TRANSFORMATION OF SPATIAL REPRESENTATION IN SCALE DIMENSION:
A NEW PARADIGM FOR DIGITAL GENERALIZATION OF SPATIAL DATA

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ABSTRACT

Generalization is a fundamental functionality in a geographical information system (GIS). It has recently become a major international research theme in cartography and GIS.

This paper describes a *scale-driven paradigm* for the generalization process. In this paradigm, scale is considered as the only factor which directly drives the transformation of spatial representation from a larger scale to a smaller scale. It is illustrated that such a transformation follows a *natural principle* and that this *natural principle* can be best depicted by the operators developed in *mathematical morphology*, which is a science dealing with shape, form and structure of spatial objects.

In this paper, the concept of *scale dimension* is introduced and generalization is considered as the transformation of spatial representation in *scale dimension*. Such a transformation simplifies the shape, form and structure of spatial data so as to bring the spatial representation from a larger scale to a smaller scale. This transformation is an objective process. The subjective aspects of generalization may be dealt with using rule-based systems. Rules can be applied before, during and after this scale-driven transformation. This paradigm allows the seemingly subjective and complex process of generalization to be greatly simplified so that a mathematical basis may be laid down.

1. INTRODUCTION

Spatial data (including map data) are usually associated with scales. At large scales, detailed information about spatial variations of a given area can be represented. If this representation is to be made at a smaller scale, then graphic space is reduced. Thus, not the same amount of detailed information can be represented due to the requirements for the clarity of graphic symbols. In this case, the contents of large scale spatial data need to be modified to suit the smaller space available on smaller scale representations, i.e. some needs to be omitted, some simplified, some displaced, some exaggerated, and so on. This modification process is referred to as generalization. In the context of this paper, generalization is considered as being a process of transforming spatial representation from a larger scale to a smaller scale.

Generalization is a vital function in spatial data handling, e.g. for geographical modelling, for efficient derivation and updating of small-scale maps and spatial databases from large scale sources, and for real-time visualisation and analysis of spatial data in a GIS.

Indeed, generalization is so important and difficult a topic that it has nowadays become a major international research theme in cartography and GIS. Over the last decade, many projects have been initiated internationally, in Canada, China, Britain, France, Germany, the Netherlands, Sweden, Switzerland, and the USA.

In the last three decades, a few conceptual frameworks have been developed by researchers (e.g. Brassel and Weibel, 1988; McMaster and Monmonier, 1989), based on which, a number of generalization operations have been identified. However, most of these operations remain at a conceptual level. In other words, there is a lack of mathematical models or algorithms to transform spatial representation from a larger scale to a smaller scale.

This paper aims to offer a new paradigm for digital generalization of spatial data. It is a scale-driven paradigm. It considers that

(a) generalization is a process of transforming spatial representation in scale dimension;
(b) this transformation process follows a natural principle, and;
(c) this natural principle can be best depicted by operators developed in mathematical morphology.

Based on this new paradigm, a mathematical (or an algebraic) basis could then be established for digital generalization of spatial data.

This introduction is followed by a scale-driven framework. In this section, the motivations of generalization are classified and scale is considered as the only direct factor which drives this transformation. In Section 3, the concept of scale dimension is introduced and the transformation in scale dimension is illustrated. Section 4 demonstrates that the transformation in scale dimension follows the natural principle for objective generalization proposed by Li and Openshaw (1993). Section

5 illustrates that the natural principle can be best depicted by the operators developed in mathematical morphology and thus the transformation in scale dimension can be best realised using morphological techniques. In Section 6, some examples are given, illustrating how morphological operators can be used for transforming spatial representation from a larger scale to a smaller scale.

2. A SCALE-DRIVEN FRAMEWORK

In order to understand the nature of generalization, it seems necessary to compare digital generalization with traditional manual generalization so that an insight into the matter may be gained.

2.1 The factor directly driving generalization: Scale

To discuss the problems with digital generalization, it seems pertinent to start with a discussion of the motivation behind generalization. Many researchers have spent efforts on this topic and identified some sets of requirements or controls have as follows:

Müller (1991) considers that generalization is promoted by four main requirements; i.e. economic requirements; data robustness requirements; multipurpose requirements; and display and communication requirements. Robinson et al (1995) has identified another four elements (but called controls), i.e. map purpose and condition of use; scale, graphic limits and quality of data. Keates (1989) has also identified 4 elements, i.e. scale and graphic requirements (legibility) and, characteristics and importance. In a more detailed manner, McMaster and Shea (1992) identified three sets of "philosophical objectives" as follows: (a) Theoretical elements: reducing complexity; maintaining spatial accuracy; maintaining attribute accuracy; maintaining a logical hierarchy; and consistently applying generalization rules; (b) Application-specific elements: map purpose and intended audience; appropriateness of scale; and retention of clarity and (c) Computational elements: cost effective algorithms; maximum data reduction; and minimum memory/disk requirement.

This is by no means an exhaustive list. It seems to the author that some kind of "generalization" (or abstraction) needs to be applied to these sets of motivation so that the problem can be simplified and useful models established. This kind of simplification is vital in scientific research. The classic example of such a simplification is the Earth being simplified by Newton as a point so that the *Law of Gravitation* could be established.

To do this, some analysis needs to be carried out. Let's take the "quality of data" as an example. The question arising is "how does this factor affect generalization?" Suppose that a set of data is for producing 1:10,000 scale map, if the quality of the data is too poor to meet the accuracy requirement for this scale, then one needs to map it at a smaller scale. Here comes out the scale of map in between "data quality" (the reason) and "generalization" (the consequence). Through applying a similar analysis to other factors, it can be observed that scale is the only factor directly driving the generalization process while others can be considered as either indirect factors or posterior factors. Indeed, the Swiss Society of Cartography has long ago made it clear in its cartographic manual that generalization is motivated only by a reduction of scale, as cited by Müller (1991). Fig.1 shows such a relationship between various motivations and the consequence.

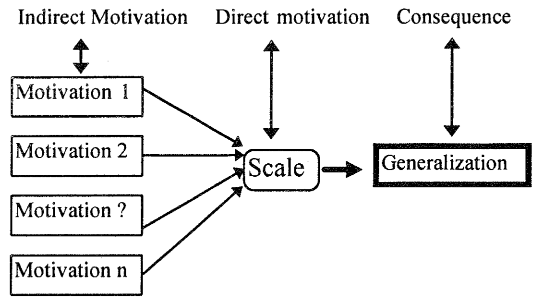


Fig.1 Scale is the only direct motivation for generalization

2.2 A scale-driven framework

Now comes the question: "When can you consider cartographic and other requirements?". To answer this question, a discussion of the difference between traditional manual generalization and digital generalization needs to be conducted.

In manual generalization, both the simplification of the shape, form and structure of map features and the consideration of graphic legibility are considered simultaneously. This makes the process appear to be very subjective. In fact, this subjectivity is mainly caused by the consideration of the "characteristics and importance" of features as pointed out by Keates (1989). On the other hand, in a digital environment, data resolution could be infinitely high, theoretically speaking. For example, two lines with a spacing much less than 0.01 mm is still separable in digital database. Therefore, graphic legibility is not an issue for digital data itself. If the spatial data is only for analytical analysis, no graphics needs to be considered. Indeed, only when a graphic presentation is considered, then comes the problem of graphic legibility, resulting in exaggeration, displacement and other complex operations. As a result of this reasoning, the relationship between traditional and digital generalization can be expressed by Fig.2.

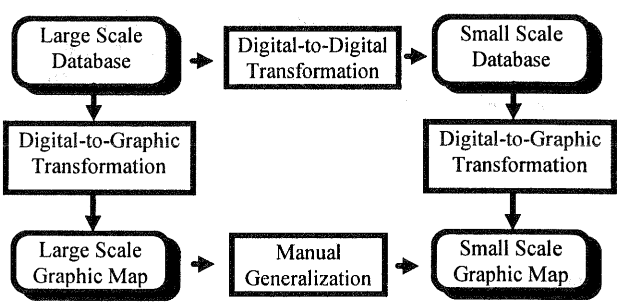


Fig.2 Relationship between digital and manual map generalization

The digital-to-digital transformation is driven by scale. Such a process will simplify the shape, form and structure of spatial representation and should be very objective so that unique solution can be achieved, given the same conditions. As will be discussed in Section 3, such a transformation can be considered as transformation in scale dimension and it follows a natural principle.

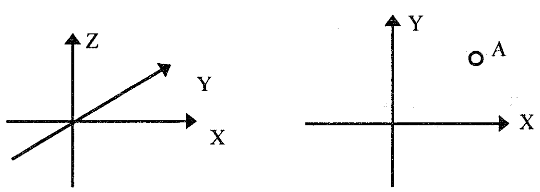
It can be noted here that the digital-to-digital transformation is the only step required if no graphic presentation is concerned. However, if graphics is considered, one needs to take into account the geographical requirements, multi-purpose requirements and cartographic requirements. It is now clear that cartographic requirements should be considered in the digital-to-graphic transformation after the scale-driven digital-to-digital transformation. Of course, one can also use some of the cartographic requirements as constraints for the digital-to-digital transformation. Some of the multi-purpose and geographical requirements may also be used as constraints for this scale-driven transformation and for selecting data layers for generalization.

3. TRANSFORMATION OF SPATIAL REPRESENTATION IN SCALE DIMENSION

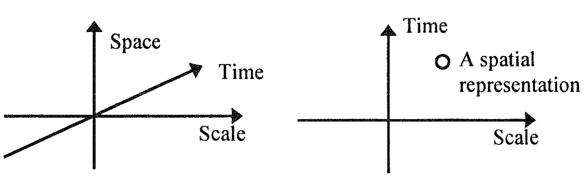
There are many mathematical models available for the transformation of spatial objects, such as conformal, affine, projective, etc. After such a transformation, the shape, size, orientation and even the topology of an object can be altered. However, these are transformations in space dimension. What will be discussed in the next two sub-sections are about the transformation in scale dimension, a concept introduced by Li (1994a).

3.1 The concept of scale dimension

It has been noted by researchers that what is supposed to be a reality is dependent on scale and time. After many illustrations, Li (1994a) introduced the concept of scale-dimension and time-scale systems, which can be illustrated in Fig.3.



(a) In 3-D space, a point represented in a 2-D plan by orthogonal projection;



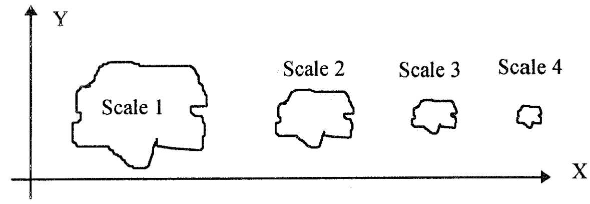
(b) In new 3-D system, a point in time-scale plan is a representation of spatial variations

Fig.3 A new 3-dimensional system

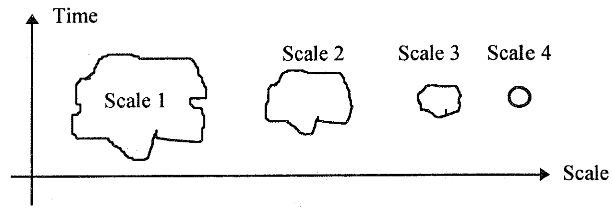
Just as a point of 3-D space can be represented in the X-Y systems, a spatial object in the new 3-D system can also be represented in the time-scale systems. In other words, a spatial representation is a record of spatial variations in the time-scale systems.

3.2 Transformation in scale dimension

Now comes the question: "why do we need to introduce the concept of scale dimension?" or "Is there any difference between scale and scale dimension". Fig.4 illustrates some examples to show the difference between scale and scale dimension.



(a) Simple scale reduction in space: complexity not reduced



(b) Transformed in scale dimension: complexity reduced

Fig.4 Difference between simple scale reduction and transformation in scale dimension

It can be noted that by scale reduction in space dimension it is meant a simple reduction in size. In this case, the complexity of spatial representation is not reduced. On the opposite side, by transformation in scale dimension it is meant that the representation is simplified to suit the representation at another (smaller) scale. It might be better to call the term scale in space dimension *size*.

The transformation in scale dimension is a transformation in time-scale systems when the time is fixed. The transformation of spatial representation in time dimension is a transformation in the same systems when scale fixed. This transformation is called temporal modelling and lies outside the scope of this article.

4. THEORETICAL BASIS FOR TRANSFORMATION IN SCALE DIMENSION: THE NATURAL PRINCIPLE

After the introducing the concept *transformation in scale dimension*, it is the time to examine the theoretical basis for such a transformation.

4.1 The natural phenomena

In order to understand the underlining problem better, some practical examples are desirable to illustrate such a transformation implied in natural phenomena. Li and Openshaw (1993) have used the Earth being viewed from various distance as an example. When a person is nearer an object, s/he can see more detail. When one gets further away from the object, less detailed information can be seen but the main characteristics of the object can be better observed, thus better overview being gained. Surveyors all have such experiences: When one stands somewhere near the peak of a hill, s/he will have difficulty in identifying the highest point. However, the observer who stands some distance away from the point can sees the peak clearly. Also when one views the terrain surface from an airplane, small details disappear and the main characteristics of the terrain variations become very clear. It is a commonplace to photogrammetrists that the stereo-models formed from high altitude photography are more generalised than those formed from low altitude photography. If one views the terrain surface from an satellite, then terrain surfaces becomes very smooth. These phenomena can easily checked by forming a stereo-model from a pair of satellite images such as SPOT images or Spacelab Metric Camera photography. These are just some out of many practical examples illustrating the transformation in scale dimension, which follows a natural principle.

4.2 The natural principle

The next question arising is "how these transformations are achieved?". In the case of human observation, it is due to the limitation of eyes' resolution. That is, all information within the limitation of human resolution disappears. In the case of stereo-models formed from images, it is due to the resolution of images. That is, all information within the image resolution (e.g. 10m per pixel in the case of SPOT images) disappears. These examples underline a universal principle, a natural principle as called by Li and Openshaw (1993), which states as follows:

> "*for a given scale of interest, all details about the spatial variations of geographic objects beyond certain limitation are unable to be represented and can thus be neglected*".

In other words, by neglecting all information about spatial variations within a given critical size (or limitation), the transformation in scale dimension which is similar to the generalization of natural phenomena can be achieved. Fig.5 illustrates how it works.

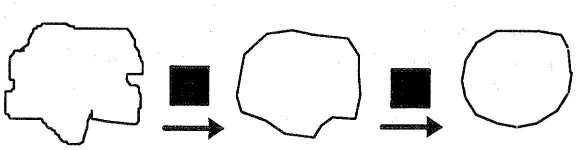


Fig.5 By neglecting the detailed spatial variations within the black square, the shape of the polygon is simplified..

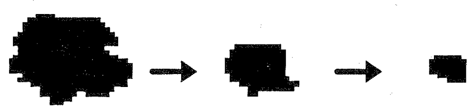
More detailed discussion of this natural principle and more practical illustrations can be found in the original paper by Li and Openshaw (1993).

5. MATHEMATICAL BASIS FOR TRANSFORMATION IN SCALE DIMENSION: MORPHOLOGICAL OPERATORS

After the introduction of this natural principle, it is the time to examine how this principle can be realized mathematically.

5.1 Examples illustrating the mathematical basis

As has been discussed previously, the shapes and structures of spatial objects are simplified when a transformation in scale dimension is applied and such a transformation follows the natural principle. Fig.6 and Fig.7 show examples which illustrate how the shape of objects can be manipulated using morphological operators in a way similar to the generalization by the natural principle. In Fig.6, a process called erosion is used. The natural principle can be best depicted by this process. The size of the structuring element (see discussion later) used in this process can mimic the critical size (within which all spatial variations can be neglected) in the natural principle However, this process does not work well in the case when there are deep channels. In this case, a process called closing should be proceeded. Fig.7 shows how such a combination works.



(a) Original image (b) Shape simplified; (c) Further simplified
Fig.6 Shape simplified by erosion process

(a) Original image; (b) Channels closed; (c) Closed image simplified
Fig.7 A combination of closing and erosion works well for even very complicated shape

5.2 The science of shape - mathematical morphology

It has been illustrated that the operators developed in mathematical morphology has great potential for depicting the digital-to-digital transformation of the generalization process. Therefore, it seems pertinent to have a more detailed discussion of mathematical morphology here.

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0
0	0	0	1	1	1	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(a) Original image A

1	①	1

(b) Structuring element B

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	+	1	1	1	+	0	0
0	0	+	1	1	1	1	+	0
0	0	+	1	1	1	+	0	0
0	0	0	+	1	+	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(c) A dilated by B ($A \oplus B$)

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	-	1	-	0	0	0
0	0	0	-	1	1	-	0	0
0	0	0	-	1	-	0	0	0
0	0	0	0	-	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

(d) A eroded by B ($A \ominus B$)

Fig. 8 Two basic morphological operators: Dilation and erosion ("+" means those becoming 1 after dilation and "-" those becoming 0 after erosion)

Mathematical morphology is a science of shape, form and structure, based on set theory. It was developed by two French geostatistical scientists -- G. Matheron and J. Serra -- in the 1960s (Matheron, 1975; Serra, 1982). It has since then found increasing application in digital image processing. The basic morphological operators are dilation and erosion. They are defined as follows (see Serra, 1982; Haralick *et al*, 1987, Li and Chen, 1991):

Dilation: $A \oplus B = \{a + b: a \in A, b \in B\} = \cup_{b \in B} A_b$ (1)

Erosion: $A \ominus B = \{a: a + b \in A, b \in B\} = \cap_{b \in B} A_b$ (2)

where A is the original image and B is called the structuring element, which can be considered to be an analogy to the kernel in a convolution operation. In Eq.(1), it is called "dilation of A by B" and in Eq.(2) "erosion of A by B".

Examples of these two operators are given in Fig.8, where the features are represented by pixels of "1"s and the origin of the structuring element is marked with a circle. The structuring element is a critical one in these operations. More discussion regarding this element will be conducted at a later stage. To show how these operators work clearly and exactly, "1" and "0" are used to represent the binary images used in this discussion. In this diagram, "+" means those becoming 1 after dilation and "-" means those becoming 0 after erosion. This convention will be used throughout this paper.

If a symmetric structuring element with origin at the centre is used for dilation, then the shape of the original image will be expanded uniformly along all directions, thus the dilation in this particular case is called expansion. Similarly, the erosion in this case is called shrink. These two special operations are illustrated in Fig.9.

```
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 1 1 1 0 0 0 0
0 0 1 1 1 1 0 0 0
0 0 1 1 1 0 0 0 0
0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
```

(a) Original image A

```
1 1 1
1 ① 1
1 1 1
```

(b) The structuring element H

```
0 0 0 0 0 0 0 0 0
0 + + + + + 0 0 0
0 + 1 1 1 + + 0 0
0 + 1 1 1 1 + 0 0
0 + 1 1 1 + + 0 0
0 + + 1 + + 0 0 0
0 0 + + + 0 0 0 0
0 0 0 0 0 0 0 0 0
```

(c) A expanded by H ($A \oplus H$)

```
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 - - - 0 0 0 0
0 0 - 1 - - 0 0 0
0 0 - - - 0 0 0 0
0 0 0 - 0 0 0 0 0
0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0
```

(d) Ashrunken by H ($A \oplus H$)

Fig.9 Special cases of dilation and erosion expansion and shrink

Another two very important operators are opening and closing. They are defined as follows:

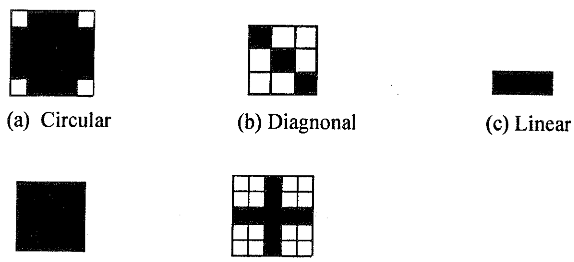
Opening: $A \circ B = (A \ominus B) \oplus B$ (3)

Closing: $A \bullet B = (A \oplus B) \ominus B$ (4)

where, A is the original image and B is called the structuring element.

Based on the two basic operators, i.e. dilation and erosion, a number of other new operators have also been developed, such as thinning, thickening, hit or miss, conditional dilation, conditional erosion, conditional thinning, conditional thickening, sequential dilation, and conditional sequential dilation, and so on. However, it is not the purpose of this paper to discuss all of them. More detailed information can be found from the book by Serra (1982).

Structuring element is the key element in a morphological operator. Structuring elements could take any shape. Fig.10 shows some of commonly used structuring elements. Indeed, it is through the proper manipulation of structuring elements that the morphological operators alter the shape, form and structure of spatial objects.



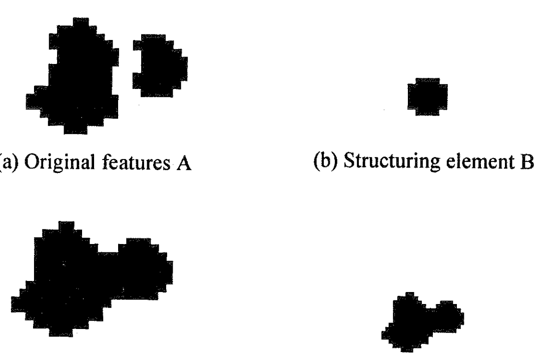
(a) Circular (b) Diagnonal (c) Linear

(d) Squared (e) Cross

Fig.10 Some possible structuring elements

6. Discussion

After the introduction of so many new concepts such as scale dimension, natural principle and morphological operators, it seems appropriate to usev an example to illustrate how morphological operators can be used to depict the transformation of spatial representation in scale dimension. Fig.11 is one of those example (Su et al, 1996). In this diagram, the size of the structuring element B is determined by the natural principle. After applying some morphological operators, the representation shown in Fig.11(a) is transformed into that shown in Fig.11(c). Features are smoothed and combined. The reduced image is shown in Fig.11(d).

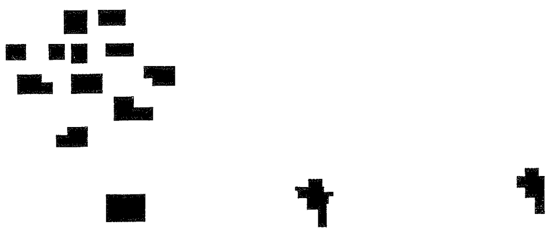


(a) Original features A (b) Structuring element B

(c) Areas combined $C = A \bullet B$ (c)Combined area reduced $C = A \bullet B$

Fig.11 Transformation of the representation of area features in scale dimension: area aggregation (Su *et al*, 1996)

Fig.12 is another example which shows the transformation of the representation of area features in scale dimension.



(a) Original feature (b) Result of 10x reduction (c) final result

Fig.12 Another example of area aggregation (Su *et al*, 1996)

7. CONCLUDING REMARKS

Digital generalization of spatial data can be decomposed into two processes, i.e. a digital-to-digital transformation and a digital-to-graphic transformation. The latter is about cartographic presentation, thus cartographic knowledge can be formalised and knowledge-based systems used at this stage. Some of multi-purpose requirements might be also be applied here.

The digital-to-digital transformation is a transformation in scale dimension and thus the process itself should be objective. It has been argued in this paper that the transformation in scale dimension is guided by a *natural principle* (Li and Openshaw, 1993) and this *natural principle* can be best depicted by the operators developed in mathematical morphology, which is a science dealing with shape, form and structure of objects. It means that, upon the two basic operators -- i.e. dilation and erosion -- in mathematical morphology, some basic mathematical models for transforming spatial representation in scale dimension can be built. These models, like affine, projective, conformal transformation *etc* in space dimension, will be of fundamental importance to generalization. If and only if these basic transformation models are developed, will one be able to develop a system which will be capable of producing consistent results.

Indeed, the development of such basic mathematical models for transformation in scale dimension based on morphological operators has been carried out by the author and his collaborators since the first study by the author (Li, 1994b) and some promising results have also been obtained (Li and Su, 1995; Su and Li, 1995; Su *et al*, 1996).

It is a commonplace that there are many routes available for travelling from Hong Kong to Vienna although some are with longer distance while others may have shorter distance. But there is only one unique shortest distance between these two cities, i.e. the geodetic line. In practice, this line may be either difficult to determine or difficult to travel along. This paper is yet another attempt to find a feasible route for digital generalization in GIS environment but certainly this route is still not the geodetic line of digital generalization. Indeed, it is the author's hope that this paper will somehow contribute to the discovery of this geodetic line.

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REFERENCES

Brassel, K. E. and Weibel, R., 1988. A review and conceptual framework of automated map generalization. *International Journal of Geographical Information Systems*, 2(3): 229-244.

Haralick, R., Sternberg, S. and Zhuang, X., 1987. Image analysis using mathematical morphology. *IEEE Transactions of Pattern Analysis and Machine Intelligence*, 9(4): 530-550.

Keates, J., 1989. *Cartographic Design and Production.* 2nd ed. Longman. 261pp.

Li, D. and Chen, X.-Y., 1991. Automated generating triangular irregular digital terrain model networks by mathematical morphology. *ISPRS Journal of Photogrammetry and Remote Sensing*, 46: 283-295.

Li, Z., 1994a. Reality in time-scale systems and cartographic representation. *The Cartographic Journal*, 31(1): 50-51.

Li, Z., 1994b. Mathematical Morphology in digital generalization of raster map data. *Cartography*, 23(1): 1-10.

Li, Z. and Openshaw, S., 1993. A natural principle for the objective generalization of digital maps. *Cartography and Geographic Information Systems*, 20(1): 19-29.

Li, Z. and Su, B., 1995. Algebraic models for feature displacement in the generalization of digital map data using morphological techniques. *Cartographica,* September Issue.

Matheron, G., 1975. *Random Sets and Integral Geometry.* Wiley, New York.

McMaster, R. and Monmonier, M., 1989. A conceptual framework for quantitative raster-mode generalization. *Proceedings GIS/LIS'89*, 390-403.

McMaster, R. B. and Shea, K. S., 1992. *Generalization in Digital Cartography.* Association of American Geographer. 134pp.

Müller, J. C., 1991. Generalization of Spatial Databases. In: *Geographical Information Systems: Principles and applications*, edited by David J. Maguire, Mechael Goodchild and David Rhind. Longman. 457-475.

Robinson, A., Morrison, J., Muehrcke, P. and Guptill, S., 1995. *Elements of Cartography*, 6th ed. John Wiley & Sons, Inc. 674pp.

Serrea, J., 1982. *Image Processing and Mathematical Morphology.* Academic Press, New York. 610pp.

Su, B. and Li, Z., 1995. An algebraic basis for digital generalization of area-patches based on morphological techniques. *The Cartographic Journal*, 32(2): 148-153.

Su, B., Li, Z., Lodwick, G. and Müller, J. C., 1996. Algebraic models for the aggregation of area features based upon morphological operators. Technical Report, Curtin University of Technology. 20pp. Also Submitted to *International Journal of Geographical Information Systems.*