An Analysis of Polarization Coherence Tomography Using Different Function Expansion

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Abstract

In this paper we investigate the Polarization Coherence Tomography technique and propose a different function expansion to reconstruct vertical profile function. Firstly we develop a new orthogonal family and then the coefficients are estimated by matrix inversion for the specific series. Finally we represent the vertical profile function using these polynomials. Further we analyze the stability in new expansion compared to Fourier-Legendre approximation. In the end this method is validated using simulated dual-baseline data.

Keywords: PCT, dual-baseline

I. Introduction

Polarization Coherence Tomography (PCT) is an advanced approach used to reconstruct the vertical profile function in penetrable volume scattering [1], which overcome the limitation of traditional methods in 3-D SAR imaging that they suffer from an inherent requirement for collection of a large number of operational baselines. The vertical profile function can be represented by the Fourier-Legendre (F-L) polynomials if only single- baseline data is available. As a new application technology in radar remote sensing, PCT is important for improved vegetation species discrimination, component biomass analysis and biodiversity studies [2].

Assuming a priori knowledge of volume depth and ground topography, Tomography reduces to solution of a set of linear equations for the unknown coefficients using PCT technology. For single-baseline data, only two unknown coefficients can be obtained in F-L series, which limits the approximation resolution. In order to achieve higher accuracy of the estimation, we must employ multi-baseline data. In addition, while dual-baseline data can be used to introduce another two F-L polynomials, its poor condition that defines the stability of inversion and sensitivity to noise confines the application. In [3] a regularization technique has been proposed at cost to loss of precision. F-L polynomials are orthogonal on [-1,1] by weight of 1, however, in practice for various two-layer models mixed surface plus volume case, scattering amplitude is stronger on the top of canopy corresponding to volume scattering and near the ground corresponding to surface-canopy dihedral response.

Therefore, we investigate the feasibility of reconstructing the vertical profile function in a new orthogonal family in this paper. Firstly in sectionⅡwe summarize the main procedure of formulating tomography problem as a new orthogonal series expansion. Then In section Ⅲ the validity of this expansion is demonstrated using simulated dual-baseline data. Further we evaluate the stability of this approximation approach and compare it with the results in Fourier-Legendre series by Cloude in simulated scenario. Finally in section Ⅳ we provide our conclusions and discussions.

Ⅱ. PCT Using New Function Expansion

PCT is a radar image processing technology that employs function expansion to reconstruct normalized vertical profile of arbitrary polarization channel using F-L series. Here instead of generating profile in F-L series, we deduce orthogonal functions on [-1,1] by weight of x^2, the first few polynomials are shown as:

$$\begin{aligned} P_0(z) &= 1 \\ P_1(z) &= z \\ P_2(z) &= \frac{1}{2}(5z^2-3) \\ P_3(z) &= \frac{1}{2}(7z^3-5z) \\ P_4(z) &= \frac{1}{8}(63z^4-70z^2+15) \end{aligned} \tag{1}$$

Now we turn to the basic definition of interferometric complex coherence [1]:

$$\gamma = e^{ik_z z_0} \frac{\int_0^{h_v} f(z) e^{ik_z z} dz}{\int_0^{h_v} f(z) dz} \tag{2}$$

Where Z_0 is the position of the bottom of scattering layer, K_z is vertical wave number. To retrieve the vertical structure $f(z)$ in functions we first normalize the range of integral by a change of variable , results are shown as:

$$\int_0^{h_v} f(z) e^{ik_z z} dz = \frac{h_v}{2} e^{i\frac{k_z h_v}{2}} \int_{-1}^{1} g(z') e^{i\frac{k_z h_v}{2}z'} dz' \tag{3}$$

We rescale the range $g(z')=1+q(z')$ so that $q(z') \geq -1$.

Instead of expanding the function $q(z')$ directly in Legendre polynomials, we make an assumption $q(z') = a(z')z'^2$. Then the real polynomials used to approximate can be written as:

$$\begin{aligned} Q_0(z) &= z^2 \\ Q_1(z) &= z^3 \\ Q_2(z) &= \frac{1}{2}z^2(5z^2-3) \\ Q_3(z) &= \frac{1}{2}z^2(7z^3-5z) \\ Q_4(z) &= \frac{1}{8}z^2(63z^4-70z^2+15) \end{aligned} \tag{4}$$

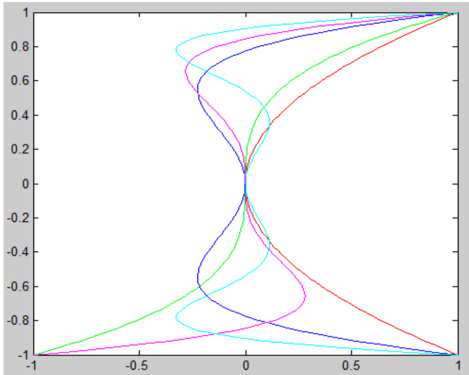


Figure 1. Plot of basis functions

For the function $a(z')$, we can develop it with series:

$$a(z') = \sum_n a_n P_n(z) \tag{5}$$

By a chain of the transformation as mentioned above coherence can be written as:

$$\begin{aligned} \tilde{\gamma} &= \frac{\int_{-1}^{1}(1+a_0P_0(z')+a_1P_1(z')+a_1P_1(z')+\ldots)z'^2 e^{ik_v z'} dz'}{\int_{-1}^{1}(1+a_0P_0(z'))z'^2 dz'} \\ &= \frac{(1+a_0)\int_{-1}^{1} z'^2 e^{ik_v z'} dz' + a_1\int_{-1}^{1} P_1(z')z'^2 e^{ik_v z'} dz' + a_2\int_{-1}^{1} P_2(z')z'^2 e^{ik_v z'} dz' + \ldots}{(1+a_0)\int_{-1}^{1} z'^2 dz'} \\ &= 3\frac{(1+a_0)f_0 + a_1 f_1 + a_2 f_2 + \ldots a_n f_n}{(1+a_0)} \\ &= 3\left(f_0 + a_{10} f_1 + a_{20} f_2 + \ldots a_{n0} f_n\right) \end{aligned} \tag{6}$$

where $k_v = \frac{h_v k_z}{2}$, $\tilde{\gamma} = \gamma e^{-ik_z z_0} e^{-ik_v}$, the unknown coefficients by zero-order term are normalized: $a_{n0} = \frac{a_n}{1+a_0}$. So Evaluation of the each component involves determination of the function f_i, which is defined from the product of interferometric wave number k_z and vegetation height h_v. The even index functions are real while odd are pure imaginary. If we can calculate the unknown coefficients a_{n0} by inverting this relation, the function of relative scattering density can be obtained as:

$$f(z)=(2z/h_v-1)^2(1+a_{10}P_1(2z/h_v-1)+a_{20}P_2(2z/h_v-1)+..a_{n0}P_n(2z/h_v-1)),\ 0\le z\le h_v.$$

As for dual-baseline data, the first five polynomials will be used to approximate. Linear formulation can be written as shown in equation (7) :

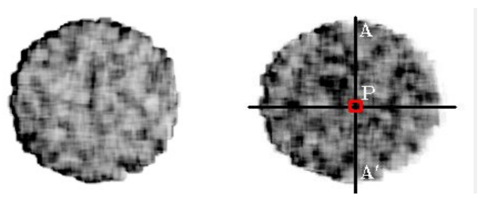
$$3\begin{bmatrix} f_1^x & 0 & f_3^x & 0 \\ 0 & f_2^x & 0 & f_4^x \\ f_1^y & 0 & f_3^y & 0 \\ 0 & f_2^y & 0 & f_4^y \end{bmatrix} \cdot \begin{bmatrix} a_{10} \\ a_{20} \\ a_{30} \\ a_{40} \end{bmatrix} = \begin{bmatrix} \mathrm{Im}ag(\tilde{\gamma}^x) \\ \mathrm{Re}al(\tilde{\gamma}^x)-3f_0^x \\ \mathrm{Im}ag(\tilde{\gamma}^y) \\ \mathrm{Re}al(\tilde{\gamma}^y)-3f_0^y \end{bmatrix} .(7)$$

where x and y are identification of two baselines. Four coefficients can be inverted using dual-baseline data.

III. Tomography Reconstruction Using Dual-Baseline POLInSAR Data in New Function Expansion

In order to demonstrate the effectiveness of the new function expansion in the generation of coherence tomography, we employ L band dual-baseline POLInSAR data simulated by ESA released POLSARPro. The forest is initialized deciduous with the height of 10m and the ground phase of 0.

Tomography of arbitrary polarization channel can be developed. We mainly investigated two representative polarization: cross polarized HV channel (volume scattering dominated) and copolarized HH-VV channel (surface dihedral response dominated). The coherence of HV and HH-VV polarization is shown in Figure 2.



HV HH-VV

Figure 2. Complex coherence (L = 11)

We retrieve forest height using the Three-Stage method, the estimated height are as shown in Figure 3. Then we employ the estimated forest height and topography phase to develop the vertical structure, which is illustrated by the vertical scattering function of one pixel P in Figure 4 and tomography along range line AA' in Figure 5.

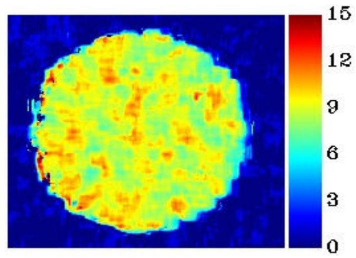


Figure 3. Estimated tree height

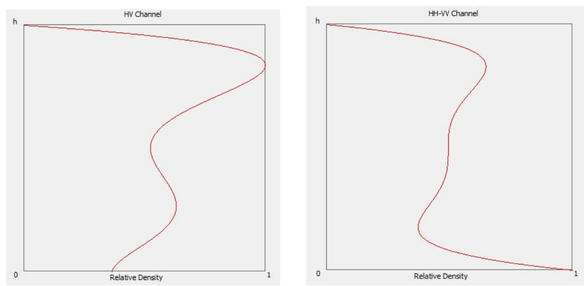


Figure 4. Profile at Point P

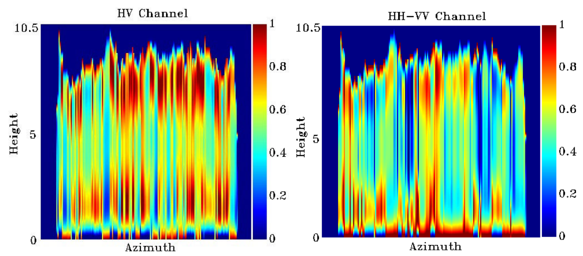


Figure 5. Polarimetric tomography along line AA'

Note that in the HV polarization channel scattering amplitude close to the crown is stronger, while in HH-VV scattering amplitude close to the ground is stronger, corresponding to the volume dominated and dihedral response dominated in practice. The reconstruction of CT is inevitably subject to the effect of noise, such as temporal decorrelation, statistical fluctuations in coherence estimation and coherence bias with limited data samples, incurring some ambiguity of the results such as the volume dominated in HH-VV and dihedral response dominated in HV shown in Figure 5. So the sensitivity to noise is a key point we should consider. From the matrix inversion formula $[F]a=g$ we find that the stability of inversion is attributed to the condition number (CN) of matrix [F]. As for single-baseline in Fourier-Legendre expansion CN can be expressed as:

$$CN = -\frac{1}{f_2} = -\frac{k_v^2}{3\cos(k_v) - (3-k_v^2)\frac{\sin(k_v)}{k_v}} \tag{8}$$

and in term of the new expansion,CN can be written as :

$$CN = -\frac{1}{f_2} = -\frac{1}{\frac{3\sin(k_v)}{k_v} + \frac{21\cos(k_v)}{k_v^2} - \frac{81\sin(k_v)}{k_v^3} - \frac{180\cos(k_v)}{k_v^4} + \frac{180\sin(k_v)}{k_v^5}} \tag{9}$$

We draw the two function curves in the same coordinates in Figure 6 and find that values of CN in the approximation method of this paper are smaller than in Fourier-Legendre by Cloude. That is to say, inversion is better conditioned and system is less sensitive to errors. In order to validate the advantage of this approximation, we compare the standard variance of coefficients using dual-baseline data above in two expansion method in TableⅠ, from which it can be seen that std in the new expansion is less than that in Fourier-Legendre series.

TableⅠ. Standard variance of profile coefficients for HV channel in two function expansion

	Fourier-Legendre	New Expansion
a_{10}	0.42	0.23
a_{20}	0.85	0.66
a_{30}	1.54	1.25
a_{40}	13.20	0.003

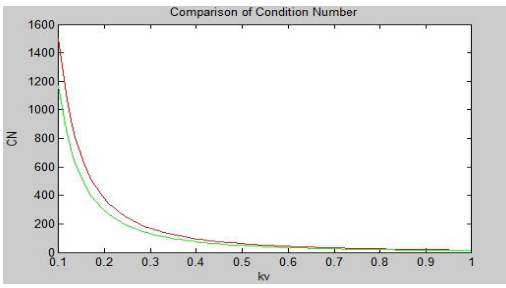


Figure 6. Comparison of condition number: Red (equation (8)), Green(equation (9))

Ⅳ.Conclusion

In this paper we have employed PCT technology to reconstruct the vertical profile function of forest and introduced a new function expansion approach. This method has been validated using POLSARPro simulated dual-baseline data. Since dual-baseline provides two complex coherences, four coefficients can be estimated and the highest polynomial order used to approximate is four in F-L expansion. In the new expansion, however, the highest order is six and the results can exhibit the internal fine structure. In addition, due to better conditioning of the matrix inversion, the measurement vector a is less susceptible to noise, which is always present in synthetic aperture radar interference processing.

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References

[1] S. R.Cloude,“Polarization coherence tomography”, Radio Sciences, 41:1-27,2006

[2] Mette, T., “Forest Biomass Estimation from Polarimetric SAR Interferometry”, DLR Research Report 207-10, ISSN 1434-8454, 2007

[3] S. R. Cloude, “Dual-Baseline Coherence Tomography”, IEEE Geoscience and Remote Sensing Letters, 4(1):127-131, 2007

[4] Papathanassiou, K. P., and S. R. Cloude, “Single baseline polarimetric SAR interferometry”, IEEE Trans. Geosci. Remote Sensing., 39, 2352-2363, 2001

[5] S. R. Cloude, and Papathanassiou, K. P., “Forest Vertical Structure Estimation Using Coherence Tomography”, IGARSS2008,2008.