A UNIFIED SPATIAL MODEL FOR GIS

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ABSTRACT:

GIS (Geographic Information Systems) are concerned with the manipulation and analysis of spatial data at a "Geographic" scale. Apart from issues of storage, database query and visualization, they must deal with several significantly different types of spatial information. These may be roughly classified as: discrete objects; networks; polygonal maps; and surfaces. Each of these has a specific set of assumptions associated with it, a specific data structure, and a specific set of algorithms. This produces a high level of complexity in the construction, manipulation, analysis and comparison of these datasets. This paper reports the results of an attempt to integrate these based on a slight modification of the fundamental spatial query: "What is here?" where "here" is usually an (x, y) location. It summarizes our previous work on this topic, and presents the "big picture" for GIS.

We demonstrate that when "here" is replaced by "closest to here" the resulting proximal query (a Voronoi diagram) may be used to manipulate the four categories described above, with a resulting simplification of the system. All discrete objects become "fields", with a value at any location. The catch is that in order to represent non-point objects a line-segment Voronoi diagram is required, and this has been shown to be extremely complex to construct on digital computers. Nevertheless, with the availability of several potential solutions, including our own, we can show the implementation of a basic GIS with a common data structure and set of operations. In addition, several other "awkward" spatial queries can be shown to be simplified.

1. INTRODUCTION

Most GIS systems use separate thematic "layers" (pages) to store different types of data. Layers can be composed of object or field type data. Each layer contains specific features or characteristics of the area, so there is a separate layer for the road network, the distribution of buildings or the terrain relief. The layers can be stacked on top of each other and various operations can be performed on them. Some of the simplest are queries using a single layer, including finding what is present at a specified location, what is the elevation at a specified point or what is the area of a selected parcel. Using more than one layer the operations can be performed on different objects and characteristics, so we can ask what is the nearest mailbox to a selected house, find houses located within a specified postcode area, or find the areas where tobacco is cultivated or if the land value is higher than a specified value.

However, although it is possible to compare and perform operations on different layers, there is no consistent method applicable to all data types. GIS has traditionally separated field and object layers and used different data structures to manage them (Burrough and McDonnell (1998)).

The solution used in this work is to represent field and object models using proximal maps. This would standardize the GIS operations and make the topology information available in every model. Each object in a proximal map is associated with a region enclosing the part of the map closer to that object than to any other. The adjacency between objects is clearly defined, as neighbouring objects share boundaries. Converting discrete object layers into proximal maps transforms the query "What is here?" to "What is closest to here?", so the answer is available at any location. We have previously reported on various aspects of this project (Gold and Dakowicz (2006), Dakowicz and Gold (2006)). In this article we integrate the various components and

show the validity of the "big picture" for GIS. We should perhaps emphasize that this is an exercise in producing a good "model of space" to form the framework for a good and consistent set of operations and algorithms. Given the many years of traditional GIS development there is no claim that the current work will immediately produce faster execution times. However, a consistent spatial model leads to a consistent spatial data structure appropriate to all the examples given. We believe the current work to be a good demonstration of the model's viability: it is based on the extremely difficult problem of implementing the Line Segment Voronoi Diagram (LSVD) in a finite-precision computing environment, especially in its kinetic and interactive form.

2. A UNIFIED SPATIAL MODEL

The main objective of this research is to develop a unified spatial model for objects and fields and demonstrate its usefulness. In this work, a vector Voronoi Diagram (VD) considered to be "the fundamental spatial data structure" (Aurenhammer (1991); Okabe et al. (2000)) is used. The VD is a proximal map of the input data. The creation of the Voronoi diagram of a set of discrete points converts the model to a continuous field. The resulting diagram covers the whole map, the attribute value at any location inside each cell is available from its node and the adjacency relationships between nodes are clearly defined. The VD is associated with the Delaunay Triangulation (DT), which is its dual graph with edges connecting neighbouring points. Both are well studied and there are many algorithms allowing their construction and modification (Guibas and Stolfi (1985); Devillers (1998)). Two other structures - the crust and skeleton (Amenta et al. (1997); Gold and Snoeyink (2001)) - can be easily extracted from the VD/DT. These have many interesting properties and can be used in various GIS operations, including digital terrain construction or watershed generation. Additionally, it is possible to convert VD/DT models to rasters

using various interpolation techniques to perform traditional analysis on them, such as slope estimation.

However, modelling line and polygon objects with the ordinary point Voronoi diagram is problematic, as the connectivity of two nodes in the VD depends on the distance between them and the configuration of neighbouring nodes. The ordinary VD does not guarantee preserving connectivity between selected nodes and this is why the Constrained Delaunay Triangulation (CDT) (Lee and Lin (1986); Chew (1987)) and the Line Segment Voronoi Diagram (LSVD) (Gold et al. (1995); Imai (1996); Held (2001) and Karavelas (2004)) were developed. Both structures preserve the configuration of the input line segments, but in different ways. In the CDT the input segments (constraints) are "forced" into the triangulation as edges and the only thing distinguishing constrained edges from ordinary edges is a flag, stating whether the edge is constrained or not. The resulting triangulation is not fully Delaunay and the structure of its dual Voronoi diagram is different from the ordinary VD. On the other hand, in the LSVD, the input segments are separate objects and each of them has an associated Voronoi region, just like point objects.

This work presents methods that can be used to construct and update the ordinary VD/DT as well as the more complex CDT and LSVD diagrams. Depending on the applications, the input data and intended operations, the ordinary VD/DT, CDT or LSVD can be produced from the input set of points or segments and points. Dynamic methods for mesh management are proposed. These allow the insertion and deletion of points and line segments in the VD/DT/CDT/LSVD, making local updates possible at any time. The same idea and mechanism are used to construct different types of Voronoi diagrams. The method is based on the idea of moving points in the VD, and in the case of the CDT/LSVD the moving point leaves a trace behind, which becomes the line segment.

In most GIS there are four major categories of features and related data structures:

1. Discrete objects. These can be points, lines or polygons. Points are used to represent locations of features or objects having relatively small size. Examples include lamp posts locations or mailboxes. Lines consist of series of connected points and can be used to represent linear features, e.g. roads or geological faults. Polygons store relatively large single objects, usually with clearly defined boundaries, such as buildings or lakes. Some objects may be mobile, changing their position in the map, e.g. people or cars.
2. Networks. These are a special case of connected lines with defined topologies. Networks consist of segments, with nodes where segments join. They are used for example to model flow in rivers or roads.
3. Polygonal maps. These are space exhausting, so the whole area is covered by non-overlapping polygons. Examples include postcode boundaries or land ownership information.
4. Surfaces. They are space-filling, and store information about the relief of the terrain or other "field" information.

We provide examples of these four categories of spatial data types represented by Voronoi diagrams, as in Figure 1. Discrete objects may be points – which are converted to fields by calculating the Voronoi diagram. They may also be polygonal objects, such as houses – which may be converted to fields by

calculating the LSVD or CDT from boundary segments. Networks, the second category, can be converted to fields by the LSVD or CDT constructed for their segments. These preserve connectivity of the links, so flow can easily be simulated, and adds the possibility of additional analyses, such as the proximity to the nearest network segment. The third category is polygonal maps, which are fields covering the whole map area. Representing them with the LSVD adds topology (connectivity) to the map automatically as edges are added. The fourth category, surfaces, can be constructed from point, contour or raster data, giving a field model based on the DT. Mobile objects can be handled in any of these four modes by using the kinetic Voronoi diagram: applications include collision detection and map updating and downdating.
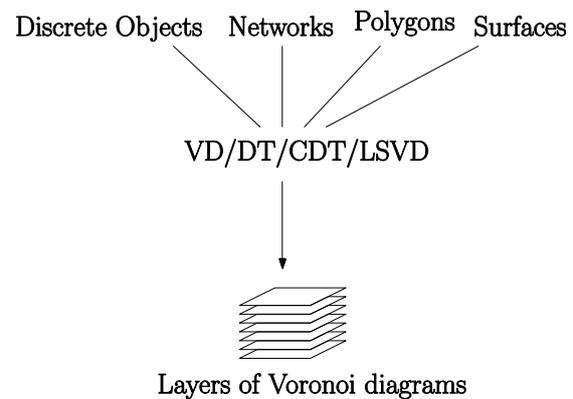

Figure 1. Integration of various models using Voronoi diagrams.

3. DISCRETE OBJECTS

We can distinguish three main types of discrete objects: points, polygons and mobile objects. Points can be converted to fields simply by constructing the Voronoi diagram. Figure 2 shows the VD of a set of rock outcrops of various types distinguished by the numbers assigned to different locations. The heavy boundary lines separate cells with different values of labels, and thus gives an approximate geological map.
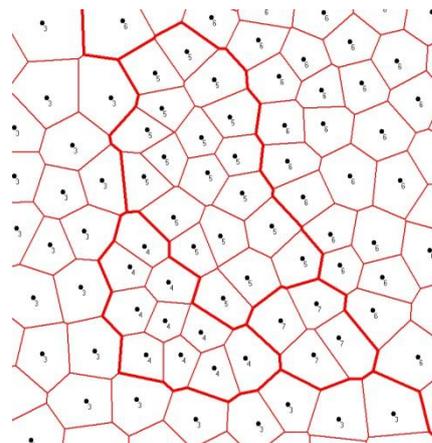

Figure 2. Boundaries between different types of rocks.

It should be noted that the VD could be applied to other types of point data: for example forestry. In this case the cell area may be an approximation of the tree crown size, and the tree density

(trees per unit area) may be determined as the reciprocal of the cell size (area per unit tree), eliminating the traditional "counting circle" approach, emphasizing the generality of the proximal model.

Discrete polygons can be converted to fields by constructing the CDT or LSVD for the boundary nodes and lines, so each polygon can be represented using line segments or constrained edges. Figure 3 presents two different representations of the same polygonal map of buildings. In Figure 3a the map is converted to a Line Segment VD (Gold and Dakowicz (2006)), and buildings are represented using polygons formed by closed loops of line segment objects. Each complete polygon has a general Voronoi cell associated with it, and neighbourhood relationships are established. Figure 3b shows the same set of polygons converted to the CDT, where boundaries of polygons are represented by constrained edges. However, polygons here are not objects as in the LSVD.
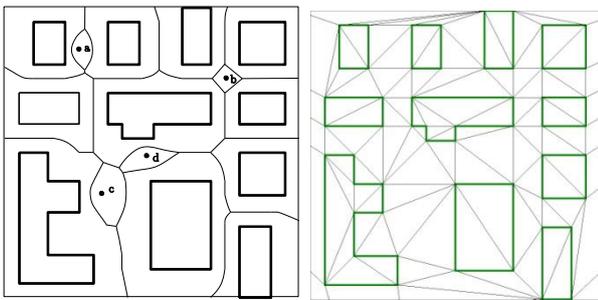

Figure 3. Polygonal objects. a) Represented by the LSVD, plus points. b) Represented by the CDT

Mobile objects can also be managed by incorporating them into the Voronoi diagram and using the moving point approach to change their locations. Figure 3a shows four point objects, which could be cars or people: objects c and d are adjacent and share a common Voronoi edge.

4. NETWORKS

Figure 4. The LSVD of a road network.

Networks using various structures associated with the VD can be modelled in several ways. Figure 4 shows a drawing of an imaginary road network. Such an image is usually converted to a digital network by digitizing it. However, this often leads to

errors at junctions, where several segments of the network meet, and the challenge is to assure that they are joined. Our kinetic algorithm (Gold and Dakowicz (2006)) eliminates problems at the junctions, as all objects have buffer zones associated with them and are merged when their buffer zones overlap. The result is a space-filling tessellation with all cells connected, including those for adjacent road segments. Figure 4 shows the LSVD created from the data. Each part of the network is represented by a line segment object and they are correctly joined at the junctions. The topology is defined and various analyses, such as shortest path queries, can be performed. Additionally various proximity relationships are readily available within the 2D space, and not just in the network, so it is simple to calculate the distance from point x in Figure 4 to the nearest road segment, for example.

4.1 River Networks: The Crust and Skeleton

If a set of points represents samples on the boundary of a polygon or along a network then constructing the VD/DT permits reconstruction of the boundary or "crust" if the samples are close enough (Amenta et al. (1997)). The medial axis, or "skeleton" (Blum (1967)) can be easily formed in a similar manner (Thibault and Gold (2000)).

The river network, even without any elevation data, provides enough information to construct a very reasonable watershed model (Gold and Dakowicz (2005)). The idea of Blum (1967), who assigned elevation values to the medial axis based on the distance from the crust, may be used here. The watershed is often roughly equidistant between river segments, as slopes on each side of the river are often similar, so it can be approximated with the medial axis.
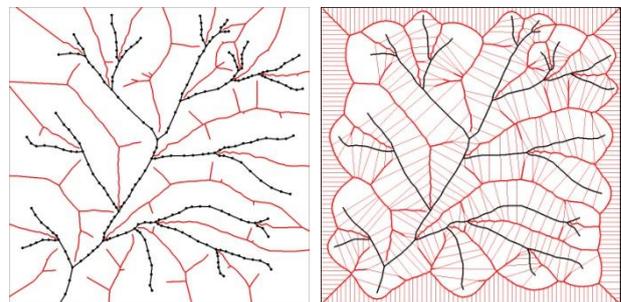

Figure 5. The river dataset. a) Crust and skeleton (watersheds). b) Flow cells.

Figure 5a shows the crust (river channel) and skeleton (watershed) constructed from the samples. Figure 5b shows the Voronoi cells associated with each sample. Traditionally this is estimated from grids, failing to preserve the Euclidean metric and making various dubious approximations of flow between adjacent cells. Here full connectivity of the original data is preserved by the data structure.

4.2 Cumulative Catchment Areas

Assuming a fixed rainfall throughout the map, the river network data can be used to compute cumulative catchment areas (Gold and Dakowicz (2005)). In the TIN model constructed from the samples each node of the river network has a Voronoi cell associated with it. The volume of water in each cell is

proportional to the cell size. The cumulative sum of these volumes downstream gives an estimate of the total water flow at any river node. This can be seen in Figure 6a, where the height of the bars represents the volume of water. This cumulative catchment model provides a first-order approximation of total flow, using the proximal model to associate rainfall areas with river samples. Clearly our proximal model has transformed a network into a surface model, while preserving the network connectivity for subsequent flow analysis.

Note the buffer zones around the river system in Figure 6b. These are simple partitions of the Voronoi cells, and may be used to select regions within a fixed distance of the network (or of any other cartographic object). Itself a polygon object, it can be used for a variety of analyses, such as limiting forestry close to a river. This is often used in GIS analysis by overlaying it with choropleth (polygonal) maps of soils, political zones, etc. This is considerably more efficient than traditional buffer zone calculation if the VD has been previously calculated.
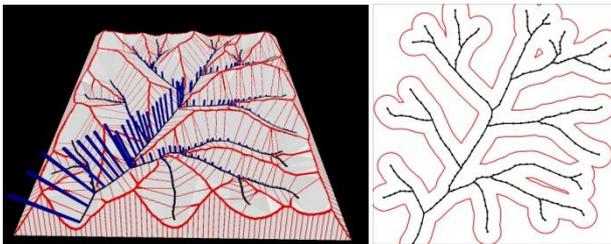

Figure 6. Modelling river networks. a) The cumulative catchment areas. b) River with buffer zones.

5. POLYGONAL MAPS

Polygonal maps can be converted to fields by making a CDT or LSVD from the edges defining the polygon boundaries. Figure 7 shows a sample polygon map and the resulting LSVD, with line segment objects approximating shapes of polygons. Each map polygon is the sum of the Voronoi cells of its boundary interiors. These may be overlaid for subsequent analysis.

Figure 7. A polygon map and the resulting LSVD.

6. SURFACES

Surfaces form the fourth category of spatial models discussed. There are three main types of surface data: elevation data points, grids and contours. These can readily be converted to fields by generating the VD/DT. Contours can be converted to the TIN model by extracting their samples and triangulating them. Cases of "flat triangles" where all three vertices are on the same contour in peaks, pits, ridges and valleys can be handled by generating skeleton points from the contours and assigning elevations to them based on the principle of constant slope (Thibault and Gold (2000)). More generally, interpolation assumes that a value may be estimated at any location, whether from the VD/DT or by more complex methods that attempt to preserve slope continuity. While "counting circles" and the "gravity model" are sometimes still used, the VD approach is based on a consistent spatial model and produces good results. The VD based Sibson method is also called "area-stealing" or "natural neighbour" interpolation ((Gold (1989), Sibson (1982), Watson (1987)). It is based on the idea of measuring the areas that a dummy point inserted at the interpolated location would "steal" from its neighbours, and then using them as the weights for the weighted-average. These neighbours (called natural neighbours) are well defined, since the insertion of a point in the VD/DT produces a unique result, and the Sibson method is particularly appropriate for poor data distributions.

6.1 Runoff Modelling

The two most widely used formulations for water flow modelling are the Finite Element Method (FEM) and the Finite Difference Method (FDM). FEM methods use irregular meshes, FDM methods are often grid based. A form of the FDM, the Integrated Finite Difference Method (IFDM), is based on non-structured meshes. A manual method of irregular cell construction originally suggested by MacNeal (1953) was fully defined by Narasimhan and Witherspoon (1976), and was automated using the Voronoi diagram by Lardin (1999), showing that an iterative finite difference scheme could be developed for Voronoi cells ("buckets") to hold the water, and using the DT to define the slopes between cells. The volume of water moved between adjacent cells depended on the gradients of the triangle edges. Water is distributed irregularly, based on the distribution of cells adjacent to the processed cell.

The efficiency and stability of IFDM simulation is determined by the shape of the cells. The method is based on the idea of moving water volumes between neighbouring Voronoi cells with the amounts determined mostly by the gradient of the cells and the width of the common edge. A problem occurs when there is a large elevation difference between two cells sharing a very narrow common Voronoi edge. Then a large amount of water may accumulate in that cell but only leave it slowly through this narrow edge, giving a poor simulation. To avoid such cases a random pattern of points is generated and inserted into the existing TIN with a guaranteed minimum spacing, as in Figure 8a. A relatively large disk radius is assigned to points and used for collision detection to prevent insertion of points too close to each other. This leads to a fairly regular distribution of Voronoi generators. Additional points have elevation values assigned using Sibson interpolation (Sibson (1982); Gold (1989)), although other interpolation methods leading to a natural and smooth surface can be used. Figure 8a shows the 3D view of the resulting TIN model with the original contour lines

draped over the terrain. Figure 8b shows the 3D view of the Voronoi cells of the same model part-way through a simulation.
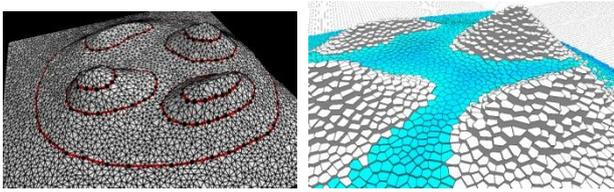

Figure 8. Runoff modelling. a) The 3D view of the Delaunay triangulation with superimposed contour lines. b) Voronoi view, part way through the simulation.
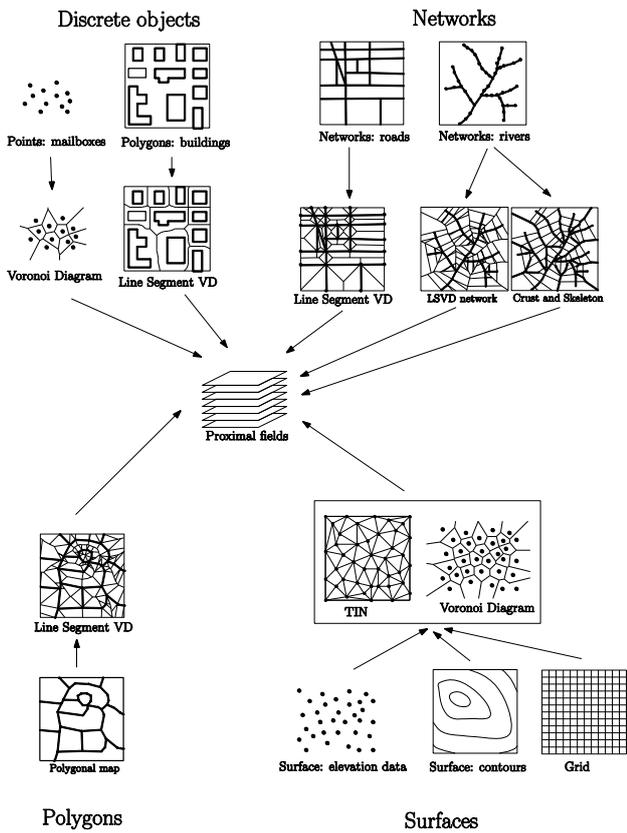

Figure 9. Transforming various data types into proximal maps.

7. INTEGRATION

Figure 9 summarizes the main types of data that may be transformed into proximal field models (Voronoi diagrams or Delaunay triangulations). The result is a set of layers or overlays: different data sets covering the same area and using the same spatial structure. Spatial analysis, which in GIS frequently consists of overlaying data sets to identify potential conflicts, may then be performed in a consistent and straightforward fashion. Merging two layers is performed by drawing the secondary layer onto the primary layer, snapping lines or points together whenever collisions, or close collisions, occur. For the classical polygon-polygon overlay care must be taken to preserve the attributes associated with each half-edge: the resulting overlaid map must have polygons with one attribute set from each original layer. Other combinations of overlays are equally straightforward: locating points within polygons, road segments within counties, houses within city wards, mailboxes adjacent to roads, roads on terrain models,

trees on landscapes, cars on two-lane highways, surface-water runoff, and many more. Conversion of Voronoi (TIN) to raster is trivial; the inverse is frequently done in terrain or 3D model simplification. The resulting data structure (a graph) is in a form known to be well handled by computer.

Albrecht (1996) suggested a set of 20 universal analytic GIS operations, in six categories: Search (including interpolation); Location Analysis (including buffer, overlay, Voronoi); Terrain Analysis; Neighbourhood analysis; Spatial analysis (including pattern and shape) and Measurements (metric properties). Our spatial model directly addresses most of these.

Figure 10 shows the process of extracting buffer zones from a network map represented by the LSVD (Figure 10a). Figure 10b shows buffer zones drawn on top of the line segment Voronoi diagram.
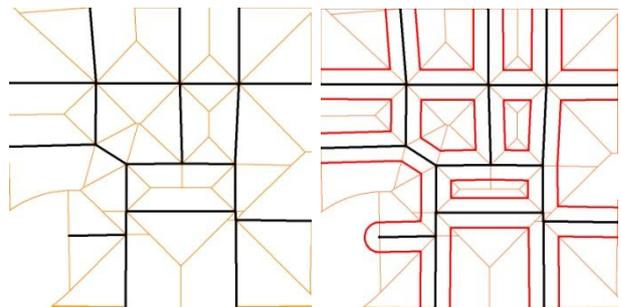

Figure 10. Road network and buffer zones. a) The LSVD. b) Buffer zones of network segments

Figure 11 shows the process of polygon overlay. Figure 11a shows a LSVD created from polygons defining building boundaries. Figure 11b shows the diagram after adding the road network of Figure 10a to the map of buildings. Figure 11c shows the result of overlying the map of buildings with the buffer zones of roads from Figure 10b. Note that the overlaps of buildings and buffer zones (perhaps where the proposed road widening causes a conflict) are themselves polygons.
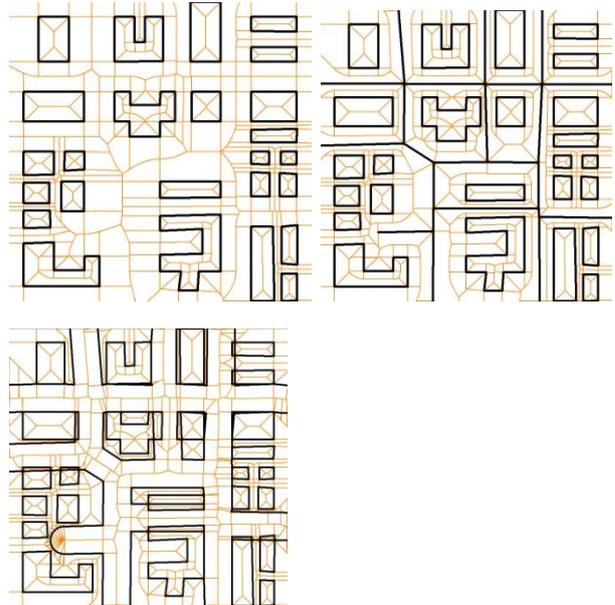

Figure 11. Polygon overlay. a) buildings, b) merged buildings and roads, c) merged buildings and road buffer zones.

8. CONCLUSION

In conclusion, a single spatial model for all (or many) types of spatial data provides a solid algorithmic framework, as well as a clarification of many types of spatial query and analysis that are currently performed with a wide range of frequently inconsistent heuristics. We believe that the proximal (Voronoi) model greatly simplifies the formulation and architecture of geographic spatial analysis. We suggest that the associated spatial model may replace many of the varied data structures used by traditional GIS: while conversion is straightforward this is not usually necessary, and our data structure permits the direct implementation of most basic forms of GIS analysis.

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