RADIATIVE TRANSFER MODEL INVERSION BASED ON MULTI-TEMPORAL CHRIS/PROBA DATA FOR LAI ESTIMATION

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ABSTRACT:

Leaf area index (LAI) is a key variable for the understanding and modelling of several eco-physiological processes within a vegetation canopy. The LAI could thus provide vital information for the management of the environment and agricultural practices when estimated continuously over time and space thanks to remote sensing sensors such as CHRIS/PROBA.

The spaceborne ESA-mission CHRIS/PROBA provides multi-temporal observations of the land surface in the spectral and directional information dimensions. This system represents a rich source of information for Earth observation purposes specifically adapted for monitoring the high dynamic of agricultural crops. For this purpose a radiative transfer model (RTM) is coupled to a canopy structure dynamic model (CSDM). The coupled models are used to exploit the complementary content of the spectral and temporal information dimensions for LAI estimation over a maize canopy. The resulting estimation of the temporal and spatial variation of LAI is improved by integrating multi-temporal CHRIS/PROBA data and ground meteorological observations. Further, the presented method provides the continuous LAI time course over the season, which is required by crop growth and land surface process models.

1. INTRODUCTION

Leaf area index (*LAI*) as defined by the single sided area of green leaves per unit of horizontal soil (Privette et al. 2001) is a key variable governing several processes such as photosynthesis, transpiration or rain interception. Estimates of *LAI* could be assimilated within vegetation process models to provide more accurate description of canopy functioning with emphasis on important environmental and economical outputs such as, carbon, water and nitrogen fluxes and stocks, canopy state and yield for crops (Chen et al. 2003; Matsushita and Tamura 2002). Remote sensing allows for detailed and frequent observations of the vegetation necessary to monitor the spatial and temporal variations of canopy characteristics (Koetz et al. 2005; Myneni et al. 1997).

At the top of the canopy, the interaction of radiation within the vegetation depends on the contribution of several components such as leaves, stems, soil background as well as the illumination and view geometries (Goel and Thompson 2000; Verhoef and Bunnik 1981). Radiative transfer models (RTM) provide an explicit connection between the canopy biophysical variables, the view and illumination geometry and the resulting canopy reflectance by exploiting our knowledge of the involved physical processes (Baret et al. 2000). The RTM have to be inverted to retrieve the biophysical variables from the measured canopy reflectance (Bacour et al. 2002; Kimes et al. 2000; Weiss et al. 2000). However measurement and model uncertainties are often leading to a large range of possible solutions, which prohibits the inversion to be properly solved (Combal et al. 2002). The regularization of such an ill-posed problem requires input of additional information to obtain more reliable and stable solutions (Combal et al. 2002; Combal et al. 2003).

Knowledge of the canopy structure dynamics is highly desirable as ancillary information to constrain the RTM inversion for the estimation of canopy characteristics. The dynamics of the canopy structure are strongly depending on crop growth processes, which result in a relatively smooth and typical temporal profile of LAI. Simple semi-mechanistic models have been proposed to describe the LAI time course (Baret 1986; Werker and Jaggard 1997). Such models could consequently be used to exploit the information on canopy structure dynamics and get more robust and reliable estimates of LAI. The use of a canopy structure dynamics model (CSDM) allows also to derive a continuous estimation of LAI which is required in some applications, particularly those based on the forcing of agricultural growth or land surface models (Delecolle et al. 1992; Moulin et al. 1998). The coupling of radiative transfer and canopy structure dynamics models offers consequently a great potential for the interpretation of remote sensing data since it integrates several sources of information (Baret et al. 2000):

- the knowledge of radiative transfer processes within RTM
- the knowledge on some biological processes within CSDM
- the temporal and spectral dimension of radiometric information
- ancillary information such as the climatic variables partly governing the CSDM including temperature, and the prior knowledge on the canopy type.

The CHRIS instrument operated on the spacborne platform PROBA observes the canopy reflected radiance in the spectral, directional, spatial and temporal dimensions, thus describing the canopy reflectance based on independent but complementary information sources (Barnsley et al. 2004). We propose to exploit synergistically two of these information dimensions, namely the spectral and temporal, based on multi-temporal CHRIS observations. A RTM coupled to a CSDM is used to improve the estimation of biophysical canopy characteristics relevant for dynamic land surface processes.

2. DATA

CHRIS multi-angular data sets were acquired over a test site in central Switzerland on eight different dates between 26 May 2005 and 22 September 2005 in Mode 5 (see Tab. 1). Out of this data sets, four dates that represent major steps in phenology of the selected agricultural fields were selected for further processing and data exploitation. The selected dates are 26 May 2005 (DOY 171), 20 June 2005 (DOY 196), 17 August 2005 (DOY 229) and 22 September 2005 (DOY 265). Within this study the multi-temporal aspect of nominal nadir acquisitions has been further exploited. The full directional information content of the data set is described in the a separate study (Kneubühler et al. 2006).

Spatial Sampling	Image area	View angles	Spectral bands	Spectral range
17m @ 556 km altitude	6.5x13 km (372x748 pixels)	5 nominal angles @ +55°, +36°, 0°, -36°, -55°	37 bands with 6-33 nm width	447-1035 nm

Table 1. CHRIS Specifications for Mode 5

2.1 Test Site and Field Data

The test site for this study is located in the rural border region between the two Swiss Cantons of Lucerne and Aargau in central Switzerland near the villages of Vordemwald (7°53' E, 47°16'N), St. Urban and Pfaffnau. The hilly area is dominated by agricultural fields in the lower parts (450-500 m a.s.l.) and mixed forests mainly on the hilltops (elevations up to 700 m a.s.l.). Agriculture mainly concentrates on barley, wheat, maize, sugar beet and pasture land.

Ground truth data were collected in a maize field parallel to the CHRIS data takes on most dates. Ground data collection included spectroradiometric measurements using a FieldSpec Pro FR, LAI measurements using a Licor LAI-2000 Plant Canopy Analyzer (Fig. 1) and hemispherical photographs, as well as determination of leaf water and chlorophyll content in the laboratory (Fig. 1). An operational meteorological station in the close vicinity of the study site provided basic meteorological observations such as air temperature (Fig. 2). The land use type was recorded for a large amount of agricultural fields.
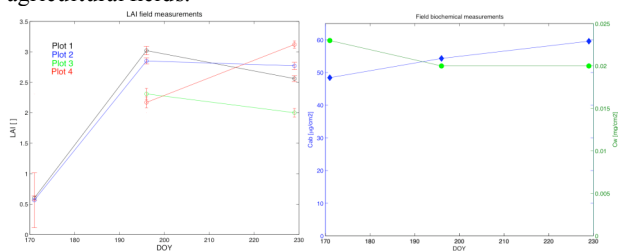

Figure 1. Biophysical (LAI) and biochemical (Cab: leaf chlorophyll content, Cw: leaf water content) field measurements
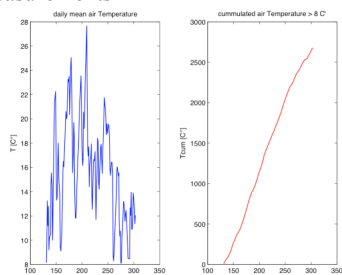

Figure 2. Daily (right: accumulated) air temperature over the maize growing season

2.2 Geometric and Atmospheric Processing

Geometric and atmospheric correction of the multi-angular CHRIS data sets under investigation was performed following an approach described in Kneubühler et al. (2005). Geocorrection is therefore based on a 3D physical model developed by Toutin (2004) which is implemented in the commercially available image processing software PCI/Geomatica. High locational accuracy of the respective multi-angular products after geometric correction is a prerequisite for reliable retrieval of HDRF information from the data set. The root mean square errors (RMSE) for the specific region of interest do generally not exceed one pixel. Figure 4 shows a subset of a geocorrected CHRIS nadir scene for the specific region of interest in this study. The accuracy of the geocorrection may be seen from the overlying pixelmap. Subsequent atmospheric correction of the CHRIS radiance data products is performed using ATCOR-3 (Richter 1998), which is based on MODTRAN-4. ATCOR-3 accounts for terrain effects by incorporating DEM data and their derivatives such as slope and aspect, sky view factor and cast shadow. ATCOR-3 is capable of processing data from tilted sensors by accounting for varying path lengths through the atmosphere and varying transmittance. Atmospheric correction results in the retrieval of HDRF (Hemispherical Directional Reflectance Factor) data sets for the various CHRIS view angles.
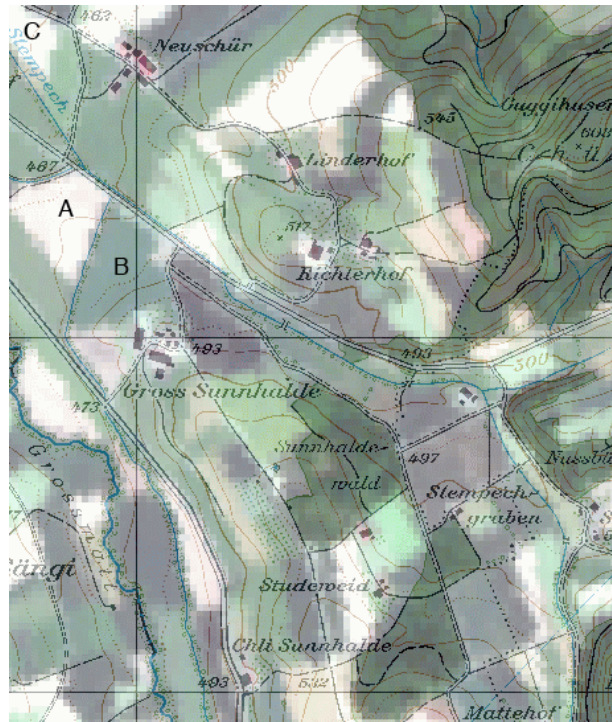

Figure 4. Geocorrected subset of the 26 May 2005 CHRIS nadir scene for the region of interest in this study with overlying pixelmap (1:25'000, © swisstopo). The investigated maize field is indicated with A, further fields with ground data B: winter barley and C: sugar beet

3. METHODS

A coupling scheme (Fig. 4) to combine two models, the RTM and CSDM, is implemented to estimate LAI based on the multi-temporal remote sensing observations (Koetz et al. 2005). The

models are first separately introduced. The coupling and LAI retrieval are subsequently presented.
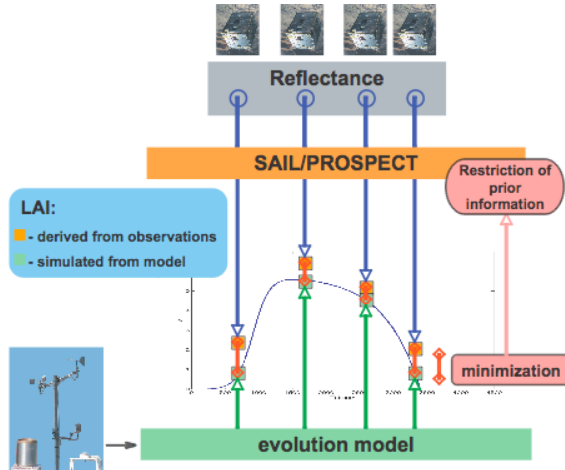

Figure 4. Concept of coupled LAI retrieval scheme (RTM & CSDM) exploiting multi-temporal CHRIS and ground meteorological observations

3.1 Canopy Structure Dynamics Model

The LAI temporal profile is governed by the net effect of growth and senescence, which are genetically programmed. However, the expression of this genetic potential is strongly influenced by environmental factors. The leaf area of an annual canopy typically shows first an exponential rise corresponding to dominant cell multiplication and elongation processes while the effects of competition for resources are limited. Then, this increasing absolute growth rate is rapidly modulated by senescence and competition for resources.

A simple semi-mechanistic model that describes LAI dynamics was proposed by (Baret 1986):

$$LAI = LAI_{Amp}\left[\frac{1}{1 + e^{-b(T-T_i)}} - e^{a(T-T_S)}\right] \quad (1)$$

The independent variable T is defined as the accumulated daily mean air temperature above 8 °C starting from sowing (Durand et al. 1982). This variable was chosen since both seedling emergence and leaf area expansion are temperature dependant (Hesketh and Warrington 1989). The CSDM describes LAI in two parts, growth and senescence (Eq. 1). The growth period is defined by a logistic equation with parameter b being the relative growth rate at the inflexion point T_i. The senescence is determined by an exponential equation with parameter a being the relative growth rate and T_s the time expressed in temperature when leaves have all senesced. The parameter LAIAmp describes the amplitude of maximal leaf area. The parameters b, T_i describe the dynamics before the time of maximum LAI, while a and T_s focus on the period after the maximum LAI.

The distribution of the model parameters typical for the observed crop maize was derived from an extensive database acquired over 44 different sites spread over the world and spanning over different climatic and cultural practices conditions (Brisson et al. 2002; Duthil et al. 1999; Marloie et al. 2001).

3.2 Radiative Transfer Models

The turbid medium radiative transfer model SAIL (Scattering from Arbitrarily Inclined Leaves (Verhoef 1984, 1985) was used, since it describes the canopy structure in a fairly simple way while producing nevertheless realistic results. The PROSPECT model (Jacquemoud and Baret 1990) was used to describe leaf optical properties. PROSPECT simulates leaf reflectance and transmittance spectra required by the SAIL model as a function of leaf biochemical contents and leaf structure.

The soil reflectance was assumed to keep the same spectral pattern and to exhibit predominantly variations in magnitude due to changes in soil moisture and roughness, which was described by a soil brightness factor s. A reference soil reflectance spectrum retrieved from CHRIS observations over bare soil of the same field, was considered for the cases investigated. The soil was also assumed to be a Lambertian surface.

3.3 LAI estimation based on Look up tables

The estimation of LAI from RTM inversion was based on a LUT (Look Up Tables) approach. It is a conceptually very simple technique, that potentially overcomes limitations of iterative optimization algorithms associated to important computation time and the risk of converging to a local minimum that is not necessary close to the actual solution (Combal et al. 2002; Kimes et al. 2000).

The generation of a look up table consists first of sampling the space of the p input variables V of the RTM (LUT_V). A total of 100000 canopy realizations have been generated following a uniform distribution and specific ranges for the respective canopy variable. Then, the RTM was used to simulate the corresponding reflectance table (LUT_R) with m numbers of measurement configurations, corresponding to the bands and directions considered. The range of each variable was defined according to previous experiments performed over maize crops under a range of conditions (Baghdadi and Baret 1998; Espana et al. 1998; Jacquemoud and Baret 1990). Note that the generation of the LUT_V allows already to define some prior information on the respective variable by restraining it to vary within a limited range.

The measurement configuration used represented the actual conditions of observations of CHRIS (Table 1). The view zenith angle was adapted to the actual view angle of the respective CHRIS nominal nadir scene. Because the illumination geometry was varying from date to date, four LUTs were created, each corresponding to a specific date of observation and the associated sun zenith angle. For the sake of simplicity, the fraction of diffuse irradiance was assumed to be 15% independent of the wavelength.

The selection of the solution within the LUT was achieved in two steps: the first one considered only the radiometric information. The second one used the CSDM fitted over the first estimates of LAI derived from the previous step to constrain the possible solutions based on the radiometric information. The later process could be iterated several times to reach convergence. The coupling of the RTM and CSDM models was based on the hypothesis that the remotely sensed observations of LAI (step 1) had to be consistent with the time profile of LAI generated by the CSDM. Consequently the remotely sensed LAI was recalibrated, when necessary, relative to the phenologically sound LAI provided by the CSDM (step 2). These two steps will be briefly described here after.

3.3.1 Step one: exploiting the radiometric information

The LUT was sorted according to the cost function χ^2_{rad} corresponding to the simple squared-sum of differences between the measured reflectance R and the simulated reflectance R_{LUT} found in the LUT_R (Eq. 2).

$$\chi^2{}_{rad} = \sum_{\lambda=1}^{11} \left(R_{LUT}^{\lambda} - R^{\lambda} \right)^2 \qquad (2)$$

The possible solutions considered were those that were within 20% of the best radiometric match. This ensemble of possible radiometric solutions was noted $[S_{rad}]$. The 20% threshold was derived after test and error trials and is consistent with what (Combal et al. 2002) proposed in an earlier study. The initial solution value, LAI_0 was then set to the median value of the ensemble $[S_{rad}]$ of best radiometric cases.

3.3.2   Step two: exploiting the prior information on canopy variables

The prior information was introduced here by refining the selection within the possible radiometric cases $[S_{rad}]$ according to the following cost function, $\chi^2{}_{var}$:

$$\chi^2{}_{var} = \sum_{i=1}^{n} \left(\frac{V_i - V_i^P}{\Delta_i^P} \right)^2 \qquad (3)$$

where n was the number of canopy characteristics on which prior information was exploited, V_i^P was the most probable value of the canopy variable i, and Δ_i^P the corresponding confidence level. The LAI value of the case ensuring the minimum of $\chi^2{}_{var}$ over $[S_{rad}]$ was selected as the solution, LAI_1. Because the LUT already incorporated some prior information on the range of variation of all the canopy variables, only the LAI and soil brightness, s, were considered in equation 3. For the soil brightness, no temporal constraint was used and V_s^P and Δ_s^P were set respectively to 0.9 and 0.2. Conversely, temporal constraints were used for LAI to get the most probable value V_{LAI}^P at a given point in time. This was achieved by exploiting the CSDM adjusted over the set of first estimates $LAI_0(t)$. The adjustment of the CSDM was performed using the simplex minimization algorithm (Nelder and Mead 1965). The confidence value Δ_{LAI}^P was assumed to be within 20% of the most probable value. This value was in agreement with the magnitude of the standard deviation observed when estimating LAI from radiometric measurements without using much prior information (Combal et al. 2002). This second step was iterated a number of times up to the convergence. Experience showed that the convergence was quickly reached and only three iterations were used in the following part of the study. At last the CSDM adjusted over the most phenologically sound cases of $[S_{rad}]$ was used to produce the final results.

4.   RESULTS AND DISCUSSION

For the evaluation of the LAI retrieval performance estimated LAI were compared to the LAI field measurements (Fig. 5). The simple root mean square error (RMSE) was calculated to quantify the agreement between actual LAI in the field and estimated values. A linear regression was used to quantify the possible biases of the estimates. The results showed a robust performance of the LAI estimation with reasonable RMSE of 0.73. Constraining the RTM inversion with dynamic prior information on the canopy structure lead to relative low inversion uncertainties. Nevertheless, when interpreting the best linear fit a consistent underestimation of the LAI estimates is evident. The LAI underestimation is probably due to the typical row structure of the maize canopy, which is not consistent with the RTM assumption of a homogenous canopy. Simulations of a 3D-RTM comparing canopy gap fraction of heterogeneous maize canopies with turbid, homogeneous canopies support this observation.   The results showed that gap fractions of a heterogeneous canopy observed from nadir were generally higher than that of a corresponding homogeneous canopy (Lopez-Lozano et al. 2007 (accepted)). Consequently

interpreting the nadir remote sensing signal of a maize canopy based on a turbid RTM would lead to an underestimation of LAI values. Forward simulation using SAIL parameterized with the field measurements lead to a significant overestimation of the near-infrared canopy reflectance relative to CHRIS observations, which further demonstrates the limitation of SAIL for the representation of maize canopy structure.

The integration of the CSDM to the retrieval algorithm allowed a continuous description of the LAI time course over the growing season. As the CSDM is capable of representing realistically the growing and senescence phases of a maize canopy the LAI values follow a phenologically sound evolution (Fig. 6). The CSDM was able to fit very well to the estimated LAI values due to the low temporal resolution of CHRIS observations.
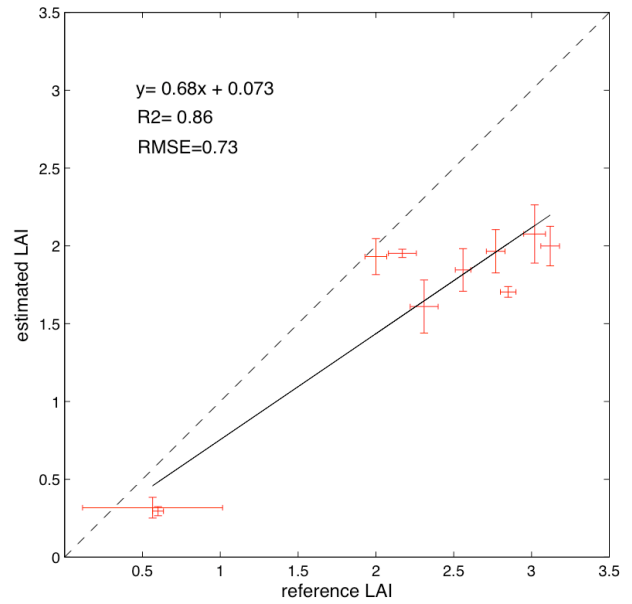

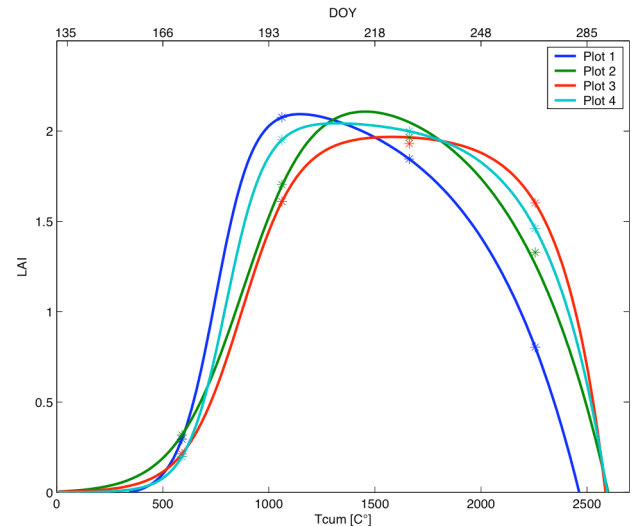Figure 5. Estimated LAI derived from CHRIS observations validated against field data

Figure 5. Continuous LAI evolution over the season described by the CSDM fitted to CHRIS observations (*).

5.   CONCLUSIONS

The presented analysis of the multi-temporal CHRIS data set focused on the interpretation of HDRF (Hemispherical Directional Reflectance Factor) changes contained in the

various acquisitions over time. A radiative transfer model (RTM) coupled to a canopy structure dynamic model (CSDM) was used to exploit the complementary content in the spectral and temporal information dimensions for the LAI estimation over a maize canopy. The knowledge of the canopy structure dynamic provided by the CSDM is used as ancillary information to achieve an improved robustness of the RTM inversion. Further, the presented coupled models integrate spaceborne remote sensing data with ground meteorological observations providing a continuous LAI time course over the season. Crop growth as well as surface process models require such a continuous description of the vegetation evolution. The presented methodology could also filter effects of poor cloud screening or atmospheric correction affecting operational derived biophysical products. Finally the future exploitation of the off-nadir view angles of CHRIS/PROBA could help to improve the clumping issue affecting the LAI estimation, as row clumping effects are most sensitive to nadir view angles (Lopez-Lozano et al. 2007 (accepted)). The simultaneous exploitation of spectro-directional behaviour of agricultural crops over time, sampled by the CHRIS/PROBA mission, bears thus the potential to improve the estimation of biophysical canopy characteristics relevant for applications such as precision agriculture and ecological modelling.

The proposed methodology prepares for the assimilation of remote sensing observations into land surface process models. Nevertheless the effectiveness of such an approach relies on remote sensing data in relatively high temporal frequency at a pertinent spatial resolution. The planned spaceborne missions Rapideye, ESA's SENTINEL2 and Venµs will hopefully provide such observations.

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