00'00.0000"N | 22°15'00.0000"E | 2109205.42447 | 526999.68357 |

| Transformation rectangular coordinates to geographic coordinates | | | | |
|--|---------------|---------------|------------------|-----------------|
| 1 | 3753391.51695 | 931090.05563 | 33°29'59.99994"N | 27°15'00.0000"E |
| 2 | 2131959.38724 | 1057198.23312 | 19°00'00.0000"N | 27°15'00.0000"E |
| 3 | 3719480.43007 | 46852.55635 | 33°30'00.0000"N | 22°15'00.0000"E |
| 4 | 2109205.42447 | 526999.68357 | 19°00'00.0000"N | 22°15'00.0000"E |

Table 1 give very good results without errors in operation of transformed coordinates into geographic coordinates; these results give proof for new law of indirect coefficients in north Africa.

Europe (some countries European)- characteristic projection: (general latitude) $B_0=50^{\circ}00'00.0000"N$, (center meridian) $L_0=10^{\circ}00'00.0000"E$, wide of (zone) 19.5°

Data coordinates system in edges and medium zone.

- 1- $\varphi=55^{\circ}00'00.0000"N$ $\lambda=19^{\circ}30'00.0000"E$
- 2- $\varphi=45^{\circ}00'00.0000"N$ $\lambda=19^{\circ}30'00.0000"E$
- 3- $\varphi=55^{\circ}00'00.0000"N$ $\lambda=15^{\circ}00'00.0000"E$
- 4- $\varphi=45^{\circ}00'00.0000"N$ $\lambda=15^{\circ}00'00.0000"E$

Transformation these geographic coordinate to rectangular coordinates system, and also inverse from rectangular coordinates to geographic coordinate's results in table-2.

Table 2 (Transformation geographic coordinates in rectangular coordinates and inverse)

| Transformation geographic coordinates in rectangular coordinates | | | | |
|--|--------------------------|--------------------------|--------------------------|--------------------------|
| | φ | λ | | |
| | | | X | |
| | | | y | |
| 1 | $55^{\circ}00'00.0000"N$ | $19^{\circ}30'00.0000"E$ | 6138715.77576 | 606992.08417 |
| 2 | $45^{\circ}00'00.0000"N$ | $19^{\circ}30'00.0000"E$ | 5029145.86841 | 749048.79402 |
| 3 | $55^{\circ}00'00.0000"N$ | $5^{\circ}00'00.0000"E$ | 6108780.95944 | 319836.88934 |
| 4 | $45^{\circ}00'00.0000"N$ | $5^{\circ}00'00.0000"E$ | 4997211.49925 | 394241.01954 |
| Transformation rectangular coordinates in geographic coordinates | | | | |
| 1 | 6138715.77576 | 606992.08417 | $55^{\circ}00'00.0000"N$ | $19^{\circ}30'00.0001"E$ |
| 2 | 5029145.86841 | 749048.79402 | $45^{\circ}00'00.0001"N$ | $19^{\circ}30'00.0000"E$ |
| 3 | 6108780.95944 | 319836.88934 | $55^{\circ}00'00.0000"N$ | $5^{\circ}00'00.0000"E$ |
| 4 | 4997211.49925 | 394241.01954 | $45^{\circ}00'00.0000"N$ | $5^{\circ}00'00.0000"E$ |

Table 2 give very good results within very small errors in operation of transformed coordinates into geographic coordinates, because the distortion in scale was increased toward direction of north; these results also give proof for new law of indirect coefficients in Europe.

CONCLUSION

From analysis results of new equation "mathematical series" for indirect coefficients by twelfth coefficients get across results following:

- Possibility by new equation gets unlimited indirect coefficients for Mercator projection and Russell stereographic projection.
- This equation has check for any coefficients.
- In this paper was a used twelfth coefficient only for Libya and some countries in Europe, where the results were very good 0.001m in edges projection.
- By using equation for indirect coefficients, the possibility to apply some countries or Europe in one system coordinates of Mercator projection or Russell stereographic projection.

REFERNCE

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