

DEVELOPMENT OF AN INVERSION CODE, ICARE, ABLE TO EXTRACT URBAN AREAS GROUND REFLECTANCES

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ABSTRACT:

Automatic mapping of urban materials from remotely sensed radiance images remains difficult because of the complex physical phenomena induced by relief. Indeed, shadows and environment effects disrupt the radiance incoming the sensor. Moreover, the measured radiance depends on the illumination conditions of the observed area. This is quite problematic for multi-temporal analysis. Images are often taken in different illumination conditions thus shadows are not located in the same place.

This paper describes a new physical model, ICARE, able to solve the radiative transfer inversion problem over urban areas, in the reflective domain ($0.4 \mu\text{m} - 2.5 \mu\text{m}$), from high spatial and spectral resolution images. This new approach takes into account the complex relief of 3D structures, the spatial heterogeneity of the scene and atmospheric effects, in order to extract rigorously the ground surface reflectance, even in shadows areas. The resolution method consists in modelling separately the irradiance and radiance components at ground and sensor levels.

The validation of ICARE is checked through the CAPITOUL trial, carried out in Toulouse (France, 2004). Measurements were performed using two airborne Pelican image systems consisting in 8 high spatial (20 cm) and spectral (30 nm) resolution cameras. Results are presented over typical urban structures in Toulouse center completed by an error budget. They demonstrate that extracted reflectances correspond very well to ground reflectance measurements. Moreover, there is a good continuity between reflectance obtained over sunny and shadowed materials. That proves that shadows areas and environment effects are well corrected.

To quantify the gain brought by ICARE, a comparison between classification based on radiance and reflectance images, obtained by inversion, is introduced.

1. INTRODUCTION

Recent studies estimate that more than 75% of the world's population are living in urban areas. A better understanding of this environment is required in many applications, to study its spatial evolution, impact on the environment, or problems of town planning and organization. With the advent of high spatial resolution sensors from the visible to the short wave infrared domain [$0.4 \mu\text{m} - 2.5 \mu\text{m}$], it is now possible to get accurate information from airborne and satellite images.

Due to their highly structured geometry, an important part of urban areas exhibits shadows. Therefore, airborne or satellite acquisitions over urban areas will depend on the illuminations conditions. This drawback must be overcome to improve applications as mapping or urban change detection, which often use images from different time or different sensors. Moreover, there are important coupling effects at the interface of the 3D ground and the low atmosphere layer like multiple reflexions on wall. All of these phenomena make it difficult to classify automatically urban materials from airborne radiance measurements. Preprocessing algorithms are usually required to retrieve ground surface intrinsic properties like reflectance that is independent of the irradiance conditions and thus improve classification performance.

Currently, different methods are used to solve the inverse transfer radiative problems (i.e. retrieving ground reflectance from radiance images) but most of them do not account for ground heterogeneity. These codes often consider a flat and homogeneous landscape with a lambertian assumption, e.g., ATREM (Goetz, 1997), ACORN (Miller, 2002). Improvements

have been carried out in FLAASH (Anderson, 2002) and COCHISE (Miesch, 2005) as they consider a heterogeneous background. Nevertheless, these codes are not able to deal with urban areas where the relief plays very important part in the remotely sensed features. At present, two models currently take into account the ground topography in mountainous region: ATCOR4 (Richter, 2002) and SIERRA (Lenot, 2003). They consider a heterogeneous background and the ground topography is modelled by a digital elevation model (DEM). But these codes are not adapted to the very uneven structure of urban areas for two main reasons. First, the earth-atmosphere coupling term and the multiple reflexions are still roughly modelled which causes important errors notably in shadows where environment effects are preponderant. Secondly, the use of DEM images prevents wall modelling or all vertical objects present in an urban area whereas walls have a large contribution in the urban radiative transfer.

The need for a retrieval model that is able to understand and model the signal over urban areas led us to develop the method we report here. This method, named ICARE (Inversion Code for urban Areas Reflectance Extraction), consists in modelling separately the irradiance and radiance components at ground and sensor level by use of models derived principally from improved versions of the usual spatially resolved approaches. Each component takes into account the relief, ground heterogeneity and atmospheric corrections.

In order to check the validity of the different assumptions made, a field campaign is carried out to validate our model on real images and first results are presented. Then, usual classification algorithms are applied on the reflectance and radiance images

to quantify the gain brought by ICARE. The paper will insist particularly on the experimental validation of this model.

2. RADIATIVE TRANSFER MODELLING OVER URBAN AREAS

2.1 Physical modeling

This part describes the main characteristics of the physical model implemented in ICARE. This approach consists in determining an analytical expression for each of the component involved in the total radiance reaching a given sensor observing an urban scene. We use here a complete separation of the atmospheric and ground optical properties to lead to an easily tractable inversion. The full spectral domain of ICARE ranges from the visible to the near infrared domain $[0.4\mu\text{m} - 2.5\mu\text{m}]$. In this domain, optical properties of the ground correspond to its spatial and spectral bidirectional reflectance:

$$\rho^{dd}(\theta_i, \varphi_i, \theta_r, \varphi_r, \lambda) = \frac{\pi R(\theta_i, \varphi_i, \theta_r, \varphi_r, \lambda)}{I(\theta_i, \varphi_i) \cos(\theta_i)} \quad (1)$$

Where $I(\theta_i, \varphi_i)$ is the irradiance incident on the surface in the direction (θ_i, φ_i) and $R(\theta_i, \varphi_i, \theta_r, \varphi_r)$ the corresponding reflected radiance in the direction (θ_r, φ_r) at the wavelength λ , θ is the zenithal angle et φ the azimuthal angle.

In this paper, a lambertien assumption is made on the ground reflectance. Only spatial and spectral heterogeneity is taken into account. Thus the spectral bidirectional reflectance $\rho(\theta_i, \varphi_i, \theta_r, \varphi_r, \lambda)$ is simply noted $\rho(\lambda)$.

Considering an airborne or satellite sensor observing an urban scene and i a pixel of the sensor image, the total radiance received by this sensor can be written as the sum of three main contributors (Figure 1) :

$$R_{\text{sensor}}(i) = R_{\text{diffuse}}(i) + R_{\text{atmospheric}}(i) + R_{\text{direct}}(i) \quad (2)$$

The first component corresponds to scattered photons coming from the target neighbouring (R_{diffuse}). $R_{\text{atmospheric}}$ refers to photons coming to the sensor without reaching the surface.

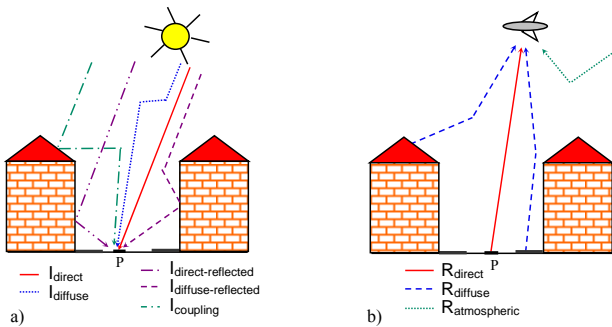


Figure 1: Irradiance components at ground level (a) and radiance components at sensor level (b).

R_{direct} is the upwelling radiance directly transmitted from the ground to the sensor. This component plays an important role as it brings the searched reflectance:

$$R_{\text{direct}}(i) = (I_{\text{direct}}(P) + I_{\text{diffuse}}(P) + I_{\text{reflected}}(P) + I_{\text{coupling}}(P)) \frac{\rho}{\pi} T_{\text{dir}}^{\uparrow} \quad (3)$$

Where P is a considered point of the ground and ρ its reflectance. I_{direct} , I_{diffuse} , $I_{\text{reflected}}$ and I_{coupling} are the different irradiance components incident to the ground according to their type of path (Figure 1). $T_{\text{dir}}^{\uparrow}$ is the upward total transmission of the atmosphere on the path ground-sensor.

Using equations 1 and 2, we can extract an analytical expression of the reflectance using a lambertian hypothesis:

$$\rho = \frac{\pi(R_{\text{sensor}} - R_{\text{diffuse}} - R_{\text{atmospheric}})}{T_{\text{dir}}^{\uparrow} (I_{\text{direct}} + I_{\text{diffuse}} + I_{\text{reflected}} + I_{\text{coupling}})} \quad (4)$$

To solve this equation, our method consists in estimate each component of equation (4).

2.2 Model implementation

The ICARE implementation is based on the radiative transfer code 6S which is used to compute the atmospheric components. ICARE needs three entry data: a digital vector model of the relief of the scene, atmospheric conditions during the acquisition and a calibrated radiance image of the scene. Three radiative components could be directly computed from these data: the direct and diffuse irradiances at the ground level and the atmospheric intrinsic radiance at sensor level. Indeed, these three components only depend on the illumination conditions of the scene. The other components of the radiative transfer equation need neighbourhood notions that mean reflectance values of the environment. To solve this problem, an iterative algorithm is carried out.

To initialise the iterative method, an average reflectance of the entire scene is computed using a flat homogenous assumption of the scene. This hypothesis implies that some environments effects like reflected irradiances are not considered in this first step. Only a basis approach of the coupling earth-atmosphere irradiance is computed. This averaged reflectance is applied on all viewed surfaces by the sensor. According to its zenithal angle, different parts of the scene are hidden. The iterative process will not be able to evaluate their reflectance. For example, walls will not be seen by a nadir acquisition. However, the optical properties of these hidden surfaces are necessary in particular to compute environment effects. Thus, spectral default values are attributed to all not viewed surfaces of the scene.

The next step of the implementation consists in computing all the different irradiance component on the ground and radiance components at sensor level. Knowing all the unknown components of Equation 4, it is possible to evaluate a first value of the ground reflectance for each pixel of the sensor. Then an iterative approach is processed until the reflectance map obtained focused on constant values. The numbers of iteration

needed for these computations depends on the scene but 6 iterations are often sufficient.

3. DESCRIPTION OF THE EXPERIMENT CAMPAIGN

An experimental validation campaign of this model took place in Toulouse in April 2004, including airborne acquisitions at high spatial and spectral resolutions and ground truth measurements within the same resolution. The airborne acquisitions were performed using simultaneously two high spatial resolution systems (PELICAN) (Duffaut, 2005) composed of four cameras each (Figure 2). Thus, eight registered images were acquired simultaneously in eight spectral bands over our test area.

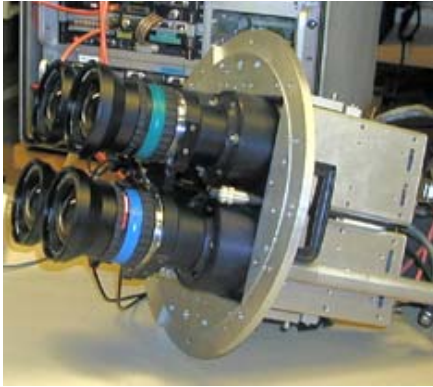


Figure 2: The PELICAN system.

The flight altitude was 2250 m and the spatial resolution at ground level 20 cm. All acquisitions were realised at nadir viewing angle. Indeed, it is the viewing angle where the largest number of urban surfaces could be viewed by the sensor. Only walls are hidden. The eight narrow filters associated to the cameras were located in the visible and near infrared spectral domain from 420 nm to 917 nm.

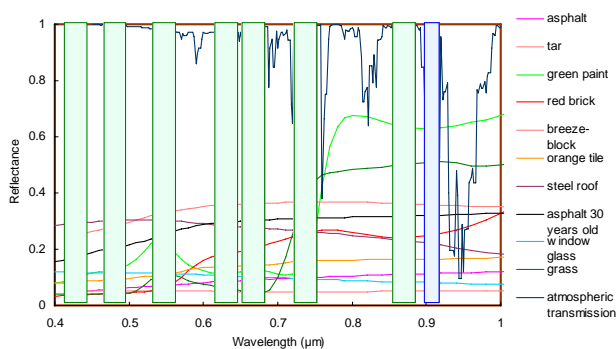


Figure 3: Location of the eight narrow filters.

The spectral location of these bands was chosen corresponding to two criteria. First, the goal of these bands was to allow a spectral discrimination of most of urban manmade materials. Figure 3 shows spectral reflectances of different urban materials coming from ASTER database (ASTER). Secondly, the different filters have to be well distributed over the spectral domain in order to validate ICARE while avoiding main gaseous absorption bands. Only the eighth band doesn't respect these rules. Unlike the others, it has been chosen to determine the water vapour content for others applications. Subsequently, only the first seven filters will be used in this paper.

The test site chosen for the experimental validation of ICARE is the préfecture area in the centre of Toulouse (Figure 4). It is composed of a closed courtyard surrounded by typical Toulouse buildings. This site corresponds to a typical urban area. The main advantage is that the courtyard has restricted access for people and cars which made easier ground truth measurements.



Figure 4: Airborne image of the Préfecture of Toulouse.

The digital vector model associated to this scene was provided by the Institut Géographique National (IGN) from stereoscopic images of Toulouse acquired in December 2003. It contains a precise geometry of the different buildings on the scene. To link this 3D vector model to airborne images, a triangulation of the model was performed to obtain triangles with a spatial resolution of 20 cm. The geometrical accuracy of the urban structure is very important as shadows areas are directly calculated from it. Airborne acquisitions were registered in the eight bands and georeferenced by IGN for the treatments.

Airborne measurements took place in April 2004 for two days running with two acquisitions per day at 14hUT and 16hUT. Ground measurements consist in irradiance, radiance and reflectance measurements on the courtyard's ground. These measurements have been done along four defined profiles in order to be able to compare measurements between two series. These series have been planned just before and after each airborne acquisition. The goal of these combined measurements was first to validate the reflectance estimated by ICARE and also to validate intermediate steps of the processing like irradiance computations. Ground measurements have been performed with an ASD FieldSpec PRO spectroradiometer.

The trial was achieved with clear sky conditions and aerosols were supposed to be urban type. A sun photometer located in the centre of Toulouse gave us information about their optical thickness. Measurements done during the experiment showed that the classical Bouguer-Lambert law could not be applied here because of the important variation of the optical depth during the day. Therefore, the aerosols' spectral optical depth was derived directly from irradiance measurements on the ground done with a sun photometer and then adjusted with Junge's law.

4. ICARE VALIDATION

4.1 Experimental validation

The goal of this experimental validation is to evaluate if the different assumptions done in ICARE are sufficient to retrieve ground reflectances from high resolution airborne images even

in shadowed areas. The results exhibit in this article corresponds to the first acquisition of the second day of the campaign, acquired at 14hUT. The four profiles present on the airborne image of the préfecture's courtyard (Figure 5) correspond to the different ground measurements profiles.

Figure 5 illustrates the results produced by the retrieval method ICARE on the préfecture area in Toulouse in the spectral band centred at 485 nm. We can see that shadows are well removed after ICARE processing. Details which were hidden in shadowed areas like cars appear in the reflectance image. Another important point is that we can see on the radiance image that roof's tiles exhibit different colours depending on their relative orientation to the incidence solar angle. Meister (Meister, 2000) shows that this phenomenon is mainly due to the variation of the direct irradiance component with the slope of the roofs. Bidirectional effects of the reflectance of tiles play also a role in the signal received by the sensor but only in a second order. This variation in the appearance of the roofs is quite a problem in classification processing. These phenomenons are no longer present in the reflectance image. ICARE takes into account the slope of the roofs in its physical modelling and the resulting reflectance obtained after processing are then independent of the illumination conditions.

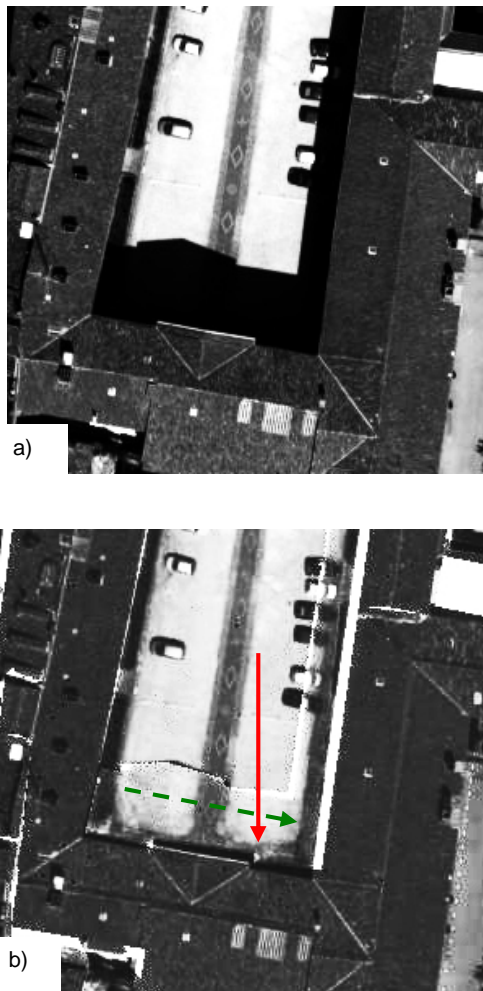


Figure 5: a) Radiance image at 485 nm, b) Reflectance image obtained after inversion at 485 nm.

The main artefact which appears in the reflectance image is the white and black areas located at the border of the shadowed areas and also at the place of a change of slope. It corresponds

to a geometric error of the 3D vector model. Indeed, the shadows map is computed from the geometric information provided by the vector model. If a side of a building is not well located, the end of the generated shadows will not be coherent with the real scene viewed by a sensor. It creates an error in the calculation of the direct irradiance component. If a pixel located normally in a sunny areas is modelling by the vector model in shadows, it will be characterised by an important radiance at the entry of the sensor (corresponding to a sunny area) and by a low irradiance level on the ground. The reflectance extracted by ICARE will be therefore very high to find coherence between irradiance and radiance levels. That explained the white areas observed in the reflectance image. Black areas are generated by the same phenomenon: a pixel really located in shadowed areas and a corresponding surface on the ground located in a sunny area. Thus, the most important errors in the reflectance image obtained by ICARE are caused by geometric errors in the 3D model of the relief. The quality of the reflectance image depends entirely of the quality of the vector model of the scene. Another important result to highlight is that the reflectance level retrieved by ICARE in shadowed areas is the same that the one obtained in sunny areas (Figure 6). There is no gap, no discontinuity in the reflectance image depending on the illumination conditions (apart from geometric errors). It means that the environment effects, preponderant in shadowed areas, are correctly modelled. This important result allows us to valid the approaches used to model the different environment components: the diffuse upwelling radiance and also the reflected irradiance components.

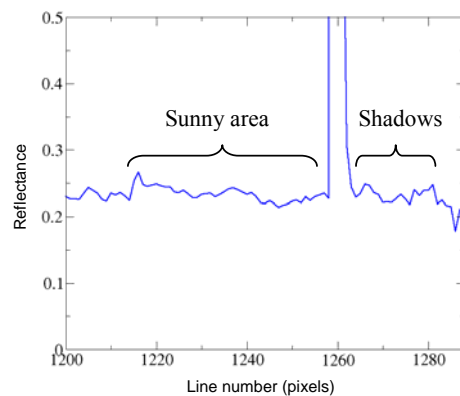


Figure 6: Reflectance values obtained along the red arrow Figure 5(b) at 485 nm.

Finally, another point to be discussed in the validation experimental, and perhaps the most important point for our experimental validation, is the comparison between the retrieved reflectances and the one measured at the ground. Previous results show that the model used to evaluate environment effects allow us to obtain a good continuity between sunny and shadowed areas but it is very important to determine if the reflectance values retrieved after inversion corresponds to ground measurements. An absolute error would not be important for classification analysis where the most important thing is the relative difference observed between different materials. But it could be very important to determine the material's name directly from the reflectance image. Many urban materials are characterised by a relative flat reflectance over the visible and near infrared region (Lachérade, 2005) and thus, the level of these reflectance is sometimes the only clue to determine the analysed material. The goal of ICARE is not only to allow an improvement in classification results but also to

give to the scientific community a tool to analyse the absolute optical properties of manmade materials directly from remote sensing images. Thus, the absolute accuracy of the retrieved reflectance is very important.

Figure 7 shows the comparison of the reflectance retrieved by ICARE at 485 nm and the reflectances measured on the ground.

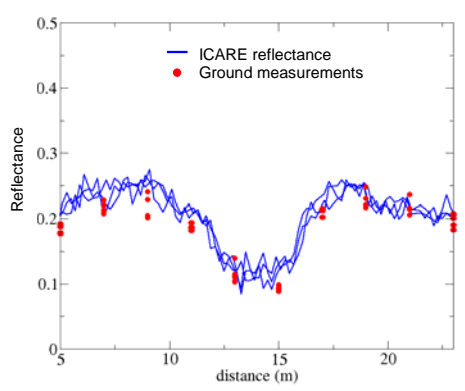


Figure 7: Comparison at 485 nm of reflectances retrieved by ICARE (blue lines) and ground measurements (red points) along the red line Figure 6b).

Red points correspond to the ground measurements before and after each airborne acquisition along a determined profile at the same points in the courtyard. Variations observed in their reflectance level are due to the spatial variability of the ground. Courtyard is composed of gravel. As the spatial resolution of the ground measurements is 20 cm, reflectance measurements include the spatial heterogeneity of the ground which explains the observed variability. In order to take into account the spatial variability also present in the retrieval image of reflectance, three close profiles to ground measurements profile were extracted. They are represented by the blue lines.

This graph shows that at 485 nm, the accuracy of the retrieved reflectance after inversion is very good. The spatial variation of the reflectance along the profile corresponds to that which was measured during ground measurements. The absolute reflectances obtained from the airborne image after the inversion method show a good agreement with the "real" reflectance of the materials at the studied spatial resolution.

As the spatial comparison between modelled and measured reflectances seems to be valid, the Figure 8 shows the spectral comparison in one point of the previous profile.

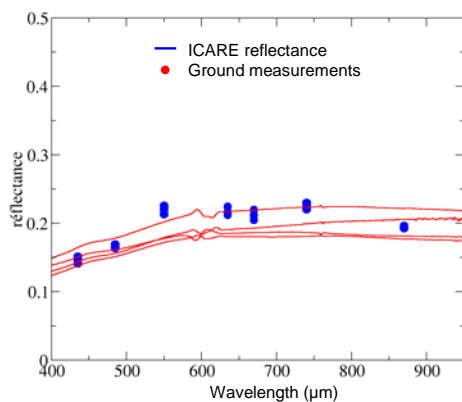


Figure 8: Spectral comparison of reflectances retrieved by ICARE in the 7 spectral bands and ground spectral reflectance measured with the ASD spectrometer.

Blue points correspond to the seven reflectance values extracted from the seven bands used during the airborne acquisition. The red lines are the corresponding spectral reflectances measured in the courtyard with the ASD spectrometer. These results are satisfactory insofar as the retrieved reflectance values show a maximum absolute error less than 0.04 whatever the spectral band. The little variation observed on the extracted reflectance between the different spectral bands is probably due to the absolute calibration of the camera. Indeed, each spectral band of the airborne instrument was separately calibrated during a vicarious calibration. This calibration has direct consequences on the radiance level in images and thus on the reflectance extracted by ICARE.

To confirm these results, similar comparisons were performed over another test site located in the surroundings of Toulouse. Analyses show that the accuracy of ICARE allows us to determine the absolute reflectance of urban materials with a performance better than 0.04 peak-to-peaks.

4.2 Classification results

This study carries out that ICARE is able to retrieve the spectral materials of urban materials directly from airborne images. The main application using these images is classification. The goal of this section is not to develop classification methods on the reflectance images but just to quantify the gain brought by ICARE with a typical classification algorithm. The classification tool used here is the unsupervised classification algorithm of the K-Means which is implemented in the software ENVI Version 4.2. The evaluation of the accuracy of the classification results was determined by comparison with the classification results realised by a photo-interpreter.

Figure 9 emphasizes the classification results obtained from the radiance and reflectance images. We find again the different problems of the radiance image, touched on previously. The shadowed areas are put in a "shadows class" and the materials in the shadows could not be identified. Moreover, the classification algorithm determines two different materials on the roof whereas it is the same. It is due to the variation of the slope of the roofs. Thus, the number of good classified pixels is only about 54%. On the reflectance image, some of the problems have been corrected because the retrieved reflectance is independent of the illumination conditions. The shadowed areas are corrected and the materials located in these areas are well identified. There are no longer problems for roofs. One material (tiles here) is identified for the roofs. Therefore the reflectance image gives us a number of good classified pixels of about 74%. The use of ICARE largely improves the results of classification.

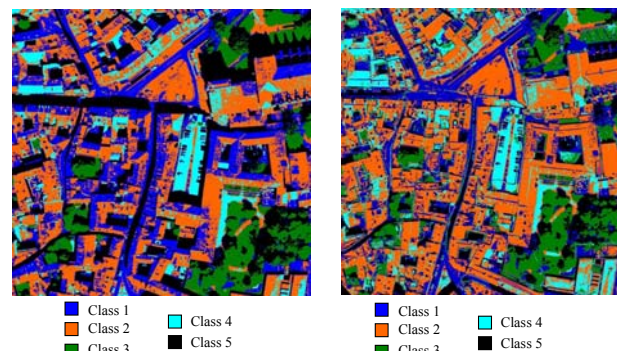


Figure 9: Classification results obtained on the radiance image (on the left) and on the reflectance image (on the right).

Nevertheless, classification results obtained with ICARE could still be improved. For example, trees are not yet modelled in the digital vector model of the relief. Therefore, the shadows generated by the tree are not computed and thus not corrected. The accuracy of the inversion and thus on the classification, will depend mainly on the accuracy and level of details of the geometric vector model used for the reflectance retrieval.

5. CONCLUSIONS AND PERSPECTIVES

In this work, an inverse radiative transfer model operating in the visible and short wave infrared is presented. The objective of the study was to develop a code able to deal with urban areas. It takes into account atmospheric corrections and environment effects in order to retrieve ground reflectance from radiance images. To solve the inverse problem, irradiance and radiance components are modelled separately. The effects of the various assumptions made were checked through an experimental campaign. This field campaign was carried out in Toulouse using two PELICAN systems. Results demonstrate that the physical modelling is well adapted to urban areas. Shadows are well corrected and there is no bias between reflectances in shadowed and sunny areas for the same materials. Moreover, the absolute reflectance of the urban materials is retrieved with an absolute accuracy better than 0.04 peak-to-peaks over the whole studied spectral region (visible and near infrared). To quantify the improvement provided by ICARE, a usual classification algorithm was applied on the radiance and reflectance images. A comparison realized between results obtained after the classification shows that the use of ICARE have the outcome to largely improve the number of good classified pixels from 54% with the radiance image to 74% with the reflectance image.

The next step of this work will be to confirm experimental results over different site tests, characterised with different architecture, with different viewing angles, and also to enlarge the experimental validation (actually limited by the camera to the silicium domain) to the whole near infrared. Moreover, two approaches are considered to improve ICARE: the improvement of the physical algorithm and the improvement or post-correction of the entry data. First, ICARE's processing actually uses a lambertian assumption. It will be necessary to take account of these directional effects in the direct-direct component. Secondly, we have seen on the validation results that the most important error comes from the digital vector model. It would be important to find methods to improve model vectors accuracy by two ways: by correcting the geometric location of different objects and also by improving the level of details included in the vector model. For example, it would be useful to model vegetation like trees. All these perspectives represent a lot of future work but the first encouraging results obtained with ICARE allow us to think that this model could be an important improvement for the study of urban areas from remote sensing images.

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