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THEORY AND EXPERIENCES OF POST-CALIBRATION

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Abstract

The theory of the post-calibration is demonstrated. The post-calibration is a method for determining the stereomodel systematic deformation using the discrepancies between the tie points of the stereomodels obtained after the block adjustment. Some practical results are shown.

1 - Foreword.

When practical works of independent models block adjustment are performed one can obviously infer that the main source of information about the systematic errors is in the discrepancies which after the block adjustment become evident between the tie points of adjacent stereomodels of the same strip. Actually information on the systematic errors cannot be inferred from the residuals on the control points because they are conveniently spread in the block aerea and when large weights are used for the control points equations the residuals become very small and meaningless; usually none of the stereomodels of the block is supplied with a sufficient number of control points which enables the determination of the systematic deformation and even when this happens one can suspect of having determined the systematic deformation of a specific stereomodel and not the systematic deformation of all the stereomodels or consistent portion of stereomodels of the block.

Nevertheless the presence of systematic deformations of the stereomodels can be detected when the absolute orientation of each strip is performed; in the strips where many control points are located the systematic behaviours of the residuals on the control points become evident whereas in the other strips the behaviour of the systematic errors can be detected on the discrepancies of the tie points between two strips; in this last case the variations of the behaviours became particularly evident.

When the strips have a sufficient number of control points in the first and the last stereomodel which allows a precise absolute orientation of these two stereomodels global information on the stereomodels deformation could be inferred from the closing errors ΔK, $\Delta\Phi$, $\Delta\Omega$, $\Delta\lambda$, TX, TY, TZ, but relationships between these closing errors and errors of the stereomodels coordinates have not yet been determined.

However the above mentioned ways of detecting the effects of the stereomodel systematic deformation could be used mainly for assessing the validity of methods for the correction of the systematic errors.

2 - Outline of the post-calibration procedure.

In the analytical formation of a stereomodel the measuring operation is determined by the stereo pointing of image points and by a computation; then the assumption can be made that the deformation of the stereomodel depends essentiably on the systematic errors of the photocoordinates; to be noted that the deformation of a stereomodel observed in an analogue instrument dependes also in some extent on the judgement of the operator which determines the residual y-parallaxes.

The 9 image points which are usually taken into account on a photograph for the relative orientation and for the tie points of two adjacent stereomodels are usually located approwimately in the same positions as those shown in fig. 1; in an analytical stereomodel the deformation depends essentially on the systematic errors in these 9 image points.

These systematic errors can be considered in the two components v and p where the first index is s = left, m = middle, d = right and the second index is 1 = high, 1 = nadiral, 3 = low.

These errors are small so that the total deformation can be considered as the sum of the two independent deformations, one given by the v error components and the other given by the p error components.

The v error components will cause y-parallax errors and consequently systematic errors in the five relative orientation parameters and a systematic stereomodel deformation which depends only on the five errors of the orientation parameters.

The p error components will cause x-parallax errors and consequently errors in the difference of height between the e.p and the low stereomodel points, which determine errors both in the heights and in the planimetric coordinates (see equations (4) and (5)).

It is obvious that these errors will cause discrepancies between the tie points because in the first of two adjacent stereomodels the errors depend on the middle and right points of the first photograph and on the left and middle points of the second photograph, whereas in the second stereomodel the errors depend on the middle and right points of the second photograph and on the left and middle points of the third photograph.

2-2-The possibility of obtaining information about the above mentioned systematic errors through the knowledge of the discrepancies DX, DY, DZ between the stereomodels tie points after the block adjustment can be assessed by a theoretical analysis and by practical experiments.

For the theoretical analysis it is necessary to define the mathematical model of the systematic deformation of a stereomodel and then the mathematical model of the discrepancies DX, DY, DZ between the tie points of two adjacent stereomodels.

Equations can be established having as unknowns the parameters which determine the stereomodel deformation and as known terms the discrepancies DX, DY, DZ after block adjustment.

This procedure has been called "post-calibration" because the information on the systematic errors on the photographs are inferred after the block adjustment; of course after the computation of the deformation parameters the stereomodels coordinates can be corrected and the adjustment can be repeated.

3 - Mathematical model of the systematic errors of a stereomodel

3-1-The topic dealt with in this paragraph is as old as the Pho

togrammetry but here it is formulated in a way suitable for determining the following mathematical model.
First of all the systematic errors of a stereomodel which depend on the errors of the relative orientation parameters are taken into consideration; in this connection the classic simmetrical relative orientation is considered with the orientation parameters φ_1, K_1, φ_2, ω_2, K_2 and only the sources of errors which affect sistematically these parameters are taken into consideration.
Some formulas have to be recalled.
Let us denote with P(X,Y,Z) a whatever tie point along the nadiral line of the exposure point (e.p) O_i. The direction tangents tan θ_x, tan θ_y which define the rays of the bundles can be denoted with m_i and n_i where the index pertains to the e.p. (fig. 2).
It is well known that the direction tangents are a functions of the photo-coordinates and of the photo orientation parameters and that

$$(1) \qquad m_i = \frac{X - X_i}{Z - Z_i} \qquad\qquad n_i = \frac{Y - Y_i}{Z - Z_i} \quad ;$$

when the photograph j undergoes rotations the direction tangents depend on $\Delta\varphi_j$, $\Delta\omega_j$, ΔK_j as it follows

$$(2) \qquad m'_i = \frac{|m_i \ n_i \ 1| \ M_{xj}}{|m_i \ n_i \ 1| \ M_{zj}} \qquad\qquad n'_i = \frac{|m_i \ n_i \ 1| \ M_{yj}}{|m_i \ n_i \ 1| \ M_{zj}}$$

where M_{xj}, M_{yj}, M_{zj} are the columns of the orientation matrix M_j ($\Delta\omega$ = first rot., $\Delta\varphi$ = second rot., ΔK = third rot.).
It is easy to find the derivatives of m_i and n_i with respect to $\Delta\varphi_j$, $\Delta\omega_j$, ΔK_j

$$(3) \qquad \begin{aligned} &\frac{\delta m_i}{\delta\Delta\varphi_j} = -(1 + m_i^2); \quad \frac{\delta m_i}{\delta\Delta\omega_j} = -m_i n_i; \quad \frac{\delta m_i}{\delta\Delta K_j} = n_i \\ &\frac{\delta n_i}{\delta\Delta\varphi_j} = -m_i n_i; \quad \frac{\delta n_i}{\delta\Delta\omega_j} = -(1 + n_i^2); \quad \frac{\delta n_i}{\delta_{\Delta K_j}} = -m_i \end{aligned}$$

The coordinates of the tie point P in the stereomodel $O_{i-1}O_i$ are

$$Z = Z_{i-1} + \frac{X_i - X_{i-1}}{m_{i-1} - m_i} = Z_{i-1} + h_{i-1}$$

$$(4) \qquad X = X_{i-1} + m_{i-1} \cdot h_{i-1}$$

$$Y = Y_{i-1} + n_{i-1} \cdot h_{i-1}$$

and the coordinates of the same point in the stereomodel $O_i O_{i+1}$ are

$$Z = Z_i + \frac{X_{i+1} - X_i}{m_i - m_{i+1}} = Z_i + h_i$$

$$(5) \qquad X = X_i + m_i \cdot h_i$$

$$Y = Y_i + n_i \cdot h_i$$

If all the stereomodels are affected by the same sistematic errors φ_1, K_1, φ_2, ω_2, K_2 the coordinates (4) and the coordinates (5) are affected in a different way because of the different values of the direction tangents and their derivatives.
Denoting with X° Y° Z° the values of the coordinates not affected by the sistematic errors, with truncated expansion in series (φ, ω, ... are small enough for neglecting the higher order terms), for both the groups of formulas (4) and (5) we have

$$Z - Z° = \frac{\delta Z}{\delta \varphi_1} \varphi_1 + \frac{\delta Z}{\delta K_1} K_1 + \frac{\delta Z}{\delta \varphi_2} \varphi_2 + \frac{\delta Z}{\delta \omega_2} \omega_2 + \frac{\delta Z}{\delta K_2} K_2$$

$$X - X° = \frac{\delta X}{\delta \varphi_1} \varphi_1 + \frac{\delta X}{\delta K_1} K_1 + \frac{\delta X}{\delta \varphi_2} \varphi_2 + \frac{\delta X}{\delta \omega_2} \omega_2 + \frac{\delta X}{\delta K_2} K_2$$

$$Y - Y° = \frac{\delta Y}{\delta \varphi_1} \varphi_1 + \frac{\delta Y}{\delta K_1} K_1 + \frac{\delta Y}{\delta \varphi_2} \varphi_2 + \frac{\delta Y}{\delta \omega_2} \omega_2 + \frac{\delta Y}{\delta K_2} K_2$$

It follows from (4) and (3), that is for the coordinates in the stereomodel $O_{i-1}O_i$, taking into account that the variations φ_1, K_1 affect m_{i-1} and n_{i-1} meanwhile the variations φ_2, ω_2, K_2 affect m_i and n_i

$$\frac{\delta Z}{\delta \varphi_1} = \frac{\delta h_{i-1}}{\delta \varphi_1} = -\frac{X_i - X_{i-1}}{(m_{i-1}-m_i)^2}\frac{\delta m_{i-1}}{\delta \varphi_1} = \frac{h^2_{i-1}}{X_i - X_{i-1}}\,(1 + m^2_{i-1})$$

$$\frac{\delta Z}{\delta K_1} = -\frac{h^2_{i-1}}{X_i - X_{i-1}}\,n_{i-1}$$

(6)

$$\frac{\delta Z}{\delta \varphi_2} = \frac{\delta h_{i-1}}{\delta \varphi_2} = \frac{X_i - X_{i-1}}{(m_{i-1}-m_i)^2}\cdot\frac{\delta m_i}{\delta \varphi_2} = -\frac{h^2_{i-1}}{X_i - X_{i-1}}\,(1 + m^2_i)$$

$$\frac{\delta Z}{\delta \omega_2} = -\frac{h^2_{i-1}}{X_i - X_{i-1}}\,m_i n_i$$

$$\frac{\delta Z}{\delta K_2} = \frac{h^2_{i-1}}{X_i - X_{i-1}}\,n_i$$

For the coordinate X:

$$\frac{\delta X}{\delta \varphi_1} = h_{i-1}\cdot\frac{\delta m_{i-1}}{\delta \varphi_1} + m_{i-1}\,\frac{\delta h_{i-1}}{\delta \varphi_1} =$$

$$= -\,h_{i-1}\,(1+m^2_{i-1}) + m_{i-1}\,(1+m^2_{i-1})\,\frac{h^2_{i-1}}{X_i - X_{i-1}}$$

and likewise

$$\frac{\delta X}{\delta K_1} = h_{i-1}\cdot n_{i-1} - m_{i-1}\cdot n_{i-1}\,\frac{h^2_{i-1}}{X_i - X_{i-1}}$$

$$\frac{\delta X}{\delta \varphi_2} = m_{i-1}\,\frac{\delta h_{i-1}}{\delta \varphi_2} = -m_{i-1}\cdot(1+m^2_i)\,\frac{h^2_{i-1}}{X_i - X_{i-1}}$$

(7)

$$\frac{\delta X}{\delta \omega_2} = -m_{i-1}\cdot m_i\cdot n_i\,\frac{h^2_i}{X_i - X_{i-1}}$$

$$\frac{\delta X}{\delta K_2} = m_{i-1}\cdot n_i\,\frac{h^2_{i-1}}{X_{i-1} - X_i}$$

Finally

$$\frac{\delta Y}{\delta\varphi_1} = -m_{i-1}\cdot n_{i-1}\cdot h_{i-1} + n_{i-1}(1+m^2_{i-1})\,\frac{h^2_{i-1}}{X_i-X_{i-1}}$$

$$\frac{\delta Y}{\delta K_1} = -m_i\ h_{i-1} - n^2_{i-1}\,\frac{h^2_{i-1}}{X_i-X_{i-1}}$$

(8)
$$\frac{\delta Y}{\delta\varphi_2} = -n_{i-1}\ (1+m_i^2)\,\frac{h^2_{i-1}}{X_i-X_{i-1}}$$

$$\frac{\delta Y}{\delta\omega_2} = -n_{i-1}\cdot\ m_i\cdot\ n_i\,\frac{h^2_{i-1}}{X_i-X_{i-1}}$$

$$\frac{\delta Y}{\delta K_2} = n_{i-1}\cdot n_i\,\frac{h^2_{i-1}}{X_i-X_{i-1}}$$

For the derivatives of coordinates X, Y, Z of the same tie point P when considered in the stereomodel O_iO_{i+1} the same relationships are valid provided that the indexes i and i-1 are substituted respectively with i+1 and i.

3-2-Now the systematic errors produced by the p components are determined; the x photocoordinates are denoted as

x_s = x coord. on the left side of the photo.

x_m = x coord. on the center of the photo.

x_d = x coord. on the right side of the photo.

and the indexes 1, 2, 3 are left out because the relationships are the same concerning the y position of the image points.
It is useless to denote the photograph where the point having a coordinate x and the error p is located because when dealing with the tie points corresponding to the exposure O_i (i.e the tie points between the model $O_{i-1}O_i$ and the model O_iO_{i+1}) the index s (left) belongs to the image points on the photograph O_{i+1}, the index m (middle) belongs to the image points of the photo. O_i and the index d(right) belongs to the image points of the photo. O_{i-1}.

The X, Y, Z obtained from the eq. (4) are denoted as X', Y', Z' and those obtained from the eq. (5) are denoted as X", Y", Z".
In determining the coefficients of equations where the deformation parameters are considered as unknowns small errors can be accepted and since the angular orientation parameters of the bundles do not exceed a few degrees it can be assumed

$$\frac{\delta m_{i-1}}{\delta x_d} \cong \frac{\delta m_i}{\delta x_m} \cong \frac{\delta m_{i+1}}{\delta x_s} \cong \frac{1}{f}$$

$$\frac{\delta n_{i-1}}{\delta x_d} \cong \frac{\delta n_i}{\delta x_m} \cong \frac{\delta n_{i+1}}{\delta x_s} \cong 0$$

The errors in a tie point P considered in the stereomodel $O_{i-1}O_i$ and in the stereomodel O_iO_{i+1} are

$$Z'-Z^\circ = \frac{\delta Z'}{\delta x_d} p_d + \frac{\delta Z'}{\delta x_m} p_m$$

$$Z''-Z^\circ = \frac{\delta Z''}{\delta x_m} p_m + \frac{\delta Z''}{\delta x_s} p_s$$

$$X'-X^\circ = \frac{\delta X'}{\delta x_d} p_d + \frac{\delta X'}{\delta x_m} p_m$$

$$X''-X^\circ = \frac{\delta X''}{\delta x_m} p_m + \frac{\delta X''}{\delta x_s} p_s$$

$$Y'-Y^\circ = \frac{\delta Y'}{\delta x_d} p_d + \frac{\delta Y'}{\delta x_m} p_m$$

$$Y''-Y^\circ = \frac{\delta Y''}{\delta x_m} p_m + \frac{\delta Y''}{\delta x_s} p_s$$

Put

$$t_{i-1} = \frac{h_{i-1}^2}{X_i - X_{i-1}} \qquad t_i = \frac{h_i^2}{X_{i+1} - X_i}$$

for writing semplification and recalled the eq. of para. 3-1 the following errors equations are obtained

$$Z'-Z^\circ=\frac{\delta h_{i-1}}{\delta m_{i-1}}\,\frac{\delta m_{i-1}}{\delta x_d}p_d+\frac{\delta h_i}{\delta m_i}\,\frac{\delta m_i}{\delta x_m}p_m \;=\; \frac{1}{f}t_{i-1}(p_d-p_m)$$

$$Z''-Z^\circ=\frac{\delta h_i}{\delta m_i}\,\frac{\delta m_i}{\delta x_m}p_m+\frac{\delta h_i}{\delta m_{i+1}}\,\frac{\delta m_{i+1}}{\delta x_s}p_s \;=\; -\frac{1}{f}t_i(p_s-p_m)$$

$$X'-X^\circ=(m_{i-1}\frac{\delta h_{i-1}}{\delta m_{i-1}}+h_{i-1})\,\frac{\delta m_{i-1}}{\delta x_d}p_d+m_{i-1}\,\frac{\delta h_{i-1}}{\delta m_i}\,\frac{\delta m_i}{\delta x_m}\,p_m \;=$$

(9) $$= -\frac{1}{f}m_{i-1}t_{i-1}(p_d-p_m)-\frac{1}{f}h_{i-1}p_d$$

$$X''-X^\circ=(m_i\frac{\delta h_i}{\delta m_i}+h_i)\frac{\delta m_i}{\delta x_m}p_m+m_i\frac{\delta h_i}{\delta m_{i+1}}\,\frac{\delta m_{i+1}}{\delta x_s}p_s=$$

$$=-\frac{1}{f}m_i t_i(p_s-p_m)-\frac{1}{f}h_i p_m$$

$$Y'-Y^\circ=n_{i-1}\frac{\delta h_{i-1}}{\delta m_{i-1}}\,\frac{\delta m_{i-1}}{\delta x_d}p_d+n_{i-1}\,\frac{\delta h_{i-1}}{\delta m_i}\,\frac{\delta m_i}{\delta x_m}p_m=\frac{1}{f}n_{i-1}t_{i-1}(p_d-p_m)$$

$$Y''-Y^\circ=n_i\frac{\delta h_i}{\delta m_i}\,\frac{\delta m_i}{\delta x_m}p_m+n_i\frac{\delta h_i}{\delta m_{i+1}}\,\frac{\delta m_{i+1}}{\delta x_s}p_s=-\frac{1}{f}n_i t_i(p_s-p_m)$$

4 - Mathematical model of the discrepancies between two stereomodels.

4-1-The deformation parameters relative to the v components of the photocoordinates systematic error are the 5 errors of the relative orientation φ_1, K_1, φ_2, ω_2, K_2; the effect of the 9 errors v in the 9 positions of fig. 1 is always that of producing the above mentioned 5 orientation errors and then well defined model deformations; in other words no assumption has to be made on the values of the components v. This is not true for the 9 components p because they produce localized errors which depend on the specific values of the components p; the unknown parameters which should be determined are the 9 components p; but it is not possible to obtain all these 9 components as it can be inferred when the equations of the discrepancies have been determined and discussed.

The discrepancy between the two coordinates of a tie point is obtained by subtracting the second deformation error from the first one, that is for example

$$DZ = Z'-Z^\circ-(Z''-Z^\circ) = Z'-Z''$$

It follows

$$DX = \frac{1}{f}m_{i-1}t_{i-1}(p_d-p_m)+\frac{1}{f}m_i t_i(p_s-p_m)-\frac{1}{f}h_i(p_d-p_m)$$

$$(10)\quad DY = \frac{1}{f}n_{i-1}t_{i-1}(p_d-p_m)+\frac{1}{f}n_i t_i(p_s-p_m)$$

$$DZ = \frac{1}{f}t_{i-1}(p_d-p_m)+\frac{1}{f}t_i(p_s-p_m)$$

Some preliminary conclusions can be drawn from the equations (10).
The term

$$\frac{1}{f}h_i(p_d - p_m)$$

has been approximated from the term

$$\frac{1}{f}h_{i-1}p_d - \frac{1}{f}h_i p_m$$

and is the only planimetric discrepancy which depends on the variations of m_{i-1} and m_i; the other terms of DX, DY are the planimetric discrepancies which depend on the two height variations

$$\frac{1}{f}t_{i-1}(p_d-p_m) \text{ and } \frac{1}{f}t_i(p_s-p_m)$$

contained in the DZ expression.
From the equations (10) it can be inferred that the values of the central errors p_m cannot be separated from the lateral errors p_s and p_d because the coefficient of p_m is always the same as the coefficient of p_s or p_d.
The equation DX allows a confident computation of $p_d - p_m$ because the coefficient $\frac{1}{f}m_i t_i$ of p_s-p_m is near to be zero and different from the coefficient of p_d-p_m.
In the equations DY and DZ the errors p_d-p_m and p_s-p_m have practically the same coefficients; this means that the values of the discrepancies depend essentially on the dissymetry of the values p_d-p_m and p_s-p_m; when $p_d-p_m=-(p_s-p_m)$ no DY or DZ can be produced by the p components of the errors; the separate evaluation of p_d-p_m and p_s-p_m is entrusted to the use of the equation DX.

The equation DY can be eliminated because the values of n_{i-1} and n_i are practically the same for the tie points in the two stereomodels; the variations of the heights in the points 1 and 3 produce proportional variations of DY whereas in the point 1 the variation of DY is practically zero.
In conclusion the unknowns which can be determined are

$$
\begin{array}{ll}
p_{d_1}-p_{m_1} & p_{s_1}-p_{m_1} \\
p_{d_2}-p_{m_2} & p_{s_2}-p_{m_2} \\
p_{d_3}-p_{m_3} & p_{s_3}-p_{m_3}
\end{array}
$$

4-2-The equations which can be established for the computation of these 6 unknowns have to be considered together with the equations which determine the discrepancies produced by the component v.
In conclusion the mathematical model for the discrepancies DZ is

$$DZ=a_z\varphi_1+b_z k_1+c_z\varphi_2+d_z\omega_2+e_z k_2+f_z(p_d-p_m)+g_z(p_s-p_m) \tag{11}$$

where

$$a_z = t_{i-1}\,(1+m_{i-1}^2) - t_i\,(1+m_i^2)$$

$$b_z = -t_{i-1}\,n_{i-1} + t_i\,n_i$$

$$c_z = -t_{i-1}\,(1+m_i^2) + t_i\,(1+m_{i+1}^2)$$

$$d_z = -t_{i-1}\,m_i\,n_i + t_i\,m_{i+1}\,n_{i+1}$$

$$e_z = t_{i-1}\,n_i - t_i\,n_{i+1}$$

$$f_z = \frac{1}{f}\,t_{i-1}$$

$$g_z = \frac{1}{f}\,t_i$$

The mathematical model for the discrepancies DX is

$$(11') \quad DX=a_x\varphi_1+b_x\kappa_1+c_x\varphi_2+d_x\omega_2+e_x\kappa_2+f_x(p_d-p_m)+g_x(p_s-p_m)$$

where

$$a_x = -h_{i-1}(1+m^2_{i-1})+t_{i-1}m_{i-1}(1+m^2_{i-1})+h_i(1+m^2_i)+ + t_i m_i(1+m^2_i)$$

$$b_x = h_{i-1}n_{i-1}-t_{i-1}m_{i-1}n_{i-1}-h_i n_i+t_i m_i n_i$$

$$c_x = -t_{i-1}m_{i-1}(1+m^2_i) + t_i m_i(1+m^2_{i+1})$$

$$d_x = -t_{i-1}m_{i-1}m_i n_i + t_i m_i m_{i+1}n_{i+1}$$

$$e_x = t_{i-1}m_{i-1}n_i - t_i m_i n_{i+1}$$

$$f_x = \frac{1}{f}m_{i-1}t_{i-1}-\frac{1}{f}$$

$$g_x = \frac{1}{f}m_i t_i$$

The mathematical model for the discrepancies DY is

$$(11'') \quad DY = a_y\varphi_1 + b_y K_1 + c_y\varphi_2 + d_y\omega_2 + e_y K_2$$

where

$$a_y = -h_{i-1}m_{i-1}n_{i-1}+t_{i-1}n_{i-1}(1+m^2_{i-1})+h_i m_i n_i-t_i n_i(1+m^2_i)$$

$$b_y = -h_{i-1}m_{i-1}-t_{i-1}n^2_{i-1}+h_i m_i+t_i n^2_i$$

$$c_y = -t_{i-1}n_{i-1}(1+m^2_i)+t_i n_i(1+m^2_{i+1})$$

$$d_y = -t_{i-1}n_{i-1}m_i n_i+t_i n_i m_{i+1}n_{i+1}$$

$$e_y = t_{i-1}n_{i-1}n_i-t_i n_i n_{i+1}$$

5 - Some considerations on the use of the mathematical model of the tie points discrepancies.

5-1-The equations (11) can be used to the computation of the five unknowns φ_1, K_1, φ_2, ω_2, K_2 and of the six unknowns $p_{s1}-p_{m1}$, $p_{s2}-p_{m2}$, $p_{s3}-p_{m3}$, $p_{d1}-p_{m1}$, $p_{d2}-p_{m2}$, $p_{d3}-p_{m3}$ because after the block adjustment the discrepancies DX, DY, DZ are available and the coefficients a_x, b_x, ... a_y, b_y, ... a_z, b_z can easely be computed.
Obviously for eleven unknowns at least eleven equations must be established, but this is not possible when three tie points are used and only nine values DX, DY, DZ are available. In this case an assumption has to be made on some unknowns; for example could be established that

$$p_{s3}-p_{m3} = 0 \text{ and } p_{d3}-p_{m3} = 0$$

or that

$$p_{s1}-p_{m1} = p_{s3}-p_{m3} \qquad p_{d1}-p_{m1} = p_{d3}-p_{m3}$$

in order to reduce the number of unknowns to 9.
In the following this last assumption has been made.
When 5 tie points are used 15 equations can be established for 15 unknowns because each additional tie point adds 3 equations and only 2 unknowns.

5-2-The mathemathical model of the tie points discrepancies has been determined taking into account a specific reference system for each model and assuming that this reference system is the same for the oriented stereomodels of a block which are referred to a unique terrain reference system.
In other words the mathematical model would be a strict mathematical model if the stereomodels could be oriented without any rotation and with the perspective centers lined up a straight line parallel to the X axis.
Actually one should take into account of the rotations $\Delta\Phi$, $\Delta\Omega$, ΔK which the stereomodels undergo when the block is computed and adjusted; from the expressions of the errors in the stereomodels coordinates X-X°, Y-Y°, Z-Z° demonstrated in 3-1 and 3-2 one should determine the expressions of the transformed errors and then the expressions of the transformed discrepancies using for semplicity approximate expressions of the direction cosines; as a result new coefficients should be obtained for the equations (11) which should contain terms in $\Delta\Phi$, $\Delta\Omega$, ΔK; since the rotations are usually limited to a few degrees and the unknowns have very small value it can be inferred that the variations of the coefficient do not affect practically the determination of the unknowns.
The demonstrated mathematical model can consequently be used for the practical applications as it happens in other analytical photogrammetric computations where for example coefficients evaluated for perfectly nadiral photographs a-

re used for the computations of the orientation of common aerial photographs.

As far as the known terms of the equations are concerned it is necessary to point out that the values of DX, DY, DZ are affected not only by the values of the rotations $\Delta\Phi$, $\Delta\Omega$, ΔK but also by the errors of these parameters and by the relative errors of the translation parameters.

It follows that the deformation parameters are to be computed using the discrepancies between many stereomodels, for the whole block or consistent part of the block. A sound procedure could be that of computing firstly the deformation parameters for each strip and after assessing the congruency of the results the deformation parameters for the whole block, taking into account that the confidence in the results relative to a reduced number of stereomodels in smaller than that relative to a larger number.

It is necessary finally to point out that the obtaining of the true values of the deformation parameters is based on the assumption that the mean value of the errors of the stereomodels orientation parameters are zero. This assumption seems to be true if the locations of the control points is considered. As an example a single strip is considered where the first and the last stereomodel are provided with a sufficient number of control points; after an independent models adjustment the first and the last stereomodel have practically no orientation errors and the sum of the relative errors in each absolute orientation parameter should be zero.

6 - Theoretical mathematical model of the tie points discrepancies.

For the validation of the equations of the deformation parameters the theoretical equations for a block where each stereomodel is flat, rectangular and has the same dimension have been determined (see fig. 4). In this case the not zero values of the direction tangents in the coefficients are denoted as m and n and t denotes the value h^2/b (h=height of flight, b=base) common to all the stereomodels.

In order to have only 9 unknowns the assumption has been made that

$$p_{s1}-p_{m1} = p_{s3}-p_{m3} = p_s-p_m$$

$$p_{d1}-p_{m1} = p_{d3}-p_{m3} = p_d-p_m$$

that is the symmetry of the p errors with respect to the axis of the strip has been assumed.

If the equations are written in the order DY1, DY2, DY3, DZ3, DX3, DX1, DZ1, DX2, DZ2 it can be pointed out that all the diagonal coefficients are defined and that no linear relationship can be found among the equations.

If the equation DZ1 is subtracted from the equation DZ3 it is obtained

$$(12) \qquad \omega_2 = \frac{DZ_3 - DZ_1}{2tmn}$$

(this is a consequence of the assumed symmetry of the p components in 1 and 3).

If the equation DX1 is subtracted from the equation DX3 (hypothesis as above) it is obtained

$$(12') \qquad k_2 - k_1 = \frac{DX_1 - DX_3}{2tmn}$$

e from the equation DY2

$$(12'') \qquad k_1 = \frac{DY_2}{hm}$$

where DY2 is to be cleared of the discrepancy on the exposure point.

If the equation DY1 is subtracted from the equation DY3 it is obtained

$$(12''') \qquad \varphi_1 + \varphi_2 = \frac{DY_3 - DY_1}{2tnm^2 + 2hmn}$$

Looking at the coefficients it seems not possible to find a simple relationship for computing $\varphi_1 - \varphi_2$; actually the effect of the error in the convergency $\varphi_1 - \varphi_2$ is that of a cylindrical deformation with the axis perpendicular to the base which obviously cannot be detected looking at the dissymetry of the discrepancies; the values φ_1 and φ_2 should be obtained with a good confidence because of the different coefficients of φ_1 and φ_2.

7 - Some practical experiments.

The computation of the deformation parameters can be performed at the end of a block adjustment when all the necessary data and mainly the discrepancies are available.

The subroutines have been written and checked but no time was available for the analysis of the results of two blocks of about 300 stereomodels.

In the fig. 5, 6, 7, 8 as preliminary results have been shown the distributions for these two blocks of the values of DZ3-DZ1, DX1-DX3, DY3-DY1 and DY2 which are inherent to the deformation parameters shown in the equations (10).

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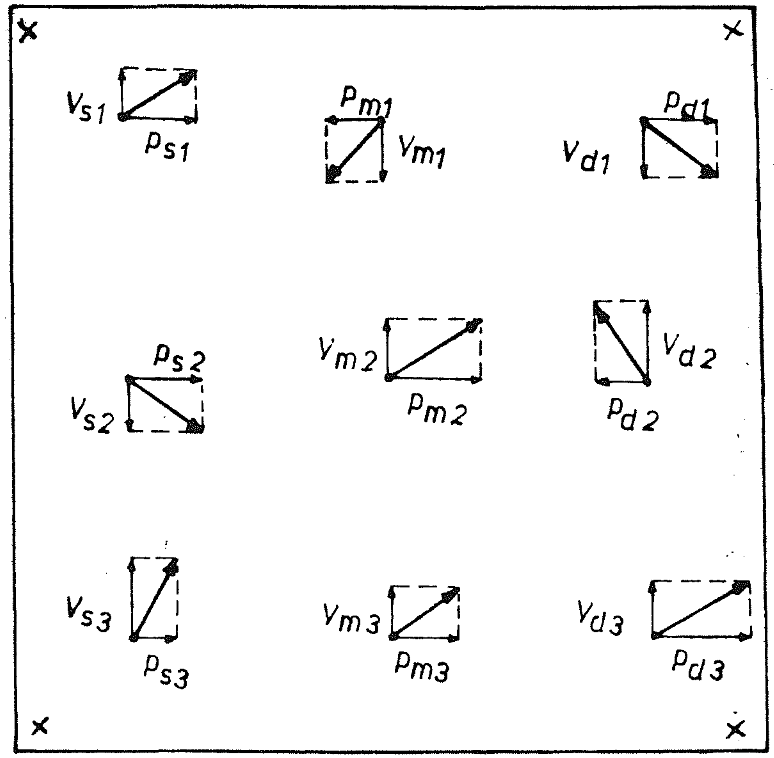

FIG. 1

Systematic photocoordinates errors in the positions where usually are located the i mage points of the tie points.

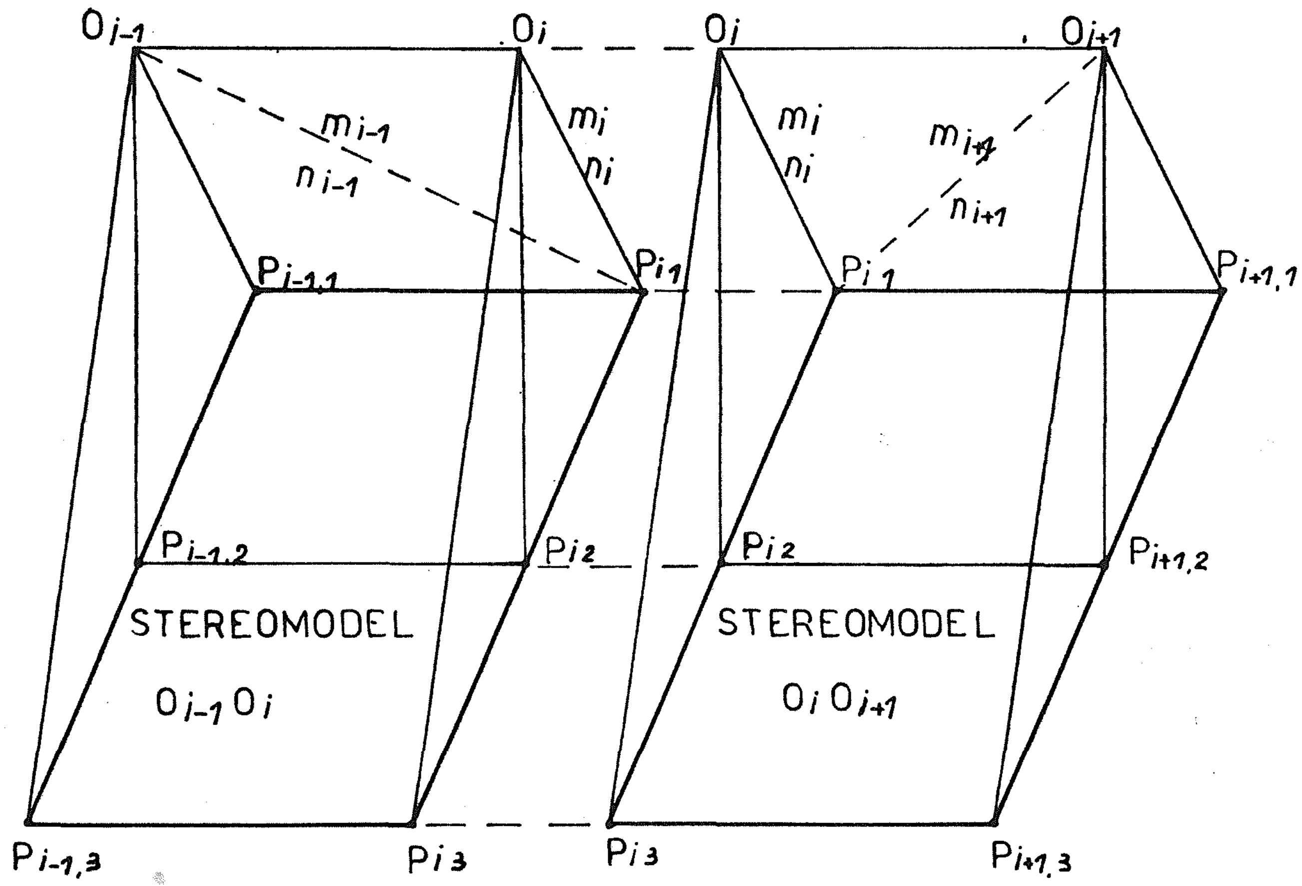

FIG. 2

Notations of the direction tangents in adjacent stereomodels

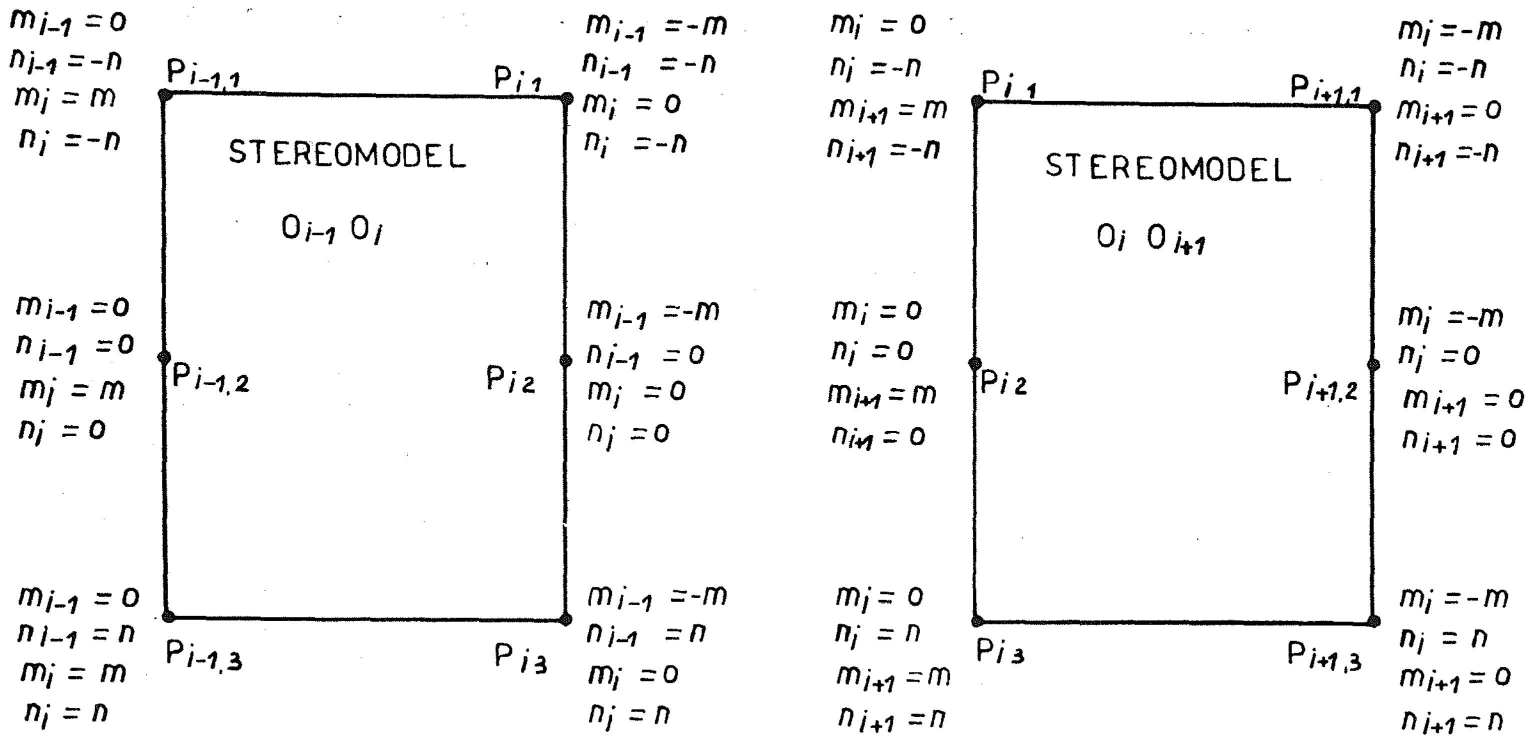

FIG. 3

Theorethical values of the direction tangents in two adjacent stereomodels

UNKN. / C.T.	φ_1	κ_1	φ_2	ω_2	κ_2	p_d-p_m	p_s-p_m	$p_{d_2}-p_{m_2}$	$p_{s_2}-p_{m_2}$
DX_1	$-hm^2-$ $-tm(1+m^2)$	$-tmn$	tm	0	tmn	$mt/f+$ h/f	mt/f	0	0
DX_2	$-hm^2-$ $-tm(1+m^2)$	0	tm	0	0	0	0	mt/f h/f	mt/f
DX_3	$-hm^2-$ $-tm(1+m^2)$	tmn	tm	0	$-tmn$	$mt/f+$ h/f	mt/f	0	0
DY_1	$-hmn-$ $-tnm^2$	hm	$-tnm^2$	tmn^2	0	0	0	0	0
DY_2	0	hm	0	0	0	0	0	0	0
DY_3	$hmn+$ $+tnm^2$	hm	tnm^2	tmn^2	0	0	0	0	0
DZ_1	tm^2	0	tm^2	$-tnm$	0	t/f	t/f	0	0
DZ_2	tm^2	0	tm^2	0	0	0	0	t/f	t/f
DZ_3	tm^2	0	tm^2	tmn	0	t/f	t/f	0	0

FIG. 4

Theoretical mathematical model of the discrepancies between the tie points.

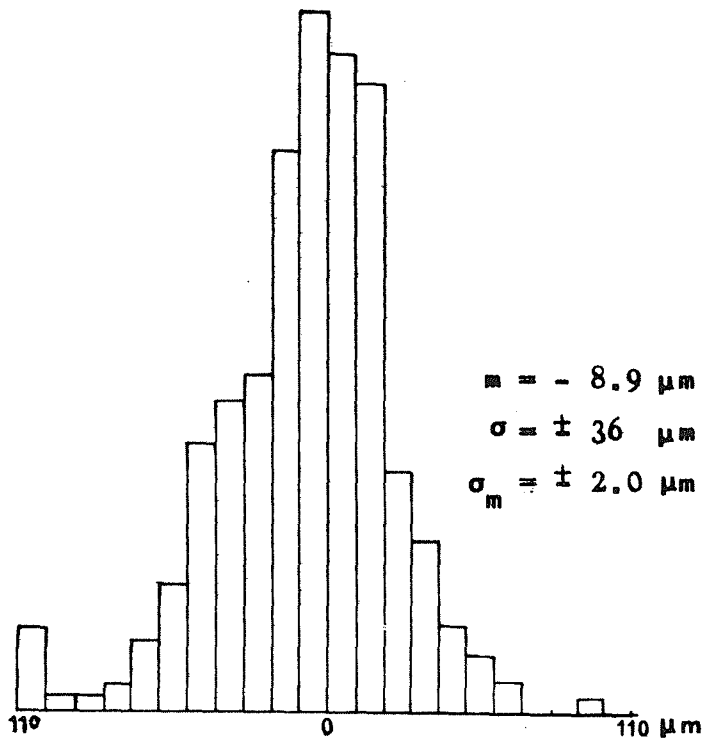

FIG. 5a)

Distribution of DY_2 (Analogue Block)

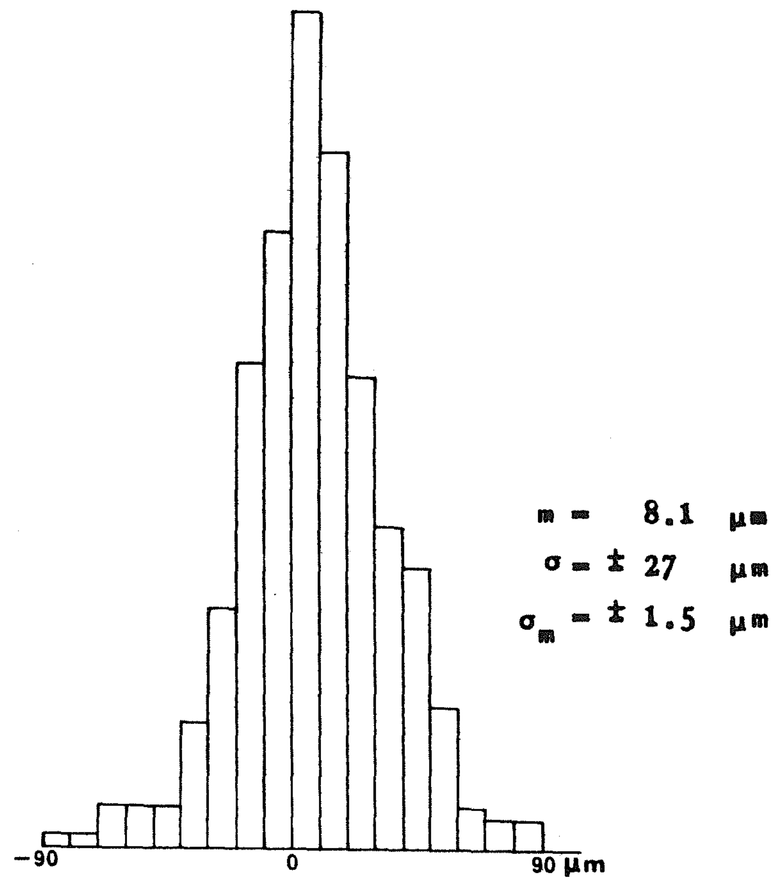

FIG. 5b)

Distribution of DZ_2 (Analogue Block)

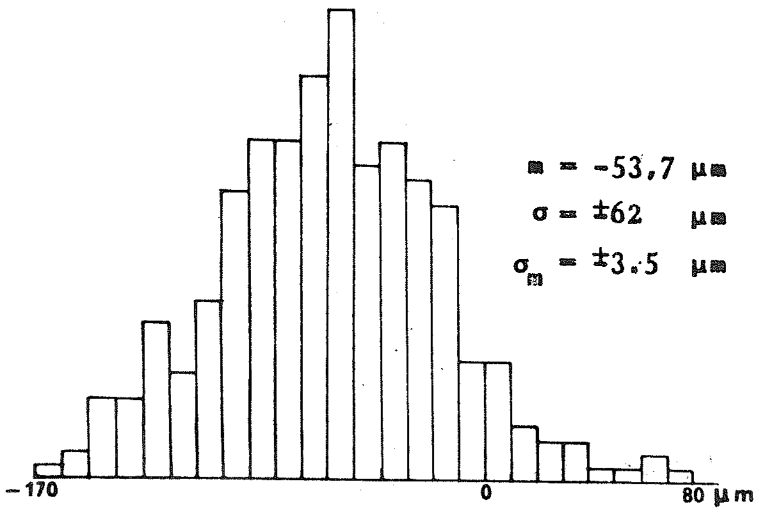

FIG. 6a)

Distribution of DX_1-DX_3(Analogue Block)

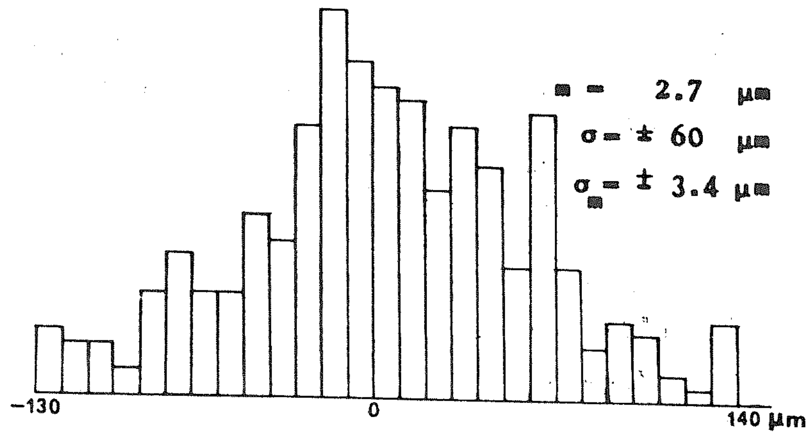

FIG. 6b)

Distribution of DZ_3-DZ_1(Analogue Block)

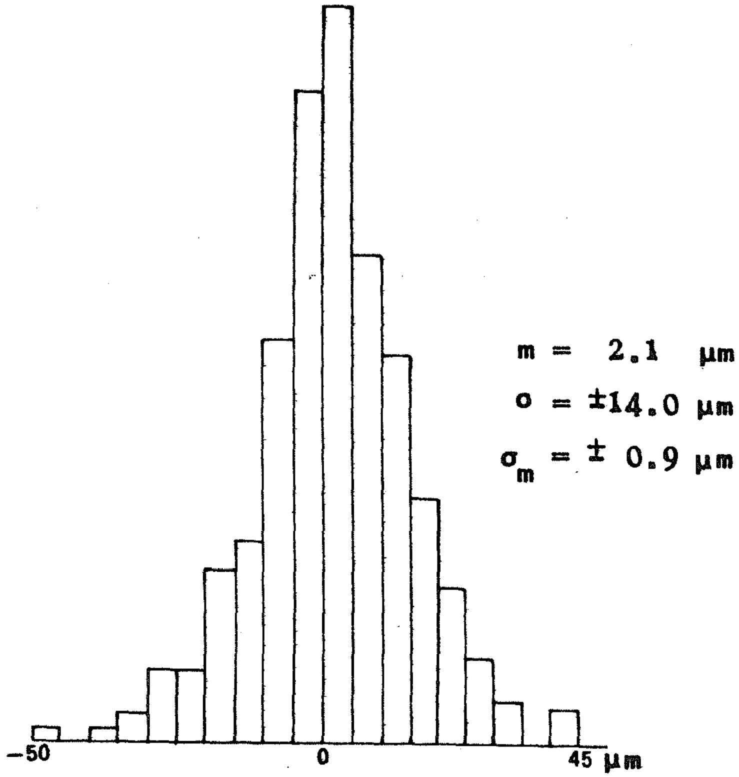

FIG. 7a)

Distribution of DY_2 (Analytical Block)

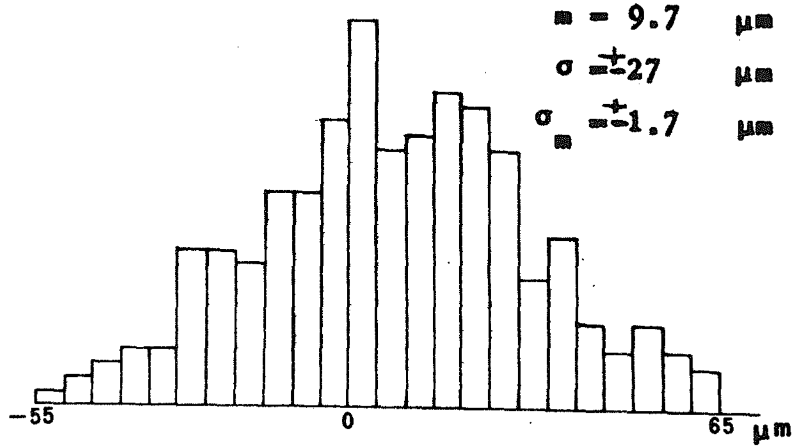

FIG. 7b)

Distribution of DZ_2 (Analytical Block)

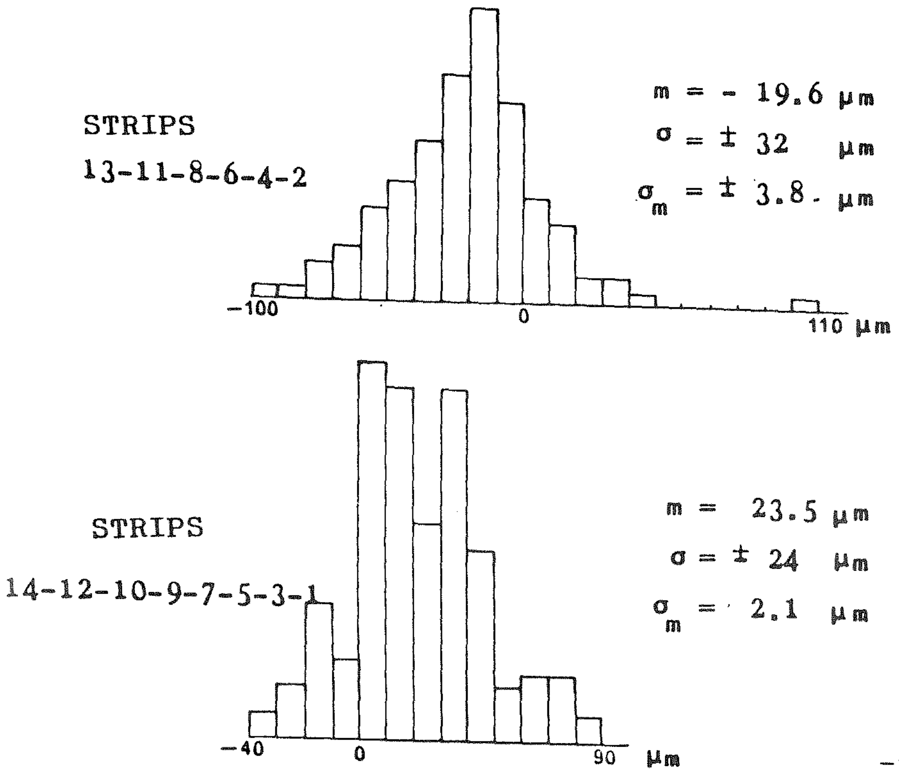

FIG. 8a)

Distribution of DX1 - DX3
(Analytical block)

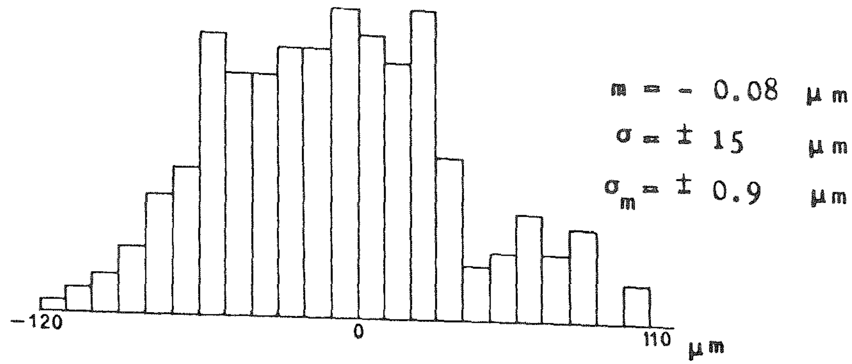

FIG. 8b)

Distribution of DZ3 - DZ1
(Analytical block)