SPACE ALLOCATION OF EDUCATIONAL CENTERS USING MULTIPLICATIVELY WEIGHTED VORONOI DIAGRAM

F. Karimi [a], M. R. Delavar [b], M. A. Mostafavi [c]

[a] MSc. student, GIS Division, Dept. of Surveying and Geomatics Eng., Collage of Eng., University of Tehran, Tehran, Iran, karimi.firoozeh@yahoo.com
[b] Center of Excellence in Geomatics Eng. and Disaster Management, Dept. of Surveying and Geomatics Eng., College of Eng., University of Tehran, Tehran, Iran, mdelavar@ut.ac.ir
[c] Geomatics Department, Laval University, Quebec City, Quebec, Canada, Mir-olfazl.Mostafavi@scg.ulaval.ca

KEY WORDS: Multiplicatively Weighted Voronoi Diagram (MWVD), Ordinary Voronoi Diagram, Educational Centers, Service Area, Allocation.

ABSTRACT:

The main problem of educational centers in a mega city like Tehran, capital of Iran, is that no enforced service areas exist to guide school selection or allow students to make the most convenient commutes to the nearest schools. Without the defined school service areas, parents seeking better and more reputable schools often have no choice but to send children to schools outside the local area. Parental-based system of schools selection has resulted in commutes that are longer than necessary for the students.
In order to reduce the time and distance of commutes to school, in this paper, a way to eliminate arbitrary choices and to create school service areas that allocate students to the closest possible public schools is proposed. In such cases education boards delimit a service area for schools in accordance with a number of criteria that includes not only the proximity of student residences, but also the size of schools and safe access to school. Ordinary Voronoi diagram and multiplicatively weighted Voronoi diagram are used in this research for space allocation of educational centers. Ordinary Voronoi diagram is used to create school service areas that allocate students to the closest possible public schools. Multiplicatively weighted Voronoi diagram considers the size of schools beside proximity. It is concluded that through the introduction of school service areas based on the multiplicatively weighted Voronoi diagram method, it would be possible to make walking commute shorter, more convenient and safe.

1. INTRODUCTION

One of the main problems of educational centers in a mega city like Tehran is that no enforced service areas exist to guide school selection or allow students to make the most convenient commutes to the nearest schools. A service area is the physical area from which these students are coming to a school.
Without the defined school service areas, parents seek better and more reputable schools and often have no choice but to send their children to the schools outside the local area. An arbitrary parental-based system of school selection may result in commutes that are longer than necessary for the students (Howe et al. 2002 and Gleave, 2001).
In order to reduce the time and distance of commutes to school, in this paper, a way to eliminate arbitrary choices and to create school service areas that allocate students to the closest possible public schools is proposed. In such cases education boards delimit a service area for schools in accordance with a number of criteria that includes not only the proximity of student residences, but also the size of schools, and a safe access to school. Safety of access entails that children do not cross any arterial road in order to reach their school.
To delimit boundaries for school service areas, the research utilized the Voronoi diagram, a widely known method for definition of spatial proximity and bounding determination technique. Voronoi diagram (VD) has been extensively used for a wide range of different applications including determining mutually exclusive partition of space in the social and environmental sciences (Okabe et al. 2000) and for the purpose of allocation by a large number of researchers including Vincent and Daly (1990), Boots and South (1997), and Cox and Agnew (1974). It has also been widely used for GIS applications and spatial analysis (Mostafavi 2002, Mostafavi and Gold 2004). Voronoi diagram for a set of points divides a given area into regions or cells so that all locations enclosed within a single cell are closest to its generating point. Since a service area was needed which allocates students within the closest commutes to the designated schools that also took into account differing school sizes in terms of student populations, multiplicatively weighted Voronoi diagram approach was useful for this research. It has been studied by Aurenhammer and Edelsbrunner (1984), Galvao et al. (2006) and Mu (2004).
The multiplicatively weighted Voronoi diagram (MWVD), however, can be utilized when points differ in size to draw polygons around them accordingly. Points can have different sizes because of some properties like capacity of centers, area of resulted polygons and load factor of districts (Galvao et al. 2006 and Restima et al. 2007). Thus larger points will have larger polygons drawn while smaller points have smaller ones.
The paper is organized as follows. In Section 2 a general paradigm for Voronoi diagram is described. In Section 3, multiplicatively weighted Voronoi diagram and some desirable properties to be met by MWVD algorithms are elaborated. In Section 4 the case study is introduced and the results are illustrated. Section 5 concludes the paper.

2. VORONOI DIAGRAM

A classic way of regionalizing or allocating space around a predetermined set of points or `generators' is Voronoi tessellation. This method is based on finding the nearest generator for every point in the space. The resultant regionalization is known as a Voronoi diagram (VD). In a comprehensive presentation on the subject including an extensive review of the literature, Okabe et al. (2000) presentd the ordinary VD as well as many of its generalizations.
In mathematical terms, let $P \equiv \{P_1, P_2, \dots, P_m\}$ be a finite set of distinct points in the 2-dimensional Cartesian space R^2

(Voronoi diagrams are also defined in R^d, with $d > 2$, but our application is limited to R^2). We denote by $\|.\|$ the Euclidean norm in R^2. Let the region $V(P_i)$ be the set of locations $X \in R^2$ verifying $\|X - P_i\| \le \|X - P_j\|$ for all j, except for j =i, which is symbolically written as (Okabe et al. 2000):

$$V(P_i) = \{X \in R^2 \mid \|X - P_i\| \le \|X - P_j\|, j \ne i, j = 1, \dots, m\}$$

The region $V(P_i)$ is called the ordinary Voronoi polygon associated with P_i, and the resulting set, $\{V(P_1), V(P_2), \dots, V(P_m)\}$, the planar ordinary Voronoi diagram generated by $P \equiv \{P_1, P_2, \dots, P_m\}$. P is a generator set, and, for each index $i \in \{1, \dots, m\}$, P_i is a generator point. The edges of Voronoi polygons in R^2 are line segments as shown in Figure 1. When the Voronoi diagram is constrained to a finite region, it becomes a bounded Voronoi diagram. Although Voronoi diagram generators can be points, lines, circles, or areas of diverse shapes (Okabe *et al*., 2000), our application deals with point generators only.

Since peripheral generators can grow unchecked in the opposite direction of the center of gravity of the generator configuration as a whole, they become infinitely large. This problem is normally resolved by introduce an artificial outer boundary and limit the growth of the external generators to these boundaries. This is the approach used in most GIS systems.

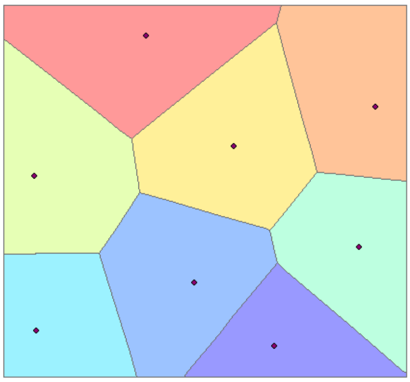

Figure 1. Ordinary Voronoi Diagram

3. MULTIPLICATIVELY WEIGHTED VORONOI DIAGRAM

There are situations where the Euclidean distance does not represent the attracting process well. In order to take other elements into account, weighted Voronoi diagrams may be used. Let $w = (w_1, w_2, \dots, w_m)$ be strictly positive weights associated to the generator points $(P_1, P_2, \dots, P_m)$. The multiplicatively-weighted Voronoi diagram (MW-Voronoi diagram) in R^2 is characterized by the weighted distance (Okabe et al., 2000):

$$d_{MW}(X, P_i) = \frac{1}{w_i}\|X - P_i\|$$

and is represented by (Okabe et al., 2000):

$$V(P_i) = \{X \in R^2 \mid \frac{1}{w_i}\|X - P_i\| \le \frac{1}{w_j}\|X - P_j\|, j \ne i, j = 1, \dots, m\}$$

The dominance region of P_i over P_j is given by (Okabe et al., 2000):

$$Dom(P_i, P_j) = \{X \in R^2 \mid \frac{1}{w_i}\|X - P_i\| \le \frac{1}{w_j}\|X - P_j\|\}$$

For j ≠ i, the dominance region becomes larger as the weight, w_i, increases. In the case with only two generator points, the locus of the points X satisfying equation is the Apollonius circle (Suzuki and Okabe 1995) as shown in Figure 2, except when $w_i = w_j$, in which the bisector becomes a straight line. In general, a MW-Voronoi region is a non-empty set and need not be convex, nor connected; and it may have holes (Okabe et al., 2000).

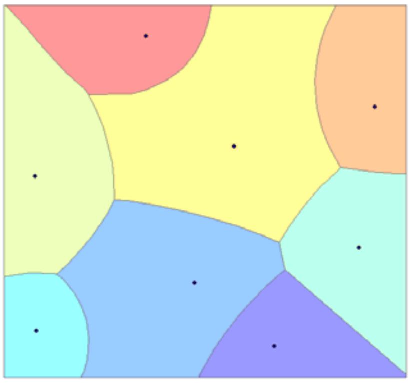

Figure 2. Multiplicatively Weighted Voronoi Diagram

For a Euclidian space S_2 with dimensionality 2 containing N generators, g_i, with weights, W_i, partitioned into N regions, r_i, each with population, P_i, an operational error metric must be selected in order to assess partitioning methods. Although many error indications are possible, the error metric used throughout this paper is proposed by Reitsma et al. 2007:

$$Error = \frac{|P_i - p_i|}{P_i}$$

where

P_i is the expected population of the generator *i* as proportion of total population which is the weight of generator g_i and

p_i is the estimated population of generator g_i as proportion of total population which is calculated as the sum of population in district i

4. CASE STUDY

The proposed methodology is implemented in a space for a road district resides in municipal District 6 of Tehran. Figure 3 represents the scope of the study area. A topographic map of Tehran at the scale of 1:2000 is used which is updated in 2006, and census data extracted from demography carried out in 2007.

Figure 3. The study area

Figure 4 shows the population density map of the study area using Kriging interpolation method.

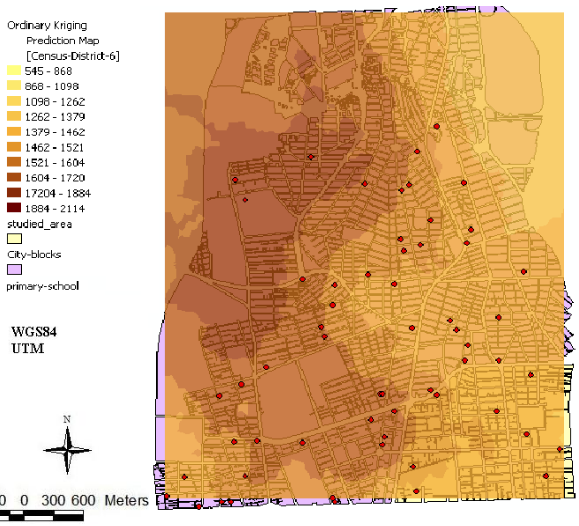

Figure 4. Population interpolation

Ordinary Voronoi diagram is utilized to create school service areas that allocate students to the closest possible public schools is proposed in this research. Multiplicatively weighted Voronoi diagram was implemented to represent more realistic picture of quality of service of primary schools at the scale of urban districts. The goal is to define the population which is unsustainable by the space and resources available level of primary schools that is the main problem of primary schools especially in mega cities like Tehran. It is done by estimating the current service area of the schools and comparing their estimated and expected population.

Delimitation of a service area for schools is done in accordance with criteria that include the proximity of student residences, the size of schools and safe access to school. Among these three rules, the safety of access rule introducing road district as an area bounded by roads which endanger children's access to the schools is adopted. Proximity of students' residences and size of the schools are applied through MWVD. Size of the schools is considered to estimate the expected population for each school, which is resulted from division of school's area into Service per Capita for primary schools. Service per Capita is the area required by a child in a school. According to the regulations of Iranian Ministry of Housing and Urban Development, the capita per student is about 4 square meters (Habibi and masaeli 1999). Weight values, W_i, are simply the proportion of the expected population of the generator g_i to its total population. Each polygon was then constructed in relation to the number of students to represent the weighted factor. Following this approach, larger schools tended to have larger service areas and vice versa.

Figures 5 and 6 show districting, using Ordinary Voronoi Diagram (OVD) and Multiplicatively Weighted Voronoi Diagram (MWVD).

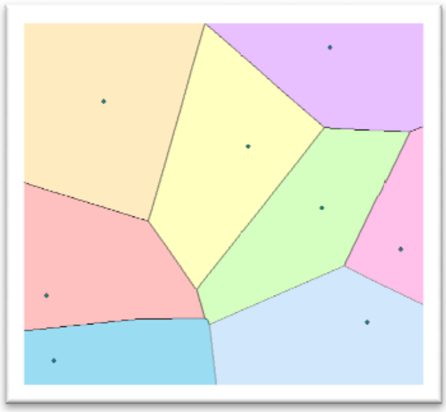

Figure 5. Ordinary Voronoi Diagram on educational centers

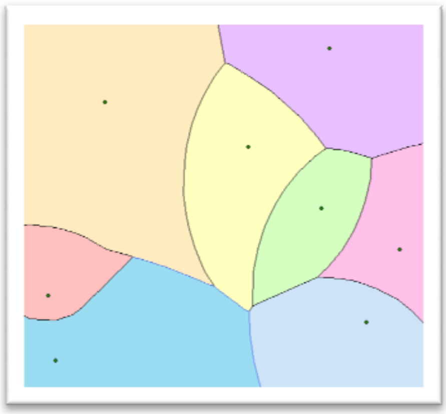

Figure 6. Multiplicatively Weighted Voronoi Diagram on educational centers

The boundaries that would represent each school service area were completed and analyzed using geospatial information system (GIS). Table 1 shows three types of error including minimum, average and maximum error for Ordinary Voronoi Diagram and multiplicatively weighted Voronoi diagram. They are calculated using error equation introduced in section 3. The error is the ratio of the difference of the estimated and expected student population in each district. Whatever this difference is less, districting is done better.

	OVD	MWVD
Min. Error	0.026	0.008
Mean Error	0.392	0.24
Max. Error	0.674	0.545

Table 1. Performance of OVD and MWVD data

5. CONCLUSION

The results verified that the ordinary Voronoi diagram is not useful for this application. Considering the population density creates a much more realistic districting using multiplicatively weighted Voronoi diagram. As presented the error of MWVD becomes less than that of for OVD. By using proposed method for defining school service areas, the student population can be partitioned to have a sustainable space and resources available by primary schools. Through the introduction of school service areas based on the proposed method, it would be possible to make walking commute shorter, more convenient and safe.

REFERENCES

Aurenhammer, F. and Edelsbrunner, H., 1984, An optimal algorithm for constructing the weighted Voronoi diagram in the plane, Pattern Recognition, Vol. 17, No 2, 251-257.

Boots B.N. and South R., 1997: Modeling retail trade areas using higher-order, multiplicatively weighted Voronoi diagrams. Journal of Retailing 73: 519-536.

Cox, K., and Agnew, J.A. 1974. Optimal and non-optimal territorial partition: a possible approach towards conflict. Papers of the Peace Science Society (International) 23: 123-138.

Galvão, L.C., Novaes, A.G., de Cursi, J.E.S. and Souza, J.C., 2006, A multiplicatively-weighted Voronoi diagram approach to logistics districting, Computers & Operations Research, Vol. 33, 93-114.

Gleave D.S., 2001: Factors Leading to Increased School Journey Length. Department of Transport, Local Government and the Regions. Scotland (DTLR), Scotland.

Habibi, M., Masaeli,S., 1999. Urban Land use per Capita's, Ministry of Housing and Urban Development, National Land & Housing Organization (in Persian).

Howe K., Eisenhart M. and Betebenner D., 2002: The price of public school choice. Educational Leadership 59: 20-24.

Mu L., 2004: Polygon characterization with the multiplicatively weighted Voronoi diagram. The Professional Geographer 56: 223-239.

Mostafavi, M.A., 2002, Development of a Global Dynamic Data Structure. PhD thesis, Laval University.

Mostafavi, M.A. and Gold, C.M., 2004, A global kinetic spatial data structure for a marine simulation. International Journal of Geographical Information Science, 18, 211-228.

Okabe A., Boots B. and Sugihara K., 2000, Spatial Tessellation Concepts and Applications of Voronoi diagrams, (second edition). Wiley, Chichester.

Reitsma, R., Trubin, S., Mortensen, E., 2007. Weight-proportional space partitioning using adaptive Voronoi diagrams. Geoinformatica, (11): 383-405.

Suzuki, A. and Okabe, A., 1995, Using Voronoi diagrams. In Z. Drezner Z.(ed.). Facilty Location: A Survey of Applications and Methods. Springer, New York.

Vincent P.J. and Daly R., 1990: Thiessen polygon types and their use in GIS. Mapping Awareness 4: 40-42.