A GAMMA-CONVERGENCE APPLIED TO MULTISPECTRAL IMAGE CLASSIFICATION AND RESTORATION

M.Iddir Zait [a,*], Y.Smara [b]

[a] INI National Institute of Computing Science , BP 68M Oued Smar Algiers Algeria - m_iddir@ini.dz
malika_iddir@yahoo.fr
[b] Laboratory of image processing and radiation, university of Sciences et Technologies Houari Boumedienne, BP32 El-Alia Bab-Ezzouar 16111 Algiers Algeria- y.smara@mailcity.com

PS WG VII/1

KEY WORDS: Image Classification, Image Restoration, Multispectral satellite image, Remote Sensing, Variational model.

ABSTRACT:

The main objective of this paper is to develop a model which combines in the same process image classification and restoration. Image classification consists of assigning a label to each site of an image to produce a partition into homogeneous labeled areas. The classification problem concerns many applications, like in the field of remote sensing: land use management, monitoring, urban areas.
Observed images are often affected by degradations. The purpose of restoration is to find an original image describing a real scene from the observed one. This problem can be identified by inverse problem. In general, it is ill-posed in the sense of Hadamard. The existence and uniqueness of the solution are not guaranteed. It is therefore necessary to introduce an a priori constraint on the solution. This operation is the regularization. We can distinguish two types of regularization: the linear one and the non-linear. In this paper, we develop a model proposed by C.Samson, combining classification and restoration with non linear regularization. It's based on works developed for phase transitions in fluid mechanics by Van der Walls-Cahn-Hilliard, and uses a Gamma-convergence theory. This model is named variational model, due to the fact that calculus of variations is its main tool. The classification-restoration is obtained by minimizing a sequence of functionals. The result is a classified and restored image, and corresponds to an image composed of homogeneous classes, separated by minimum length boundaries. The minimization problem is transformed by Euler-Lagrange equations into PDEs (Partial Differential Equations) resolution problem.
We have experimented this model on synthetic and satellite images. For real images, we have considered images from SPOT-1 satellite representing the regions of Blida in south-west of Algiers (capital of Algeria). We will discuss at the end of the paper the results we have obtained.

RESUME :

L'objectif principal de ce papier est développer un modèle qui combine dans un même processus une opération de classification d'image et une opération de restauration. La classification consiste partitionner une image en régions repérées par des étiquettes différentes. Le problème de classification concerne beaucoup d'applications telles que la gestion de la couverture terrestre en télédetection, le suivi de l'urbanisation etc…
Les images observées sont souvent dégradées. Le but de la restauration est de retrouver l'image originale à partir de celle observée. Ce problème est un problème inverse mal posé au sens d'Hadamard. L'existence et l'unicité de la solution ne sont pas assurées. Il est alors nécessaire de régulariser la solution par l'introduction d'un a priori. Nous pouvons distinguer deux types de régularisation: linéaire et non-linéaire. Dans ce papier, nous développons un modèle variationnel, proposé par C.Samson, qui combine classification et restauration avec une régularisation non linéaire. Il est basé sur les travaux de Van der walls Cahn-Hilliard développés pour les transitions de phase en mécanique des fluides, et utilise la théorie de la Gamma Convergence. La classification restauration est obtenue en minimisant une séquence de fonctionnelles. Le résultat correspond à une image composée de classes homogènes séparées par des interfaces de longueur minimales. Le problème de minimisation est transformé par les équations d'Euler-Lagrange en un problème de résolution d'équations aux dérivées partielles (EDP).
Nous avons testé ce modèle sur des images de synthèse et sur des images satellitaires de la série SPOT-1 recouvrant la région de Blida dans sud est d'Alger (capital d'Algérie). Nous présenterons à la fin du papier les résultats obtenus.

1. INTRODUCTION

The remote sensing is a multidisciplinary science that knows actually a real flight, with the use of sensors embarked more and more sensitive and more and more varied.

For the extraction of the thematic information of the remote sensed images, the classification proves to be an inescapable tool. It consists in achieving a partition of the image into labeled regions. We can distinguish two types of classification: the supervised classification for which the number of classes and their parameters are known beforehand, and the non

supervised classification that doesn't require any knowledge on the classes.
Many classification models can be found in the field of stochastic approaches (discrete models) with the use of Markov Random Field (MRF) theory (Pony and al., 2000). Structural approaches as splitting, merging and region growing have also been developed. Works on the classification by variational models (continuous models) have been conducted lately, mainly because the notion of classes has a discrete nature.
The variational approaches are always associated with resolution of partial differential equations (PDEs), and have for interest, that they allow to get in many cases the results of existence and uniqueness of the solution. They can be implemented by powerful numeric methods (Deriche, R. and Faugeras, O., 1995).
It often happens that acquired images have less than desirable quality due to various imperfections and/or physical limitations in the image formation and transmission processes. The acquired image may look blurry due to the motion of camera for example or atmospheric turbulence. Noise may be introduced owing to measurement errors, quantization, etc. The aim of restoration is to find the original image from the observed one. This problem can be identified by inverse problem.
In this paper, we present a model proposed by C.Samson that combine in the same process, image classification and image restoration (Aubert, G. and Kornprobst, P., 2002). This deterministic model is based on variational calculus and resolution of partial differential equations (PDEs). It is inspired from Van Der Walls-Cahn-Hilliard works on phase transition in mechanic, and uses Gamma-convergence theory. The classification-restoration is achieved by minimization of a sequence of functional that contains at least one term for classification and other one for restoration.
We suppose that discriminant feature between classes is the spatial distribution of intensity. Of course, other discriminant features like the texture can be used. We also assume that the distribution of intensity is Gaussian for each class. Under these assumptions, classes can be characterized by their means and its standard deviations.

2. IMAGE RESTORATION

Generally, image degradation can be modeled by a linear and translation invariant blur and additive noise. The equation relating observed image *I* and original one *f* can be written as:

$$I = Kf + n \tag{1}$$

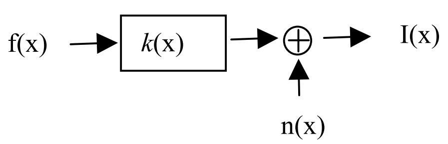

Figure 1. Linear image degradation model

Where *K* is a convolution operator with the impulse response of the system *k* ($K\, I = k * I$), and *n* is an additive white noise. In practice, the noise can be considered as Gaussian. The restoration consists of recovering the original image f from the observed one I. One simple method consists in minimizing the half quadratic error given by equation 2:

$$J_1(f, I) = \int_\Omega \left(Kf(x) - I(x)\right)^2 dx \tag{2}$$

Many restoration methods are performed under the condition that the blur operator is known. Unfortunately, the true image must be identified directly from the degraded image by using partial or no information about the blurring process and the true image. Such estimation problem is called blind deconvolution, and consists of finding alternately an estimate to the original image and the impulse response.
The problem of recovering an image that has been blurred and corrupted with additive noise is an inverse problem and is always ill-posed in the sense of Hadamard. The existence and uniqueness of the solution are not guaranteed. It is therefore necessary to introduce an a priori constraint on the solution. This operation is the regularization. We can distinguish two types of regularization: the linear one and the non-linear. The regularized solution is computed by minimizing the functional:

$$J\ (f,I) = \int_\Omega \left(Hf(x) - I(x)\right)^2 dx + \lambda^2 J_{regul} \tag{3}$$

Where λ is a real parameter.
The most important linear regularization is the Tikhonov one, (Aubert, G. and Kornprobst, P., 2002) described by equation 4:

$$J_{regul} = \int |\nabla f|^2 \tag{4}$$

This regularization leads to a solution without edge preserving. To overcome this problem, the non linear regularization is used. On the homogeneous regions that correspond to weak gradient, an important smoothing is done. On the contours that correspond to strong gradient smoothing is very weak. So the noise in the image can be minimized while preserving the contours of the objects. Among the non linear methods that have been proposed, the most successful ones are the total variation (TV) restoration (Bertalmio, M., and al., 2003) (Rudin, L., and Osher, S., 1994), (Vogel, C.R., and Oman, M. E., 1996) and the regularization with a φ function (Samson, C., and al, 2000). For our implementation, we have used the φ function regularization, and we have assumed that the image is not blurred. In this case, the equation 3 can be written as:

$$J\ (f,I) = \int_\Omega \left(f(x) - I(x)\right)^2 dx + \lambda^2 \int_\Omega \varphi|\nabla f| dx \tag{5}$$

In the table below, we present some φ functions and their property in relation to the convexity:

	$\varphi(t)$	$\varphi'(t)/2t$	Convexity
Total Variation	$\vert t\vert$	$1/2\vert t\vert$ if $t \neq 0$	Yes

Tikhonov	t^2	1	Yes
Geman & McClure	$1/(1+t^2)$	$1/(1+t^2)^2$	No
Green	log(cosh(t))	tang(t)/*2t* if t≠0	Yes
Hebert & Leahy	Log $(1+t^2)$	$1/(1+t^2)$	No
Hyper Surfaces	$2\sqrt{(1+t^2)}-2$	$1/\sqrt{1+t^2}$	Yes
Perona & Malik	$1-e^{-t^2}$	e^{-t^2}	No

Table 2. Some φ functions

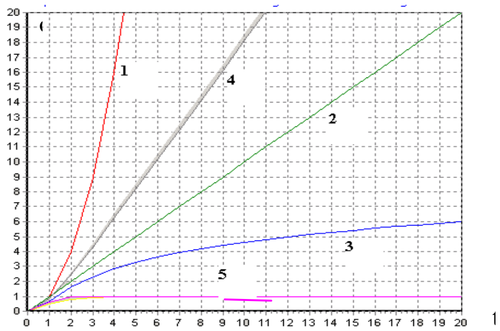

Figure 3. Graphs of φ functions

1. $\varphi(t)=t^2$
2. $\varphi(t)=|t|$
3. $\varphi(t)=\log(1+t^2)$
4. $\varphi(t)=2\sqrt{1+t^2}-2$
5. $\varphi(t)=\exp(-t^2)$

3. VAN DER WALLS-CAHN-HILLIARD THEORY

Van der waals-cahn-Hilliard theory on phase transitions has been studied extensively in mechanics to describe the steady states of the physical systems constituted of unsteady phases (Aubert, G. and Kornprobst, P., 2002). Let's consider a physical system constituted of a fluid of which the energy of Gibbs by unit of volume (potential) is a function W depending on the distribution of density of the fluid. If the fluid is constituted of two different phases described by the levels μ(x)=a and μ(x)=b, then the potential W is double well potential with two minima.

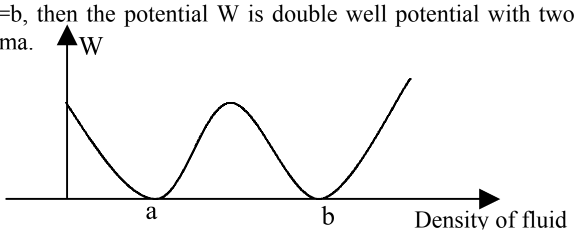

Figure 4. Double well potential

At stability, the fluid will take two values μ (x) =a or μ(x)= b. The approach consists in characterizing the stability state of the system by minimising:

$$p_\varepsilon = \inf_\mu \int_\Omega W(\mu(x))dx$$

Under the constraint $\int_\Omega \mu(x)dx = m$ (6)

The mass of the fluid m is constant and ε is positive real. The regularized solution of this problem is obtained by minimizing E_ε where :

$$E_\varepsilon(\mu) = \int \left[\varepsilon |\nabla \mu|^2 + \frac{1}{\varepsilon} W(\mu(x)) \right] \quad (7)$$

Under the constraint $\int_\Omega \mu(x)dx = m$

4. ANALOGY WITH CLASSIFICATION AND RESTORATION

The stability of a mixture of fluids is reached when each of the fluids forms a homogeneous entity separated of the other by interfaces of minimal lengths. Mathematically, this state is gotten by the minimisation of the energy E_ε.

We can note the similarity that exists between image classification and the stability of fluids in mechanics. Indeed, the classification consists in partitioning an image into homogeneous regions, of minimal interfaces.

Energy is then defined on the image, so that its minimum corresponds to a classified image. This configuration of the image is equivalent to the steady state of the fluids for which the criterion is minimal.

The potential W defined on image is K wells, where K is the number of classes (Samson, C., and al, 2000).

By analogy, the problem of classification and restoration can be deduced directly from equation 7 and can be written as:

$$P_\varepsilon = \begin{cases} \min_f J_\varepsilon(f) \\ J_\varepsilon(f) = \int_\Omega \left[\varepsilon\lambda^2 \varphi(|\nabla f|) + \frac{\eta^2}{\varepsilon} W(f) \right] dx \\ \text{under constraint} \int_\Omega (f(x)-I(x))^2 \, dx \le \sigma_v^2 \end{cases} \quad (8)$$

Where σ_v is the standard deviation of noise, and η is a real parameter.

η^2/ε *W(f)* is a classification term, that attract gray level of pixels to the K means of classes.

5. GAMMA CONVERGENCE THEORY

Let X be a metric space, and let f_ε: X ⟶ [0, +∞[be a family of functions indexed by ε>0.

We say that f_ε Γ-converge as ε ⟶ 0^+ to f: X ⟶ [0, +∞[if the following two conditions

$$\forall x_\varepsilon \rightarrow x \quad \liminf_{x \rightarrow 0^+} f_\varepsilon(x_\varepsilon) > f(x) \qquad (9)$$

and

$$\exists x_\varepsilon \rightarrow x \quad \limsup_{x \rightarrow 0^+} f_\varepsilon(x_\varepsilon) < f(x) \qquad (10)$$

are fulfilled for every $x \in X$. The Γ-limit, if it exits, is unique. The Γ-convergence is stable under continuous perturbations, that is, $(f_\varepsilon+v)$ Γ-converge to $(f+v)$ if f_ε Γ-converge to f and v is continuous (Aubert, G., and al, 2002). The most important property of Γ-convergence is the following:
if $\{x_\varepsilon\}_\varepsilon$ is asymptotically minimizing, i.e:

$$\lim_{x\rightarrow 0^+}(f_\varepsilon(x_\varepsilon)-\inf_x f_\varepsilon)=0 \qquad (11)$$

and if $\{x_{\varepsilon h}\}_h$ converge to x for some sequence $\varepsilon_h \rightarrow 0$, then x minimizes f.

6. EXPRESSION OF THE FUNCTIONAL AND IMPLEMENTATION

By applying the properties of Gamma-convergence, the solution of the equation 8 is obtained by minimizing the functional $J\varepsilon$ *when* ε is approaching the zero value.

$$J_\varepsilon(f)=\int_\Omega\left[\varepsilon\lambda^2\varphi(|\nabla f|)+\frac{\eta^2}{\varepsilon}W(f)\right]dx+\int_\Omega(f(x)-I(x))^2dx \qquad (12)$$

$$\tilde{f}=\lim_{\varepsilon\rightarrow 0}\left\{\arg\min_f J_\varepsilon(f)\right\}$$

We can note that this functional is composed by three terms: regularization term, classification term and data fidelity term. The first term is weighted by a parameter proportional to ε, and the classification term is weighted by a parameter proportional to 1/ε. The convergence of the criterion given by equation 12 is reached for little values of ε, so that the regularization and the classification are not achieved simultaneously. For high values of ε, the regularization is privileged, and progressively with ε decreasing, the process changes its behavior, and becomes classification process.
The power of the regularization by φ functions lies in its nonlinearity. This later criterion leads to difficulties for optimization calculation. If φ is quadratic, the function to be minimized is quadratic, therefore the minimum is single and easy to calculate. To bring back itself to a quadratic model, the semi quadratic theorem is used, and consists of introducing an auxiliary variable b.

$$\forall t,..\varphi(t)=\inf_{L\leq b\leq M}\left(bt^2+\psi(b)\right) \qquad (13)$$

$$b_{\inf}=\frac{\varphi'(t)}{2t} \qquad (14)$$

where: $\lim_{t\rightarrow\infty}\frac{\varphi'(t)}{2t}=L$, $\lim_{t\rightarrow 0+}\frac{\varphi'(t)}{2t}=M$

$\psi(b) = g((g')^{-1}(b)) - b((g')^{-1}(b))$ and $g(t) = \varphi(\sqrt{t})$

The equation 12 can be rewritten as:

$$J_\varepsilon(f)=\int_\Omega(f(x)-I(x))^2dx+\varepsilon\lambda^2\int_\Omega\left[b|\nabla f|^2+\psi(b)\right]dx+\frac{\eta^2}{\varepsilon}\int_\Omega W(f)dx \qquad (15)$$

For minimizing the sequence of functional 15, we use Euler Lagrange equation and the minimization problem is transformed to a problem of resolution of partial differential equation (PDE), given by:

$$[f(x)-I(x)]+\frac{\eta^2}{2\varepsilon}W'(f(x))-\varepsilon\lambda^2 div(b\nabla f)=0 \qquad (16)$$

Where div is the divergence operator.

7. EXPERIMENTAL RESULTS

To validate the approach of classification suggested, we initially tested it on a synthetic image before applying it to the real satellite image. The synthetic image contains 4 classes detailed in table 2.

classe	μ_i	σ_i
1	22.46	4.66
2	63.62	5.07
3	107.13	4.58
4	232.02	4.73

Table 5. Characteristics of synthetic image classes

The figure 6 illustrates the image to be classified and the figure 7 the graph of the potential W, with 4 wells. In figure 8 we can see the localization of training areas, and in figure 9 we show the classified image.

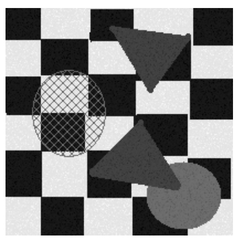

Figure 6. Original image

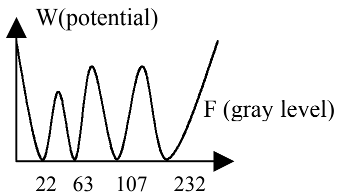

Figure 7. The potential W

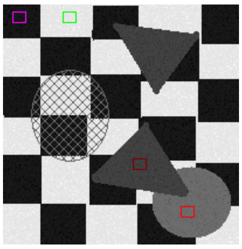

Figure 8. Training areas

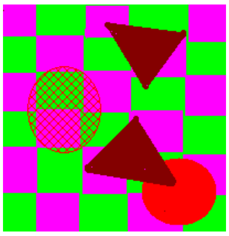

Figure 9. Classified image

The multispectral image Spot-1 of which we lay out consists of three channels XSi (20m X 20m), i=1, 2, 3 resulting from scene 50-282 of February 23, 1986. The image of size 256x256 pixels represents the area of Blida in Algeria as shown in figure 10.

Figure 10. Map of North of Algeria situating the region of Blida.

The classification is supervised in the sense that the algorithm assumes the knowledge of number of classes and their characteristics. The training on the satellite image allows us to define seven (07) classes detailed in table 11, and their localization is shown in figure 14.

Classes	Themes
1	Less dense urban zone
2	Less dense natural vegetation
3	Naked ground, aerodrome of Blida
4	Non cultivated fields
5	Dense urban zone (city of Blida)
6	Cultivated fields
7	Dense natural vegetation

Table 11. The classes of the scene of BLIDA (ALGERIA)

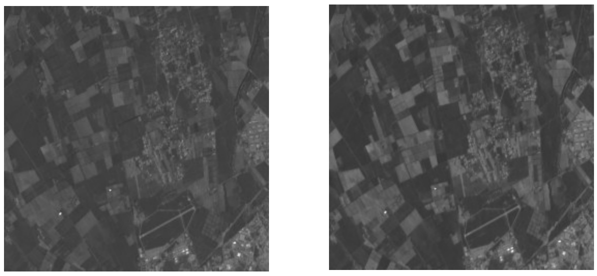

Figure 12. The three bands of the satellite image of Blida.

Figure 13. Color composite of the three bands. (RGB: XS3, XS2 et XS1).

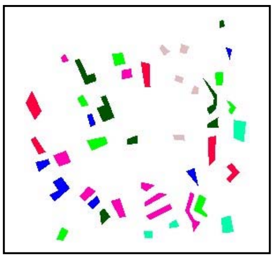

Figure 14. The training areas of the satellite image.

The resolution of equation 16 necessitates the knowledge of classification and restoration parameters. For our implementation, we have made the choice experimentally. Since the performance of the method depends not only on the choice of parameters, but it also depends on the function φ, we have tested the algorithm with different functions and we present in figure 15 the result obtained with the function of Hebert & Leahy. It has been proved mathematically that convex functions leads to convergence of criterion. The experimentation shows that the non convex functions may give better results, but the convergence is not guaranteed.

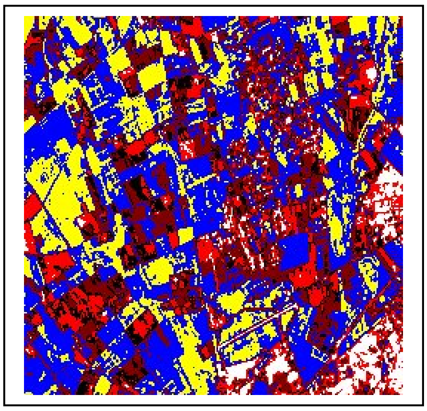

Figure 15. Classified image

The variational algorithm convergences always and rapidly to the global minimum, this is not the case for Markov models. Moreover, this method showed its performance to classify correctly an image after its restoration. We can see in the figure 9 and 15 that the contours of objects as well as small structures are well preserved.

8. CONCLUSION

In this paper, our first objective was to develop a robust model for remote sensing image classification. Because images are often corrupted with additive noise, we opted for a model that combines image classification and edge preserving restoration. The edge preserving restoration is not a fortuitous choice, because the boundaries information is important for classification process. The method we have developed is based on the use of van der walls theory in mechanic of fluid, and the gamma convergence theory, and consists in construction and minimization of a sequence of functional. To avoid smoothing on object contours, the regularization chosen is the φ function one.

The minimization of the functional is achieved by resolution of Partial Differential Equations and the use of descent gradient algorithm. The implementation of the algorithm showed that the method is effective; the image is well restored and classified in few iterations.

To implement the algorithm, we assumed that the image is only corrupted with additive noise. To complete this work, we project to take in consideration the effect of blur.

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