A NEW CONSTRUCTION METHOD FOR CIRCLE CARTOGRAMS AND ITS APPLICATION

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ABSTRACT:

An area cartogram is a transformed map on which areas of regions are proportional to statistical data values; it is considered to be a powerful tool for the visual representation of statistical data. One of the most familiar cartograms is a circle (or circular) cartogram, on which regions are represented by circles. Dorling first proposed a circle cartogram construction algorithm in 1996 according to the requirements 'avoid overlap of circles' and 'keep contiguity of regions as much as possible'. The algorithm first places circles according to the geographical configuration of regions and then moves them one by one in order to fulfil the requirements. It outputs results which express a spatial distribution of data; however, the relative positions of circles on cartograms sometimes greatly differ from the geographical maps. Cartogram readers usually compare the shape of cartograms to that of geographical maps, realize the shape distortion, and recognize the characteristics of spatial distribution of represented data on cartograms; removing the excess circle reposition process from the algorithm is preferable to make the relative position of circles on resultant cartograms as similar as possible to that on the geographical map. In this paper, I propose a new construction method for circle cartograms that attaches a high value to the preservation of relative position of circles while considering the requirements proposed by Dorling. I formulated a construction problem for non-linear minimization with inequality constraint conditions and applied the method to the world population dataset.

1. INTRODUCTION

A cartogram (or area cartogram) is one of the most powerful visualization tools for spatial data in quantitative geography (e.g. Monmonier, 1977; Dorling, 1996; Tobler, 2004). Cartograms are transformed maps on which the areas of regions are proportional to the data values. Deformation of the shape of regions and their displacements assist map-readers in intuitively recognizing the distribution of data represented on cartograms.

Cartograms are classified on the basis of two characteristics: the shapes and contiguities of regions indicated on the cartograms. For the shapes of regions, some cartograms use complex shapes, whereas others use simple shapes such as circles and rectangles. The ease of comparison between cartograms with complex shapes of regions and geographical maps enables map-readers to comprehend the characteristics of spatial data presented in such cartograms. However, comparison of cartograms with complex region shapes is difficult in terms of the size of the regions; in this sense, it is better to use simple shapes to express data.

Cartograms that express regions in the form of simple shapes are classified into two types on the basis of contiguities of the regions illustrated on them. One is contiguous cartograms; a rectangular cartogram proposed by Rasiz (1934) serves as an example. Rectangles represent regions, and different rectangles representing adjacent regions are placed contiguously. A rectangular cartogram is an effective visualization tool, as the size of regions is easy to perceive. However, its construction is difficult because it is impossible to maintain all contiguities of regions in many cases; it then becomes necessary to omit some of the contiguities. Therefore, although several solutions have been proposed (e.g. Heilmann *et al.*, 2004; Speckmann *et al.*, 2006; van Kreveld and Speckmann, 2007), their applications are limited. The other type is non-contiguous cartograms; rectangular cartograms proposed by Upton (1991) and circle (or circular) cartograms proposed by Dorling (1996) are examples. They represent regions by rectangles and circles and omit the contiguities of regions. In particular, a circle cartogram is often used for visualization because of its simple construction algorithm.

Dorling (1996) proposed a circle cartogram construction algorithm according to two requirements for an easily comprehensible resultant: 'avoid overlap of circles' and 'keep contiguity of regions as much as possible'. It is also important to 'keep the similarity of configuration between circles on cartograms and regions on geographical maps'; accordingly, the algorithm first places circles according to the geographical configuration of regions and then moves circles one by one in order to fulfil the requirements. It outputs results which express a spatial distribution of data. However, the relative positions of circles on cartograms sometimes differ greatly from the geographical maps; the displacement of circles then causes difficulty in distinguishing which circles represent which regions.

I believe that there are two problems with the previous construction algorithm. One is that the algorithm does not consider maintaining the relative position of circles explicitly. The information on regions' contiguity includes the information on the relative position of regions; however, I think that it is not sufficient to keep the similarity of positions between circles on cartograms and regions on geographic maps. The other problem is that the previous algorithm moves circles one by one to search for a circle configuration that satisfies the requirements for circle cartogram construction. This means that the algorithm searches the local minimum by changing the *x*- and *y*-coordinates of one circle and repeats this procedure until all of the requirements are satisfied. This step-by-step procedure outputs results which are dependent on the order of the circles' movement and may output a local minimum which is far from the initial data. To find a local minimum near the initial data, it is better to use a solution that considers all requirements together and outputs the positions of all circles simultaneously.

In this paper, I propose a new construction method for circle cartograms that attaches a high value to the preservation of relative positions of circles while also considering the requirements proposed by Dorling (1996). I formulated a circle cartogram construction problem with non-linear minimization and inequality constraint conditions. I then confirmed the applicability of the proposed formulation using the world population dataset.

2. APPROACH TO CIRCLE CARTOGRAM CONSTRUCTION

2.1 Requirements for resultant cartogram shapes

Here, I confirm the requirements for circle cartogram construction. The following are considered in the algorithm for circle cartogram construction as proposed by Dorling (1996), although the first requirement is not made explicit.

1) Maintain similarity between the position of circles on cartograms and the position of regions on geographical maps.
2) Place the circles on top of other circles that share their border on the geographical map if possible.
3) Avoid overlapping of circles.

I agree that fulfilling these requirements enhances the readability of cartograms. When people read circle cartograms, they note the differences in the size and position of circles by comparing those regions on geographical maps and recognize the characteristics of the spatial distribution for the represented data on cartograms. Large relocations of circles make data interpretation difficult; therefore, it is important to retain the circle alignment in cartograms. Moreover, avoiding any overlapping of circles is important as overlaps hinder the recognition of circle sizes.

I consider these requirements to be essential for constructing visually elegant circle cartograms, and I propose solutions that satisfy these requirements.

2.2 An approach to circle cartogram construction

Suppose that the data to be expressed on a circle cartogram are given to every region in the target domain. Let D_i denote data given to region *i*; then r_i, or the radius of circle *i*, is given by $r_i = \sqrt{D_i/\pi}$.

Now, a circle cartogram construction can be considered as a problem where the positions of the circles' centres must be determined. The distance between the centres of neighbouring circles should be the sum of the radii of those circles, and the relative positions of the circles' centres on cartograms should resemble the corresponding regions on the geographical maps.

In fact, this description of circle cartogram construction is quite similar to that of distance cartogram construction. Distance cartograms are diagrams that visualize the proximity indices between points in a network. Its construction problem is to determine the location of points on cartograms according to the given proximity indices between points.

Shimizu and Inoue (2009) formulated the distance cartogram construction problem as follows. Let p_{ij} denote the given proximity index between points *i* and *j*; d_{ij} denote the distance on a cartogram between points *i* and *j*; (x_i, y_i) denote the *x*- and *y*-coordinates of point *i* on the coordinate system of the cartogram, respectively; (x^G_i, y^G_i) denote the *x*- and *y*-coordinates of point *i* on the geographic coordinate system, respectively; *L* denote the set of point pairs that have the proximity data; θ^G_{ij} denote the bearing of the link that connects points *i* and *j* on the geographical map; and θ_{ij} denote the bearing of the link that connects points *i* and *j* on the cartogram. The distance cartogram construction is then formulated as a least-squares problem:

$$\min_{x,y} \sum_{(i,j)\in L} \left\{ \alpha \left(\frac{d_{ij}}{p_{ij}} - 1 \right)^2 + (1-\alpha)\left(\theta_{ij} - \theta^G_{ij}\right)^2 \right\} \tag{1}$$

where $d_{ij} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2}$,

$$\theta_{ij} = \tan^{-1}\frac{y_j - y_i}{x_j - x_i},\ \theta^G_{ij} = \tan^{-1}\frac{y^G_j - y^G_i}{x^G_j - x^G_i},$$

$$0 < \alpha < 1.$$

Equation (1) consists of two objective functions: one minimizes the difference between the given proximity data and point distances on cartograms, and the other minimizes the difference between the direction of links that connect the points on cartograms and geographical maps. The weight parameter *α* is adapted to set the balance between two objective functions. Shimizu and Inoue (2009) showed that distance cartograms can be constructed by solving equation (1).

The difference between circle and distance cartogram constructions is that circle cartogram construction requires the avoidance of circle overlapping. Thus, I propose to add constraint conditions to the formulation of distance cartogram construction in this paper.

3. FORMULATION OF CIRCLE CARTOGRAM CONSTRUCTIONS

3.1 Formulation of circle cartogram constructions

Here, I formulate the circle cartogram construction. Let circle i on a cartogram represent region i on a geographical map; (x_i, y_i), the x- and y-coordinates of the centre of circle i on the coordinate system of the cartogram, respectively; ($x^G{}_i$, $y^G{}_i$), the x- and y-coordinates of the centroid of region i on the geographic coordinate system, respectively; r_i, the radius of circle i; d_{ij}, the distance between centres of circles i and j; C, the set of pairs of regions that share borders; $\theta^G{}_{ij}$, the bearing of the link that connects the centroids of regions i and j on the geographical map; and θ_{ij}, the bearing of the link that connects centres of circles i and j on the cartogram. Figure 1 shows the definitions of the notations.

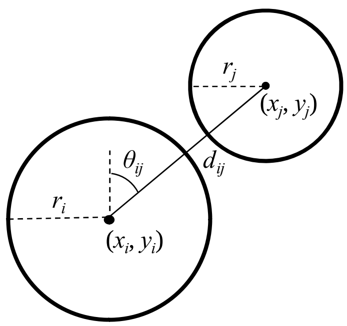


Figure 1. Neighbouring circles on circle cartogram.

The formulation for circle cartogram construction is as follows:

$$\min_{x,y} \sum_{(i,j)\in C} \left\{ \alpha \left(\frac{d_{ij}}{r_i + r_j} - 1 \right)^2 + (1-\alpha)\left(\theta_{ij} - \theta_{ij}^G\right)^2 \right\} \quad (2)$$

subject to $d_{mn} \geq r_m + r_n \quad \forall(m,n) \quad m \neq n$

where $d_{ij} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2}$,

$\theta_{ij} = \tan^{-1}\frac{y_j - y_i}{x_j - x_i}, \; \theta_{ij}^G = \tan^{-1}\frac{y_j^G - y_i^G}{x_j^G - x_i^G}$,

$0 < \alpha < 1$.

The formulation for circle cartogram construction (equation (2)) is quite similar to that for distance cartogram construction (equation (1)). Equation (2) is also composed of two objective functions as well as one constraint condition. The first term of the objective function involves keeping the circles which represent neighbouring regions closer to be contiguous on the cartogram; the second term of the objective function involves maintaining similarity between relative positions of circles on the cartogram and regions on the geographical map; α is the weight parameter for these two objective functions. The constraint conditions between all pairs of circles are set so as to avoid overlapping of circles.

3.2 Expression of formulation by *x*- and *y*-coordinates

The unknown variables of equation (2) are the coordinates for the centre of circles x_i and y_i since the radii of circles, r_i, and geographic coordinates of the centroid of regions, $x^G{}_i$ and $y^G{}_i$, are given. Equation (2) can then be rewritten as follows:

$$\min_{x,y} \sum_{(i,j)\in C} \left\{ \alpha \left(\frac{\sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2}}{r_i + r_j} - 1 \right)^2 + (1-\alpha)\left(\tan^{-1}\frac{y_j - y_i}{x_j - x_i} - \tan^{-1}\frac{y_j^G - y_i^G}{x_j^G - x_i^G} \right)^2 \right\} \quad (3)$$

subject to $\sqrt{\left(x_n - x_m\right)^2 + \left(y_n - y_m\right)^2} \geq r_m + r_n \quad \forall(m,n) \quad m \neq n$

where $0 < \alpha < 1$.

To simplify the second term of the objective function, the inverse function of the tangent is removed.

$$\begin{aligned} &\min \sum_{(i,j)\in C} \left(\tan^{-1}\frac{y_j - y_i}{x_j - x_i} - \tan^{-1}\frac{y_j^G - y_i^G}{x_j^G - x_i^G} \right)^2 \\ &\approx \min \sum_{(i,j)\in C} \left(\frac{y_j - y_i}{x_j - x_i} - \frac{y_j^G - y_i^G}{x_j^G - x_i^G} \right)^2 \end{aligned} \quad (4)$$

To avoid the denominator becoming zero, equation (4) is transformed as follows:

$$\begin{aligned} &\min \sum_{(i,j)\in C} \left(\frac{y_j - y_i}{x_j - x_i} - \frac{y_j^G - y_i^G}{x_j^G - x_i^G} \right)^2 \\ &\approx \min \sum_{(i,j)\in C} \left\{ \frac{\left(y_j - y_i\right)\left(x_j^G - x_i^G\right) - \left(x_j - x_i\right)\left(y_j^G - y_i^G\right)}{d_{ij} d_{ij}^G} \right\}^2 \end{aligned} \quad (5)$$

Finally, the circle cartogram construction problem is formulated as shown in equation (6):

$$\min_{x,y} \sum_{(i,j)\in C} \left[\alpha \left(\frac{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}}{r_i + r_j} - 1 \right)^2 + (1-\alpha) \left\{ \frac{(y_j - y_i)(x_j^G - x_i^G) - (x_j - x_i)(y_j^G - y_i^G)}{\sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}\sqrt{(x_j^G - x_i^G)^2 + (y_j^G - y_i^G)^2}} \right\}^2 \right] \quad (6)$$

subject to $\sqrt{(x_n - x_m)^2 + (y_n - y_m)^2} \geq r_m + r_n \quad \forall (m,n) \quad m \neq n$

where $0 < \alpha < 1$.

Now it is possible to construct circle cartograms by solving equation (6), a non-linear minimization problem with inequality constraint conditions. In the following section, I check the applicability of the proposed formulation.

4. APPLICATION

4.1 Investigation of applicability of proposed formulation

I applied the proposed formulation to data from the World Population Prospects by the United Nations Population Division. I solved the problems through a trust-region interior-point method using the mathematical programming software NUOPT.

NUOPT is a product developed by the software company Mathematical Systems, Inc., of Japan. It can be used to solve complex mathematical optimization problems by setting initial values to the unknown variables (x- and y-coordinates of the centres of circles) and describing the objective functions and constraint conditions.

As described previously, getting the configuration of circles on the cartogram to be similar to that of regions on the geographical map is preferable; the initial values should then be set according to the geographic coordinates. Additionally, setting the initial values to satisfy all the constraint conditions is important for making the calculations easier; however, it is also important to set the initial values close to the resultant coordinates to make the change in coordinate values, i.e., the change in positions of circles, smaller.

Considering the above conditions, I set the initial values of x- and y-coordinates for the centre of circles to touch at least one pair of circles and avoid overlaps of any other pairs of circles.

There is one additional issue to consider. The construction of circle cartograms by equation (6) requires that the set of pairs of contiguous regions C should compose one graph. However, the world country data do not satisfy this condition, as there are separate continents and island countries. I thus added dummy region data to connect all circles in one graph.

Figure 2 is the output figure of the world population circle cartograms in 2010 obtained when solving for the proposed formulation. The weight parameter α for this calculation was 0.7. The coloured circles represent countries, and the abbreviated three-letter codes indicate the country names. The white circles represent the dummy circles. The sizes of the dummy circles were decided through a trial-and-error process to achieve comprehensible outputs.

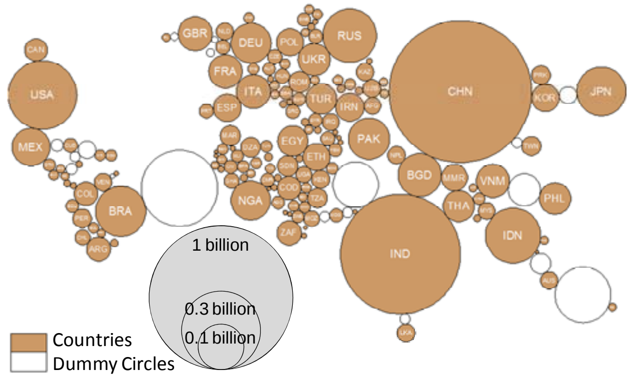
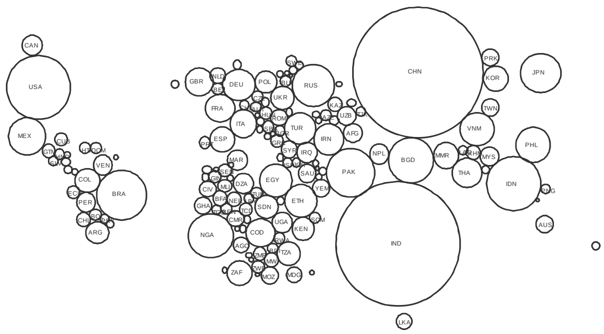


Figure 2. Resultant circle cartogram of the world population in 2010 by proposed formulation (α = 0.7).

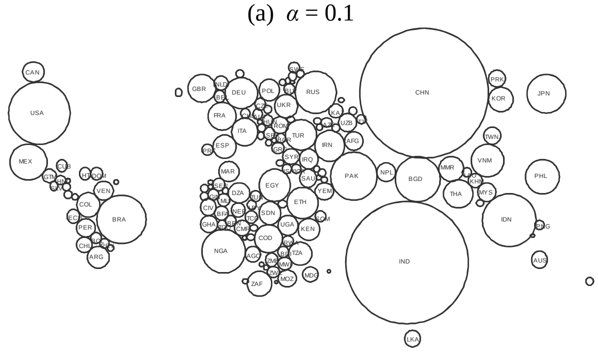
It was confirmed that the formulation developed in this study can be used to construct a circle cartogram. Since the configuration of circles on the cartogram seems quite similar to that of regions on a geographical map, it is easy to sense which circles represent which regions. The population distribution in the world is well presented in this figure without any overlaps of circles; the readers of Figure (2) can easily see that the populations of the Asian countries dominate the 2010 world population in 2010.

4.2 Evaluation of weight parameter settings

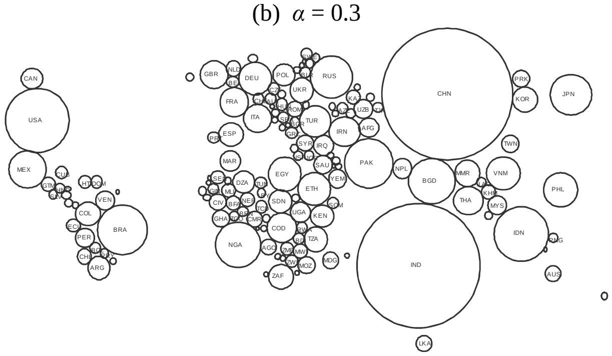
Figure 2 shows the calculation results when the weight parameter α is equal to 0.7. Without discussing and evaluating the meaning of setting α, it would be difficult to adjust its value. In this section, I clarify the meaning of α and explain the change in figures when it is adjusted by using the 2010 world population data.



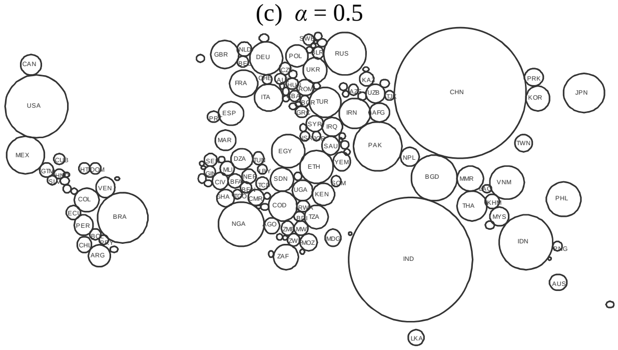
(a) $\alpha = 0.1$



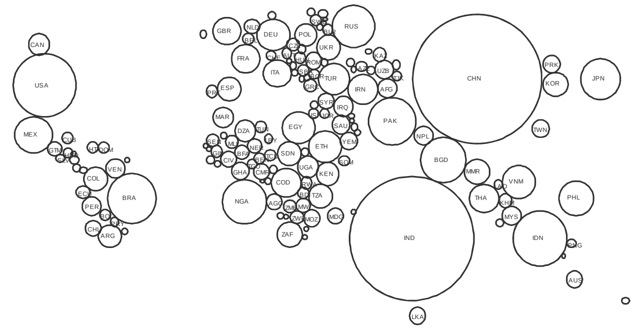
(b) $\alpha = 0.3$



(c) $\alpha = 0.5$



(d) $\alpha = 0.7$



(e) $\alpha = 0.9$

Figure 3. Differences in the resultant circle cartograms for 2010 world population by adjusting α

As shown in equation (2), when α is close to zero, the representation of contiguity information is weighted; when α is close to unity, the similarity to the geographic configuration is weighted. As a result, a smaller α should place circles closer together to express the contiguity of regions, although the deformation of circle positions becomes larger; in contrast, a larger α should place circles according to the geographic configuration, although some pairs of circles with contiguity information are placed separately.

I then checked the differences in the resultant cartograms by adjusting α. Figure 3 shows the calculation results with α equal to 0.1, 0.3, 0.5, 0.7, and 0.9. There do not seem to be large differences between these figures, but the figures calculated with larger α have larger gaps between circles and keep the similarity to the geographic configuration of regions.

I evaluated the difference in shapes using the two indices: the percentage of represented contiguity information on cartograms, and the residual sum of squares for x- and y-coordinates of the affine transformation from the cartogram coordinates to geographic coordinates. The former index corresponds to the first term of the objective functions; it should be large when α is small. The latter index corresponds to the second term of the objective functions. The affine transformation is composed of rotation, scaling, or shear, and if the two configurations of points are matched by the affine transformation, the similarity is high; the disparity between the geographic and affine-transformed cartogram coordinates is the index of similarity of configuration. The residual sum of squares for the affine transformation should be large when α is small since the smaller residuals express higher similarity.

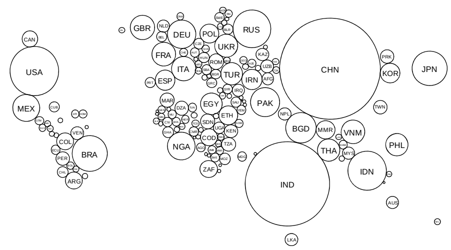
Table 1 shows the calculation results of two indices. It confirms that contiguity is well represented when α is small and that the similarity of configuration is preserved when α is large. These two favourable characteristics for circle cartograms are the trade-off; however, users who construct circle cartograms by this formulation can easily adjust α since its meaning is clear.

Table 1. Tests for representation of regions' contiguity and similarity of configuration.

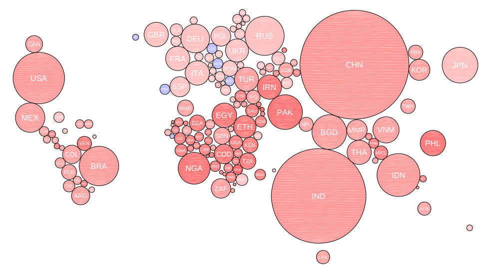
α	Percentages of represented contiguity	Residual sum of squares of affine transformation
0.1	**69.7**	89.3
0.3	67.7	37.6
0.5	59.6	30.0
0.7	54.6	21.6
0.9	53.6	***19.3***

4.3 Visualization of changes in world population distribution

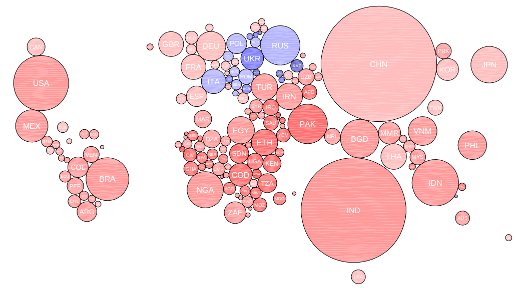
I show some examples using the proposed formulation. Figures 4 and 5 show visualization of the change in world population distribution from 1980 to 2050. The data is from the World Population Prospects by the United Nations Population Division, which contains the country population data for every five years from 1980 to 2050. Figures 4 and 5 show the population for every 10 years; the colour in the circles represents the annual population growth of the last five years for each country.



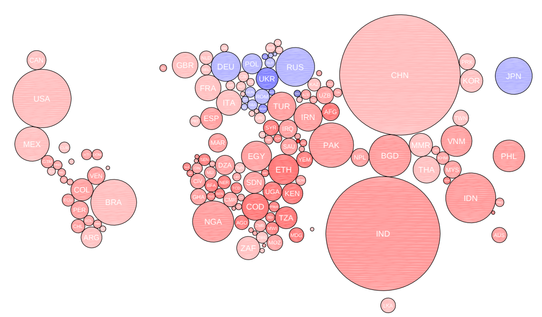
(a) 1980



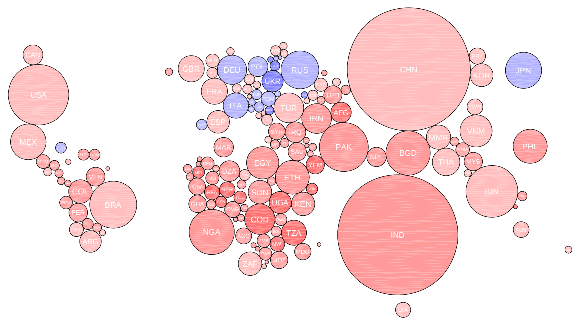
(b) 1990



(c) 2000



(d) 2010



(e) 2020

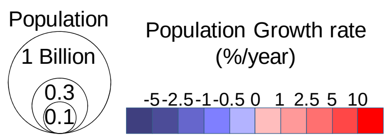
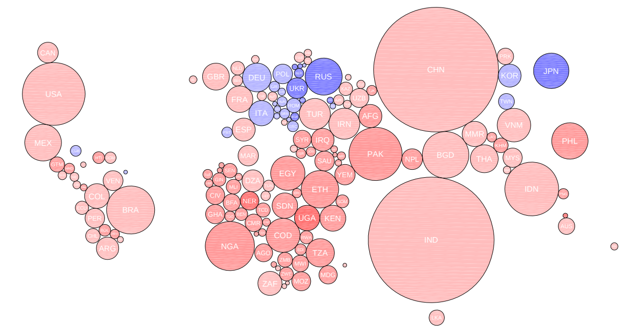


Figure 4. World population distribution from 1980 to 2020.

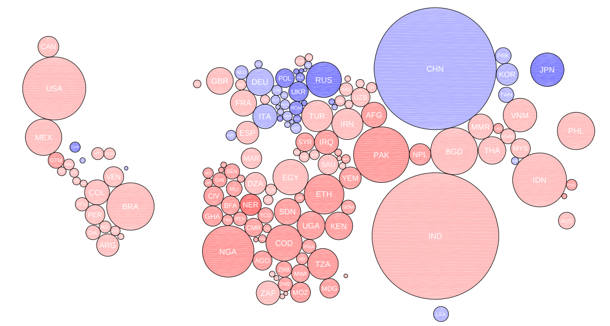
When comparing cartograms of different years, cartogram readers first note the difference in figures and then realize the change in data presented. To make the comparison of figures easier, I used the coordinate values of circle centres on the previous year's cartograms as the initial values for constructing the next year's cartogram.

First, circle cartograms were confirmed to be useful for visualizing the distribution changes of spatial data. By using the shape of previous circle cartograms as initial data for the construction of the next circle cartograms, it is possible to output similarly shaped cartograms; consequently, the change in data can be clearly visualized.

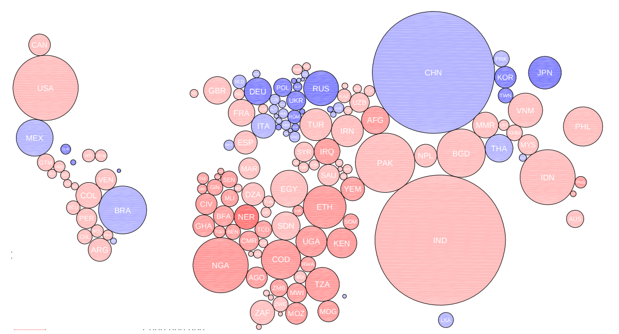
Second, Figures 4 and 5 clearly show that the total world population is increasing rapidly, particularly in Asian and African countries. The colours in the circles clearly show the change in population; however, even if the colour is not shown on these cartograms, visualizing the change in population is still possible.



(a) 2030



(b) 2040



(c) 2050

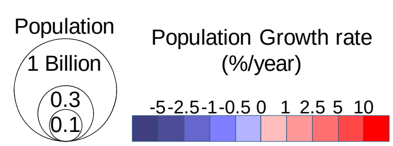


Figure 5. World population distribution from 2030 to 2050.

5. CONCLUSIONS

In this paper, I have proposed solutions for circle cartogram construction by formulating these problems as constrained non-linear minimization problems. The application of the proposed solution revealed that the solution for circle cartogram construction can output cartograms without overlapping circles. A comparison between the outputs of the proposed solutions and that of the previous method with the evaluation of outputs is left for future work.

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