# EXTRACTION OF OPTIMAL SKELETON OF POLYGON BASED ON HIERARCHICAL ANALYSIS[1]

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**KEY WORDS:** Optimal Skeleton, Algorithm, Hierarchical Structure

**ABSTRACT:**

Skeleton is an important 1-D descriptor of polygon and a useful tool for advanced geometric algorithms. However the skeleton that most existing algorithms investigated is the longest one, which is not intended in all circumstances. This paper proposes an algorithm for extracting hierarchically optimal skeleton network of polygons. The algorithm incorporates length, angularity and area of associated part of skeleton segment. The result is a hierarchical structure and each level corresponds to a specific detail of skeleton. The new algorithm has three steps. First the constrained Delaunay triangulation of polygons is constructed. Secondly skeleton segments are connected between neighbouring triangles and skeleton network is built. Thirdly, a dynamic pruning process considering the weights is employed to produce optimal skeletons at each level of detail. The weight in last step is determined by length, angularity and position of skeleton segment.

## 1. INTRODUCTION

Skeleton is the locus of centers of maximal discs interior of a polygon and the primary component of polygon shape descriptors. It has many applications in pattern recognition and computer vision. In automatic cartography and Geographical Information Systems (GIS), skeleton of polygons is very useful for automatic label placement, collapse of area features of natural phenomena to linear features in generalization and map conflations. Skeletonization is the process of finding the centers of maximal circles and connecting them to form a line.

There exist numerous implementations of skeletonization based on the original mathematical definitions. Ogniewicz and others (1995) gave four types of skeletonization algorithms: simulation of the grassfire, analytical computation of medial axis, topological thinning and ridges on the distance map. They also presented an algorithm for building hierarchic skeletons based on Voronoi analysis. Kimmel and others (1995) gave similar classifications of skeletonization algorithms. Practically the most three implementations are based on morphological thinning (as in Figure 1a), Voronoi diagram (as in Figure 1b) and Delaunay triangulation (as in Figure 1c). The latter two types produce straight line skeletons.

When part of the skeleton of a polygon is needed, most previous algorithms are intended for longest ones, which however is not always appropriate. This paper introduces an algorithm which considers properties of skeleton segments and computes a weight for each of them. The optimal skeleton is determined by pruning process based on weight.

## 2. PRIMARY SKELETON NETWORK

As indicated above primary skeleton segments of polygons can be extracted using different implementations. Mathematical morphology can simulate the original definition closely. However in order to employ this kind of implementations, the vector polygons must be converted to raster data first. The dimension of polygons representing natural phenomena varies remarkably and the raster resolution can not be determined easily. Any given resolution cannot appropriate for all polygons of different sizes and further affects shape of skeletons. Every skeleton segment is associated with some part of polygon and this relation is very useful in GIS. But it is hard to build this relation in raster data. The algorithms based on vector data can satisfy this requirement. Delaunay triangulation, which is the dual of Voronoi diagram in 2-D Euclidean plane, is used to build primary skeleton network in this paper.

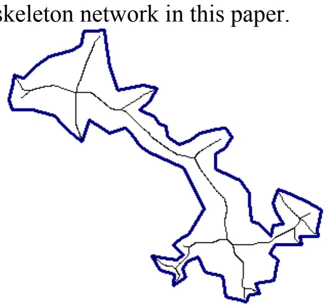

Figure 1a. Skeleton based on morphological thinning of area

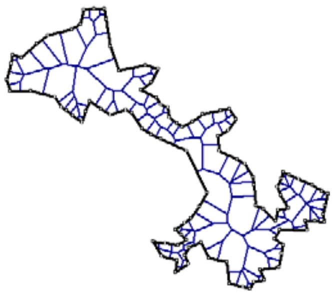

Figure 1b. Skeleton based on Voronoi of vertices of polygon

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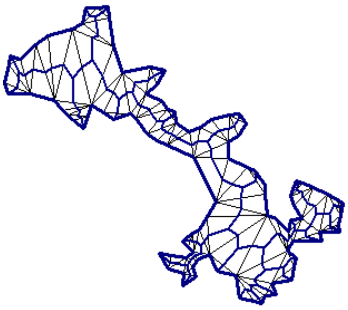

Figure 1c. Skeleton based on triangulation of polygon

### 2.1 Delaunay Triangulation of Polygons

Delaunay triangulation is one of most investigated geometric structures and has applications in many disciplines for its well-shaped characters. Numerous algorithms have been appeared. For a polygon, the Delaunay triangulation can be revised to incorporate the edges of polygon and then a so-called constrained Delaunay triangulation is produced.

In order to extract the skeleton network with enough topological information, edges and triangles in triangulation are classified first. There two types of edges in triangulation: object edges which are coincident with edges of polygon (as $P_1P_2$ in figure 2) and virtual edges which are bounded by two inconsecutive vertices of given polygon (as $P_1P_3$ in figure 2). They are denoted as $E_O$ and $E_V$ respectively. Triangles are classified into three types according to types of constituted edges. Triangles with one virtual edge and two object edges are ear-triangles (denoted as $T_E$ , as $\Delta P_1P_2P_3$ in figure 2). Those with two virtual edges and one object edge are middle triangles (denoted as $T_M$, as $\Delta P_1P_8P_9$ in figure 2). Those with three virtual edges are interior triangles (denoted as $T_I$, as $\Delta P_1P_3P_8$ in figure 2). Here we do not consider polygons which have three vertices.

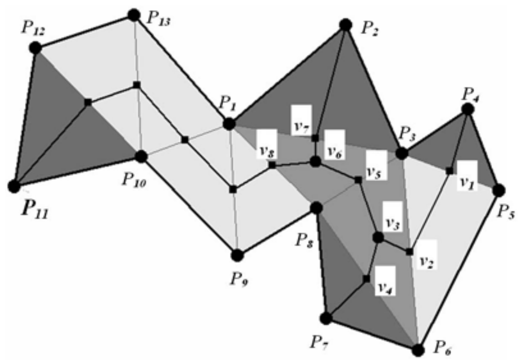


Figure 2. Triangulation of polygon

### 2.2 Primary Skeleton Network

Primary skeleton network can be constructed by threading process in polygon triangulation or by connecting straight skeleton segments.

Each triangle with type $T_E$ contributes one straight skeleton segment whose one node is the center of the only one virtual edge and another node is opposite vertex of virtual edge (as $P_4v_1$ in figure 2). Each triangle with type $T_M$ contributes one straight skeleton segment whose two nodes are the centers of the two virtual edges (as $v_1v_2$ in figure 2). Each triangle with type $T_I$ contributes three straight skeleton segments that share a common node at the barycentre and end at the centers of three edges respectively (as $v_3v_2$, $v_3v_4$ and $v_3v_5$ in figure 2).

In order to reduce the zigzag of skeleton segment, a special modification of barycentre position is needed when a $T_I$ triangle is met. For a given triangle with type $T_I$, three ratios of length of every two edges are calculated, as $P_3P_6/P_3P_8$, $P_3P_6/P_6P_8$ and $P_3P_8/P_6P_8$ in figure 2. If two of them are beyond given range, the barycentre is moved to the center of line segment which connects two center points of the two longer edges of triangle. Difference can be found by comparing figure 1c and figure 7, where the threshold range is from 0.7 to 1.4.

If an end node of a straight skeleton segment is shared by two straight skeleton segments, the two segments are merged into an edge of skeleton topological network. The new graph edge may be further extended if one of its end nodes is shared by only one another straight skeleton segment. And if one end node of a graph edge is shared by three or one straight skeleton segments, a graph node is generated. Until all straight skeleton segments are converted to edges, primary skeleton network of a polygon is generated. For the convenience of programming, the process is started from ear-triangles.

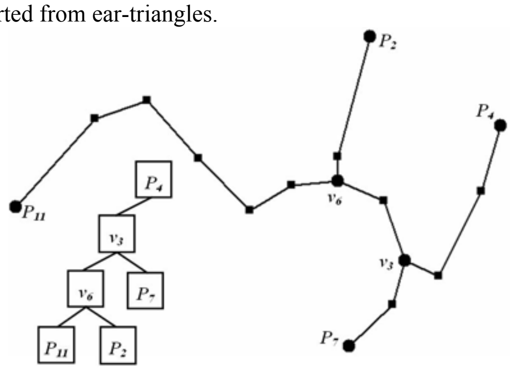


Figure 3. Primary skeleton network of polygon and the corresponding binary tree

As in figure 2, $\Delta P_3P_4P_5$ is the first triangle under consideration, the straight skeleton segment $P_4v_1$ is produced. The new segment shares $v_1$ with only one segment $v_1v_2$ produced by $\Delta P_3P_5P_6$. A new graph edge $P_4v_1v_2$ is generated. $v_2$ is shared by $v_2v_3$ and the graph edge is extended as $P_4v_1v_2v_3$. The process stops as $\Delta P_3P_6P_8$ is a $T_I$ -type triangle. The other two straight skeleton segments $v_3v_4$ and $v_3v_5$ are taken as new starts respectively and the new threading process will stop at another straight skeleton segment produced by a triangle with type $T_E$ or $T_I$. $v_3v_4$ is further connected with $v_4P_7$ and a new graph edge $v_3v_4P_7$ is produced. Then $v_3v_5$ can be taken as a start for continuing. During the process, $P_4$, $v_3$, $P_7$ are converted to graph nodes. The traverse is iterated until all triangles are visited. It may be breadth-first or depth-first, which produces equal skeleton network.

Each graph edge is one part of skeleton associated with part of the polygon. The length of an edge can represent some characteristics of associated part of polygon. During the traverse of triangulation for building skeleton network, corresponding information is recorded. As in figure 2, graph edge $P_4v_1v_2v_3$ is associated with sub-polygon $P_3P_4P_5P_6v_3P_3$ and $v_3v_5v_6$ is associated with $P_3v_3P_8v_6P_3$. Obviously, the sub-polygon is a union of triangles (or part of triangle with type $T_I$) covering the associated edge. Vertices of all triangles are organized in clockwise order, so the sub-polygon can be generated easily.

Primary skeleton network (as in Figure 3) is un-directed graph within which each node is 1-degree or 3-degree. Each edge of this graph is a string of consecutive straight-segments that only shares common intersections at end nodes with other edges.

There are two types of graph edges in skeleton network according to characteristics of end nodes. If two end nodes of a graph edge are 3-degree, the edge is an interior edge (as $v_3v_6$ in figure 3, denoted as $GE_I$). If only one end node is 3-degree and another is 1-degree, the edge is an ear edge (as $P_4v_3$ in figure 3, denoted as $GE_E$). If both nodes are not 3-degree, the edge is a single edge (denoted as $GE_S$), which appears at last step of pruning process in section 4.

The set of all ear edges is denoted as:

$$O_e=\{o_1, o_2, \cdots, o_n\}$$

And all interior edges are denoted as:

$$I_e=\{i_1, i_2, \cdots, i_m\}$$

## 3. EVALUATION OF SKELETON SEGMENTS

As indicated above, skeleton network is an un-directed graph. If any 1-degree node is taken as root, the network becomes a binary tree. Except for root and leaf nodes, all tree nodes are complete (as in figure 3). Considering a compact description of polygon, the importance of each edge which locates at different position of polygon is different. The importance is a function of its length, area of associated sub-polygon and angularity with neighboring edges. For an edge with vertices string:

$$\{P_1(x_1, y_1),\ P_2(x_2, y_2), \ldots, P_n(x_n, y_n)\}$$

its weight $W(o_i)$ can denoted as :

$$W(o_i)=\lambda_L L(o_i) + \lambda_A A(o_i)+ \lambda_S S(o_i) \qquad (1)$$

Where $L(o_i)$, $A(o_i)$ and $S(o_i)$ are factors related with its length, area of associate sub-polygon and angularity and $\lambda_L$, $\lambda_A$ and $\lambda_S$ are weights of respective factors. Because the three factors are in different measurement, they must be normalised first.
Specially, when $\lambda_A=\lambda_S=0$, equation (1) becomes:

$$W(o_i)=\lambda_L L(o_i)$$

### 3.1 Length

The length of all ear edge (with type $GE_E$) is computed in Euclidean space as:

$$L'(o_i)=\sum_{k=1}^{n}\sqrt{(x_{P_{i1}}-x_{P_{i2}})^2+(y_{P_{i1}}-y_{P_{i2}})^2}$$

Then we get a set of all ear edges' length:

$$L'=\{L'(o_1), L'(o_2), \cdots, L'(o_n)\}$$

The maxima and minima values are denoted as $L_{max}$、$L_{min}$. All values are then normalised using (1).

$$L(o_i)=(L'(o_i)-L_{min})/(L_{max}-L_m \qquad (2)$$

### 3.2 Area

In vector data model, every interior part of a polygon are regarded as homogeneous. So the area weight value of an edge is determined by the size of sub-polygon associated.

First we get a set of all ear edges' associated area in Euclidean space:

$$A'=\{A'(o_1), A'(o_2), \cdots, A'(o_n)\}$$

The maxima and minima values are denoted as $A_{max}$, $A_{min}$. All values are then normalised using following equation:

$$A(o_i)=(A'(o_i)-A_{min})/(A_{max}-A_{min}) \qquad (3)$$

### 3.3 Angularity

For three edges sharing one node, the two with similar angularities are visually connected as one according to Gestalt rules. All edges in skeleton network are simulated using linear least square equation. The straight line is defined as:

$$l:\ y=a_0+a_1x$$

We can get the slope of simulated line as:

$$a_1=(n\times\sum_{i=1}^{n}(x_iy_i)-\sum_{i=1}^{n}x_i\times\sum_{i=1}^{n}y_i)/(n\times\sum_{i=1}^{n}x_i^2-(\sum_{i=1}^{n}x_i)^2) \qquad (4)$$

All edges are processed to get simulated slopes using above equation. All vertices of the polygon are used to process to get a general slope $a_g$ using linear least square equation. The angularities of all $GE_E$ type edges are computed using following equation:

$$S_i'=|\arctan(a_1)-\arctan(a_{1i}')| \qquad (5)$$

And if $S_i'>\pi/2$, then $S_i'=\pi-S_i'$. Then we get a set of all ear edges' angularities:

$$S'=\{S'(o_1), S'(o_2), \cdots, S'(o_n)\}$$

$S_i'$ denotes the sharp angle between the simulated line of polygon and the one of skeleton segments. The smaller of this value the more important of this ear edge. The maxima and minima values are denoted as $S_{max}$、$S_{min}$. All values are then normalised using following equation:

$$S(o_i)=(S_{max}-S'(o_i))/(S_{max}-S_{min}) \qquad (6)$$

### 3.4 Weights of the factors

The values of weights of the three factors, $\lambda_L$, $\lambda_A$ and $\lambda_S$, can be determined subjectively. The length is the most important factor as previous research used length as the only one factor. In label positioning in automatic cartography, larger space is preferred. So at this context, the area of sub-polygons associated with edges in skeleton network is more important than angularity.

When a polygon is collapsed in cartographic generalisation, the resulted medial axes may be preferred to have similar angularity with original polygon. And then the angularity becomes more important.

## 4. HIERARCHICAL STRUCTURE OF SKELETON NETWORK

In order to get optimal hierarchical structure of skeleton, a bottom-up pruning strategy is employed. Firstly a set of processed edges is initialized and denoted as $O_e' = \{o_i \mid i=m, m+1, \cdots, n\}$. Initially m is 1 and $O_e' = O_e$.

(1) Compute and Normalise three factors ($L'$, $A'$ and $S'$) of each ear edge in $O_e' = \{ o_i \mid i=m, m+1, \cdots, n\}$ to get $L$, $A$ and $S$.

(2) Compute the weight of each ear edge $W=\{W(o_i) \mid W(o_i)=\lambda_L L(o_i) + \lambda_A A(o_i) + \lambda_S S(o_i), i=m, m+1, \cdots, n\}$.

(3) Find the ear edge with minimized weight $o_k$, and exchange the position of $o_k$ and $o_M$. And then m=m+1. The step prunes the edge with minimized weight.

(4) Maintenance of topological information of skeleton network. If both nodes of edge $o_k$ with minimized weight are 1-degree, go to next step. Assume the node with 3-degree of edge $o_k$ is $v_p$, because $o_k$ is moved, the other two edges will be connected to produce a new edge whose length and associated area are the sum of the original two. The new edge's angularity will be re-calculated. If both of the two original edges are $GE_I$ type, the new one is marked as $GE_I$ type. Otherwise it is marked as $GE_E$ type.

5) If $m<n-1$, go back to first step. Otherwise the process is finished.

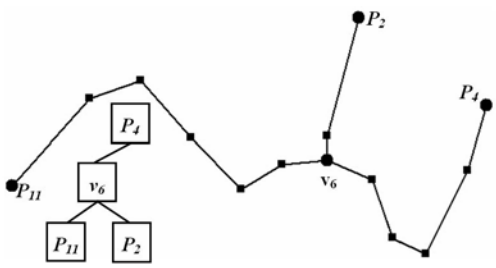


Figure 4. Ear edge $v_3P_7$ is prunedas with minimized weight


Figure 5. Ear edge $v_6P_2$ is pruned as with minimized weight

The processing of primary skeleton network in figure 3 is showed in figure 4 and 5. First, ear edge $v_3P_7$ is pruned and interior edge $v_3v_6$ and ear edge $P_4v_3$ are merged into an ear edge $P_4v_6$. Then ear edge $v_6 P_2$ is pruned and interior edge $P_4 v_6$ and ear edge $v_6 P_{11}$ are merged into a single edge $P_4P_{11}$. For only one edge left in skeleton network, the process is finished.

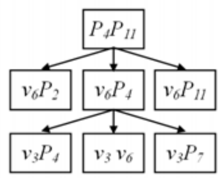


Figure 6. Hierarchical structure of skeleton network

As indicated above the skeleton network can be regarded as a binary tree. If there are $n$ ear edges in $O_e$, there are $n-1$ leaves, $2n-2$ graph nodes, $2n-3$ graph edges and $n-3$ interior edges. During pruning, one edge is marked and moved out of candidate set $O_e'$ once a time. When $m \leqslant n-3$, one interior edge is eliminated in each interation. When m = n−2, there is only one edge left in $O_e'$ which can be taken as the optimal skeleton for whole polygon. The elements in $O_e'$ are in importance-ascending order after pruning. For the skeleton network in figure 3, the result of $O_e$ is $\{ v_3P_7, v_6P_2, P_4P_{11}\}$, which is a hierarchical structure as showed in figure 6. Inversely, an incremental skeleton network can be acquired by inserting an edge to next neighbouring edge.

The polygons representing natural phenomena should always be regarded as fuzzy objects. When part of skeleton network is needed, the result can be extracted from the hierarchical structure of skeleton network. Figure 7 shows two levels of skeleton for given polygon

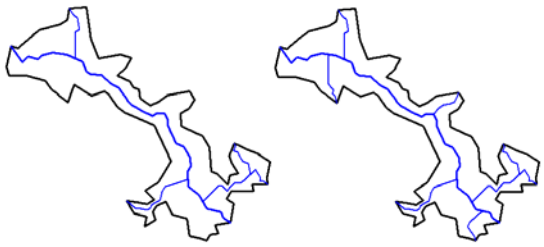

Figure 7. Two levels of hierarchical structure of skeleton network

## 5. RESULT AND DISCUSSION

The new algorithm for extracting optimal skeleton which considers more factors is a general model of existing ones which consider only length. When $\lambda_A=\lambda_S=0$, the result is the longest skeleton as left image showed in figure 8. The right one is acquired by setting $\lambda_L$, $\lambda_A$ and $\lambda_S$ with 4, 2 and 1 respectively. The right one agrees more with human being's visually cognition and is desired in many contexts.

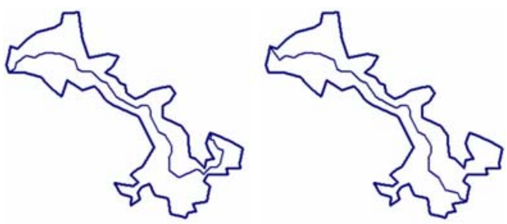

Figure 8. Longest and optimal skeleton of polygon

Further research needs consider polygons with holes. As an abstraction of polygon, the skeleton should be simplified to eliminate more details.

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