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Spectral Separability-Using Non-Parametric statistical tests, with special reference to Kolmogrov - Smirinov two tailed test

by

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Abstract

The spectral separability studies are generally carried out on the basis of histogram outputs to select suitable bands to achieve maximum classification accuracy. The objective of this study is to select suitable statistical tests to test the hypothesis whether the distributions of spectral values in different classes represent the same population and the consequent impact of rejection of such a hypothesis on the separability. In testing, normality is assumed in these samples but the histogram outputs showed deviation from normality. The application of non-parametric tests which have no valid assumption have been suggested. The Kolmogrov-Smirinov two tailed test is discussed and the maximum difference statistic S(x)-F (x) = Max/D/ indicated the degree of separability.

Introduction:

The spectral response of the objects on the surface of earth varies in different parts of spectrum. The Modular Multispectral scanner, presently in use, has the capability to collect spectral values in 11 channels including the thermal channel. The variation in spectral behaviour provides a scope for spectral separability among different objects. The spectral separability studies are generally carried out to select band or combination of bands, to achieve maximum classification accuracy and optimization of computer time. The spectral separability studies, at present, are carried outon the basis of histogram outputs of spectral values of some pre-determined cover types in different bands. These outputs, which are normally available on transparent sheets, are compared (two at a time) by superimposing one over the other on a light table and the degree of separability in each band is evaluated and recorded in the form of matrix. The assessment of the degree of separation and its coding is left to the interpreter and remains a matter of subjectivity of the interpreter. In the present case, a code 1, was adopted for completely overlapping to 25 parcent separation, a code 2 for 25 percent to 75 percent

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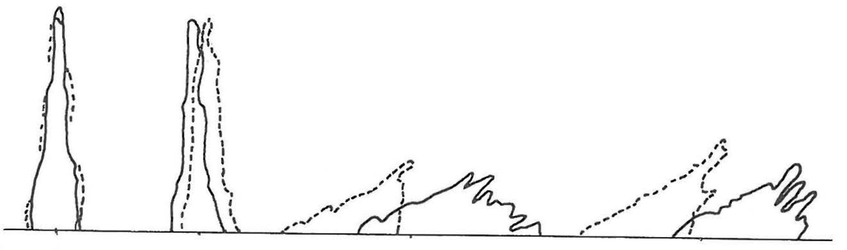
separation, code 3 for 75 percent to 99 percent separation and 4 for 100 percent separation as illustrated in the example given below:



Experience has shown that for a given cover type, some particular bands provide good separability. However visual comparison of the histograms, involves considerable computer time and the method still remains crude.

Of the selected bands, band combinations of two and more can be considered for achieving better separability. Combination of two bands at a time could be visualised in three dimensions space. For this purpose, it is assumed that the spectral values in each band are normally distributed and a combination of two such bands provide a bivariate normal distribution and determine a space for a specific covertype. A close quarter examination of these spectral value distribution of some specific types showed non-normality and have been tested for the goodness of fit by using X^2 distribution. The computed X^2 value differed significantly from the critical value at a given probability level indicating deviation from normality.

The present study is undertaken to investigate, whether the statistical comparison of two sample distributions of spectral values representing different cover types has any correlation with the separability. Whether it is possible to assess the degree of separability from any of these methods, dispensing with the histogram comparison, without any assumption of normality.

For this purpose, two test sites representing different forest types have been considered having the spectral separability indices 2,3,4 in different bands as explained in the first paragraph. Since the shifting of means has a considerable influence on the separability, it was intended to test whether the means of spectral values differ significantly.

The students 't' test is used for this purpose; To use this test, it is assumed that the samples are drawn from normal population and the population variance is same for both samples. The critical value of 't' at different probability levels is the same as in the case of normal distribution since the sample sizes exceeds 30.

By this it is evident that the critical value depends mostly on the sizes of samples under test i.e. critical values decrease with the increase in sample sizes.

The absolute maximum difference is calculated by preparing a cumulative frequency table for the sample distributions and standardizing the cumulative frequency by dividing by nl, n^2 as the case may be. However, the max. difference in this case was computed on Univac 1100 computer by using statistical package for social sciences (SPSS) system.

The table below provides the absolute maximum difference values for different test sites representing separability indices 2,3,4

Separability index	Absolute Maximum Difference	Significance
2	0.5181	Significant
2	0.4578	"
3	0.8338	"
4	0.9500	"

From the table it could easily be inferred that all the maximum difference values are highly significant even at separability index 2. The sample distributions are significantly differing from each other indicating the rejections of hypothesis that they come from the same population.

As it was already pointed out, the critical value attains higher values when the sample sizes decrease. A detailed study was carried out to study the behaviour of critical values and consequent significance by reducing the sample size by randomly selecting sub-samples at different intensities. The maximum differences arrived at, are incorporated in the table below for a test case with a xseparability index 3.

Sampling intensity (%)	Separability index 3 Case 1,	2,	3
5	.8463	.8093	.8649
10	.8754	.8312	.8792
20	.8649	.8214	.9011
100	.9189	.9461	.9471

As seen from the above table, the maximum difference values recorded a change upto 15% which is attributed to random variations inherent in the population. Though there is no substantial increase in the maximum difference, the critical value changes considerably with the change in the sample sizes. The table below gives an idea of the change in cirtical value at 5% significance level with the change in sample sizes.

The statistic $t = \frac{X_1 - X_2}{Sdm}$

is distributed as 't' distribution and attains normality beyond a sample size of 30.

X_1, X_2 are means of samples 1 and 2 under comparison

Sdm: Pooled standard deviation after affecting the correction for normality.

The table below indicates 't' values for different separability indices:

Separability index	't' value	Significance at 5% level
2	0.90	Not significant
2	0.73	"
3	2.03	Significant
4	2.04	"

(The critical value of 't' at 5% significance level is 1.96 for sample sizes $n_1 = 100$, $n_2 = 100$

Conspicuously, the 't' values for a separability index of '2' are not **significant** indicating that it is possible that the two sample distributions of spectral values represent the same population. For separability index 3 and beyond the 't' values are significant and there is a reason to believe that they represent different populations. However the 't' value is on the increase with the increase in separability. The X^2 test, which is used to test goodness of fit of a sample to the theoretical normal distribution showed a similar trend. Though the 't' test considers the most relevant parameters like mean, and variance and provides a good comparison, it was felt that assumption of normality in every case may lead to incorrectness. To avoid this it was proposed to make use of Non-parametric tests.

Though there are quite a few non-parametric tests, they are generally based on assigned ranks to the actual values. Moreover they are designed for small size samples. The only parametric test which is free from all these obstacles is the Kolmogrov-Smirinov test. In this test a comparison of cumulative frequencies of two sample distributions is made, step by step, and the maximum difference is computed.

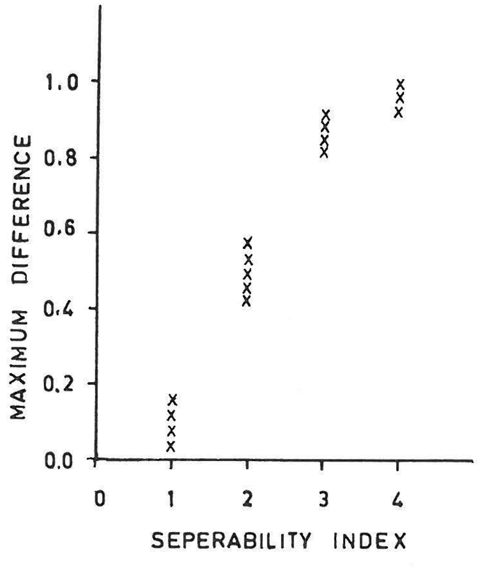
$S(x) - F(x) = \text{Max } /D/$ and $/D/$ is proved to be independent of the sample distributions under consideration. The K-S one tailed test can be used to test whether a given sample is drawn from a given population. While the two-tailed test is useful in testing whether two given sample distributions represent the same population. The critical values are available in the standard statistical tables. For 95% probability the critical value is

$$1.37 \sqrt{\frac{n1 + n2}{n1 \times n2}}$$

n1 \ n2	10	25	50	75	100
10	.6096	.5206	.4713	.4617	.4548
25		.3699	.3356	.3165	.3082
50			.2740	.2507	.2370
75				.2233	.2096
100					.1932

From this test, like the 't' test no inference could be drawn about the separability. This test is very sensitive as the results indicate the presence of significance in maximum difference with separability index 2. The increase or decrease in the sample size did not show substantial effect on the maximum difference.

Viewing at the maximum difference values between pairs of samples, the pre-determined separability indices show a good correlation with maximum difference values. The maximum difference is on the increase with the increase in separability This tendency indicates a possibility to arrive at the degree of separability by examining the maximum difference values. This method, as seen from the results indicates the degree of separability in terms of percentage instead of broad classes and removes the subjectivity and bias. The graph below indicates the correlation between the separability index and maximum difference which have been computed for some cases.



Conclusion

Statistical inference is based on testing of a preset hypothesis at a desired probability level. Depending upon the significance of test values the hypothesis is either accepted or rejected. The tests only indicate whether two given samples differ significantly or not. However, it is difficult to infer on the degree of separability. The test of mean, by using students 't' distribution and Kolmogrov-Smirinov test, where the cumulative frequency is compared step by step are most relevant tests. Comparatively K-S, two tailed test is more sensitive and shows significant difference even at separability index 1 occasionally. However the maximum difference statistic used in K-S test indicated the degree of separability and would be a helping tool for evaluating initial separability.

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