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THE AUC SELF-CALIBRATING BLOCK ADJUSTMENT METHOD  
AND ITS RESULTS FROM JÄMIJÄRVI TESTFIELD

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Abstract:

At Aalborg University Centre the Jämijärvi test block was computed with various control and flight situations. The results proved the presence of systematic image errors of tangential nature and of an average magnitude of 4  $\mu\text{m}$ . After corrections of the systematic image errors the square residuals of check points amounted to some 0.9 cm which errors are also partly due to errors in the terrestrial measurements. This meant an accuracy improvement of 15-40% as compared to a conventional bundle adjustment. The same tangential deformation was found by self-calibration (without control points) using only three images.

THE AUC SELF-CALIBRATING BLOCK ADJUSTMENT METHOD  
AND ITS RESULTS FROM JÄMIJÄRVI TESTFIELD

1. Introduction

At the laboratory for photogrammetry and landsurveying at Aalborg University Centre a programme system is developed for analytical treatment of photogrammetric data, measured in a mono/stereocomparator. This system has been used on Danish cadastral blocks for trig. net densification.

Flights over the Finnish Jämijärvi testfield have been subject to an investigation by the ISP commission III/3, in which the AUC laboratory has taken part. Parallel to the goals of this investigation, it has given us a possibility to test our system and the parameters for correction of systematic deformations on "true" values of check points, and compare with our Danish results.

2. Programme ANA

The system is split into two individual programmes:

- I On-line interactive programme for control of the observations and computation of terrain coordinates.
- II Off-line programme for a bundle block adjustment with additional parameters.

In both programmes gross error detection is made by the principles of robust estimators (see presented paper: Götterdämmerung) [3].

In the on-line programme there are the following steps:

- a mean of repeated observations,
- b conform/affine transformation on fiducial marks,
- c correction for
  - radial lens distortion (from cal. report)
  - refraction
  - earth's curvature,
- d relative orientation, and
- e absolute orientation (3 dimensional conform transformation).

In the off-line programme is made a bundle adjustment. Preliminary values are called from the on-line programme. As elements are used:

- a terrain points,
- b camera parameters, and
- c elements for correction of systematic deformations.

As observations may be used:

- a image coordinates,
  - b coordinates for control points,
  - c outer orientation elements of camera stations,
  - d values for systematic deformations, and
- all observations can be given individual weights.

Twelve additional parameters are used for correction of systematic image deformations. They are: four tangential (tg1, tg2, tg3, tg4), three radial (ra1, ra2, ra3), two for affinity (af, sh), two for displacement of principal point (dhx, dhy) and one for correction of camera constant (dc). The formulae for correction of these systematic image deformations are:

Tangential deformation:

$$\begin{aligned} dt &= r^{1.85} (tg1 \cdot \cos\alpha + tg2 \cdot \sin\alpha + tg3 \cdot \cos 2\alpha + tg4 \cdot \sin 2\alpha) \\ x'' &= x' + dt / 100000 \cdot \sin\alpha \\ y'' &= y' - dt / 100000 \cdot \cos\alpha \end{aligned}$$

Radial deformation:

$$\begin{aligned} dr &= ra1(r^3 - r \cdot r_0^2) + ra2 \cdot r \left( \sin \left( \frac{r \cdot \pi}{r_0} \right) \right)^2 + ra3 \cdot \sin \left( \frac{2r\pi}{r_0} \right) \\ x'' &= x' + dr / 100000 \cdot \cos\alpha \\ y'' &= y' - dr / 100000 \cdot \sin\alpha \end{aligned}$$

These parameters were proposed by Dr. O. Kölbl [4]. Only the power in the tangential deformation is changed from 2 to 1.85, following the idea of Dr. J. Hakkarainen [2].

Affine deformation:

$$\begin{aligned} x'' &= x' \\ y'' &= y' - (af \cdot y' + ak \cdot x') / 100000 \end{aligned}$$

Displacement of principal point:

$$\begin{aligned} x'' &= x' + dhx / 100000 \\ y'' &= y' + dhy / 100000 \end{aligned}$$

Correction of camera constant:

$$c'' = c' + dc / 100000$$

where x and y = image coordinates in decimetres

c = camera constant in decimetres

$$r = \sqrt{x^2 + y^2}$$

$\alpha$  = grid bearing

$r_0$  = second intersection of the radial deformation curve with the zero axis of the coordinate system (dm).

For each camera station a set of twelve parameters can thus be given.

### 3. Possibilities

With the "ANA" system we can compute single models, strips or blocks. The photographs can be taken by non-metric cameras, terrestrial or aerial cameras. It is also possible to use the off-line programme for camera calibration, e.g. using the methods of parallax measurement as suggested by Dr. O. Kölbl.

### 4. Jämijärvi, Computation Steps and Testdata Used

In the ISP commission III/3 investigation, observation material was received from Finland, measured in diapositives at the Zeiss PSK 1.

For this purpose, the following steps for preparation of data were executed:

Inner orientation: Affine transformation to the 4 fiducials.  
Correction for radial lens distortion and refraction, according to Bertram's formula.

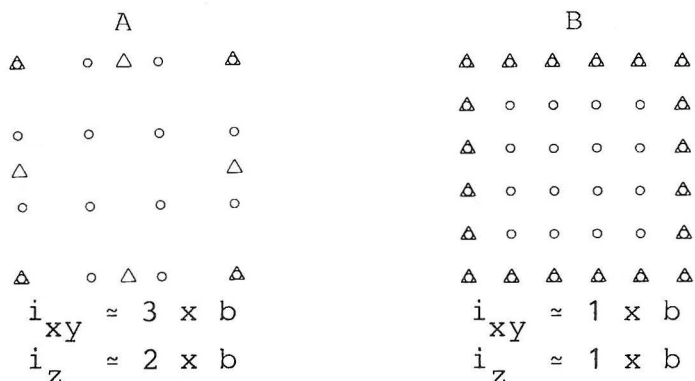
Relative orientation.  
Absolute orientation.

For the final computation a bundle adjustment with nine additional parameters was performed. These were the parameters for tangential, radial and affine deformations, which have proved to be present in Danish large-scale triangulation blocks. The last three parameters are normally used only in connection with a total camera calibration. For the Jämijärvi block the value  $r_0$  (see previous page) was set to 135 mm.

For the bundle adjustment, the a priori RMSE for the different types of observations were chosen as shown in the table below:

	without add. parameters	with add. parameters
Image coordinates $x'$ and $y'$	3 $\mu\text{m}$ ( $p=1$ )	3 $\mu\text{m}$ ( $p=1$ )
Control points X, Y and Z	0.0 cm	0.5 cm
Additional parameters (observation value 0.0)	-	3

The following control point arrangements are studied in this paper.

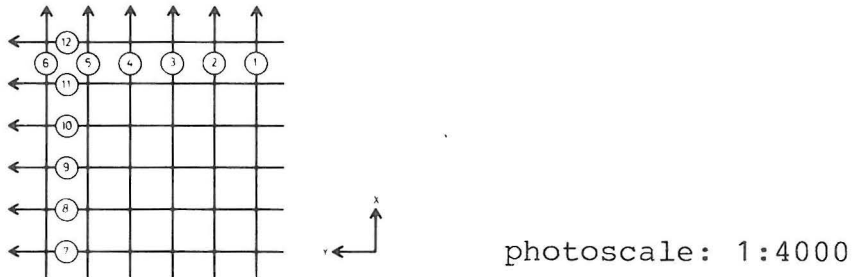


Version A is chosen because this arrangement is commonly used in Denmark. Version B is chosen to show the extreme obtainable accuracy with a maximum control. (Also used in the commission III/3 investigations.)

For test purposes appr. 100 check points were available. The control points as well as the check points have an accuracy of 0.5 cm in planimetry and 0.06 cm/km in height.

The following flight arrangements were used:

6 strips 60/60% flown from south to north and  
6 strips 60/60% flown from east to west.



The photos were taken with a Zeiss RMK 23x23/15 camera from an altitude of appr. 600 metres.

## 5. Results

### One set of additional parameters applied to all strips

Table A (page 9) shows the RMSE at check points with different flight and control situations.

For the block including cross strips, the accuracy improvement due to additional parameters only amounts to about 20%. This is due to the high internal stability of the block and because a great deal of the systematic deformation is compensated by rotating the photos for  $90^\circ$  (tg3, tg4, af and sh). The results encouraged us to perform a bundle adjustment with a set of parameters for each of the 6 strips (see page 6). The accuracy improvement for the less stable flight arrangement consisting of parallel strips amounts to 40%.

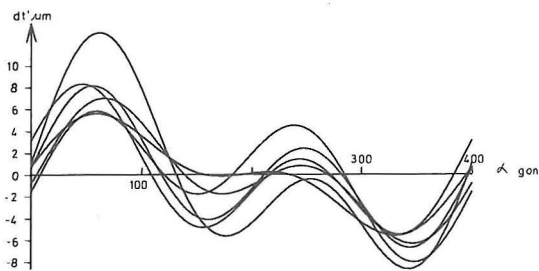
Table B demonstrates again the importance of the flight arrangement for the detection of the systematic image deformation. With cross strips the systematic image deformation can be estimated significantly better than with parallel strips. This is particularly evident in free block adjustment (without control points, see table C).

The control point arrangement studied gave no significant different results in the estimation of the systematic image errors apart from a small improvement in the determination of the affinity. This improvement caused by a greater number of control points is only seen in the flight situation with 20% sidelap.

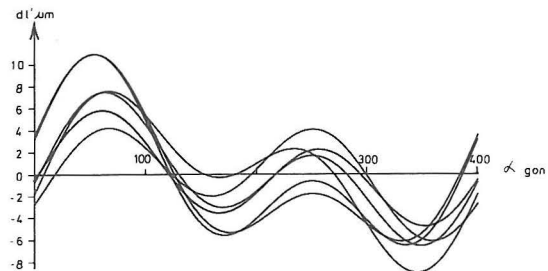
### Different sets of additional parameters for each strip

In all the preceding results only one set of additional parameters was used for the whole block. In order to test, whether this assumption is justified, the individual strips were allowed distinct sets of additional parameters. Two blocks of the Jämijärvi test block (both including cross strips and 20% sidelap) were computed with different additional parameters in each strip. For these computations the control point arrangement B was used.

The tangential deformation which was the most significant deformation in the test block is shown below for the individual strips.



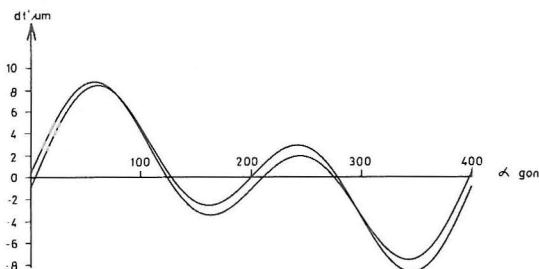
Block with odd strip numbers.



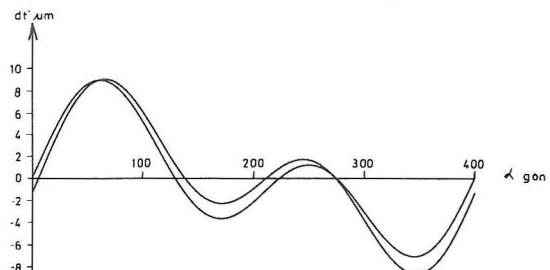
Block with even strip numbers.

The accuracy of determining this deformation for each strip is in the order of  $1-2\mu\text{m}$ . The deformations found for the individual strips show a very large similarity, thus justifying the use of only one set of additional parameters in the block. This conclusion is confirmed by the fact that the residuals at the check points are similar for both adjustments.

To exclude the possibility that these systematic deformations are induced by the control points, a free adjustment of the block was executed including one set of additional parameters. Comparison with a computation on control points (B) showed no significant difference in the deformation parameters. Thus the deformations found are not induced by control points.



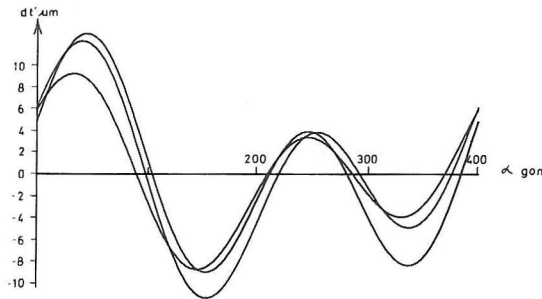
Block with odd strip numbers.  
Free adj./control version B.



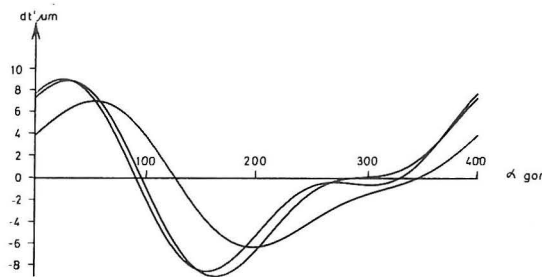
Block with even strip numbers.  
Free adj./control version B.

A similar constancy of the systematic errors for blocks was found in Danish blocks.

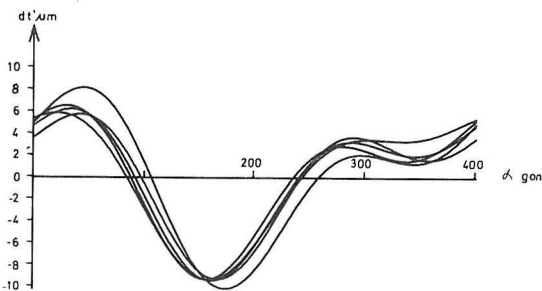
The tangential deformation for three Danish blocks from practical work in cadastral survey is shown in the example below.



camera: Zeiss RMK 2323/15  
 overlap: 60/30  
 flying height: 1500  
 number of strips: 3  
 number of terrain points: 223  
 number of redundant observations: 772  
 $\sigma_0$ : 3.1



camera: Zeiss RMK 2323/15  
 overlap: 60/30  
 flying height: 1250  
 number of strips: 3  
 number of terrain points: 192  
 number of redundant observations: 421  
 $\sigma_0$ : 3.4



camera: Wild AG 2323/15  
 overlap: 60/30  
 flying height: 1250  
 number of strips: 5  
 number of terrain points: 1250  
 number of redundant observations: 2462  
 $\sigma_0$ : 3.1

With this internal stability it is possible to determine the systematic effect with a high accuracy, even when using only a small number of images of the block. One of the authors has for example used this principle to perform a calibration of the camera without use of control points and using 3 images only.

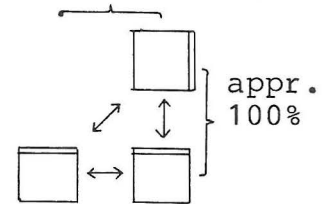
## 6. Camera Calibration

A small block, containing three photos: numbers 91, 92 and 120 from the test block Jämijärvi have been measured for calibration purposes. These photos are included in a flight situation with cross strips.

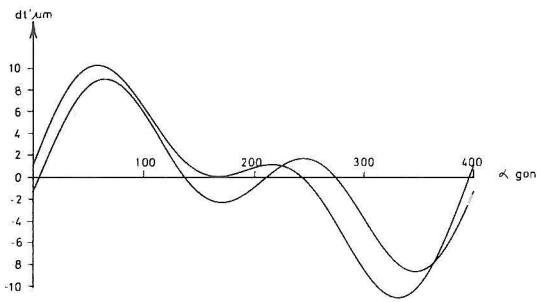
The flight situation for the three photos is:

All three possible models have been measured stereoscopically (about 150 points in every model according to the ideas of Dr. O. Kölbl) in a stereocomparator.

appr. 60% overlap



In this way it was possible to define the systematic image parameters without using any control points, and without any transfer points.



The tangential distortion from the same flight situation in the original test (whole block, no control points) and from the calibration are shown. No significant difference was found between the tangential deformations of the two determination methods.

The RMSE of the systematic deformations were 1.5-2.0 times larger than in the original test, which is partly due to the lower accuracy in the stereoscopic measurements (RMSE of unit weight is increased from 3.1 to 4.2).

### 7. Danger of Using Additional Parameters

contr.version	A			B		
add.param.	none	af+sh	all	none	af+sh	all
				2.5	2.4	2.2
$\mu_x$				2.5	2.3	2.0
$\mu_y$				5.1	6.0	3.6
$\mu_z$				4.1	3.8	3.1
$\sigma_o$						
$\mu_x$	5.5	4.6	3.8	3.9	3.4	2.9
$\mu_y$	7.4	5.7	4.4	5.0	3.7	3.2
$\mu_z$	10.9	11.0	9.9	9.1	10.1	7.4
$\sigma_o$	3.6	3.4	2.9	4.0	3.6	3.0

RMSE of the check points (notations see table A)

From the table it will be seen that the RMSE in Z may increase if only the parameters for affine and shear deformations are included.

Likewise, the RMSE in X and Y increases if only tg1 and tg2 from the tangential deformation are included. The parameters can be explained physically as centering errors in the lens system |1|.

Consequently, there is a significant danger in correcting for systematic image deformations. If these deformations are corrected with a wrong parameter the results will easily deteriorate. It is important to be completely sure of the deformation type before extra parameters are included in the block adjustment.

### 8. References

- [1] D. Brown: Decentering Distortion of Lenses. Photogrammetric Engineering Vol. XXXII, 444-462, 1966.
- [2] J. Hakkarainen: On the Use of the Horizontal Goniometer in the Determination of the Distortion and Image Quality of Aerial Wide-Angle Cameras. Helsinki 1976.
- [3] T. Krarup, J. Juhl and K. Kubik: Götterdämmerung over Least Squares Adjustment. Presented Paper 14th ISP Congress Hamburg 1980.
- [4] O. Kölbl: Tangential and Asymmetric Lens Distortion, Determined by Self-Calibration. BuL 43, 35-42, 1975.



TABEL A

Overlap	Control version		Flight situation		without add. p.		with add. p.							
	Flight situation		without add. p.		with add. p.		without add. p.		with add. p.					
2*60/20		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			3.3	2.3	2.5	2.0	3.4	2.6	2.5	2.0	5.9	5.6	5.1	4.0
			4.0	3.1	4.0	3.1								
2*60/20		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			3.3	2.6	2.5	2.2	3.5	2.6	2.5	2.0	4.5	3.9	5.1	3.6
			4.0	3.1	4.1	3.1								
60/60		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			2.8	2.1	2.4	2.0	4.5	2.5	3.3	1.8	6.4	5.0	5.2	3.9
			3.9	3.1	4.1	3.1								
60/60		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			4.4	2.6	3.1	2.1	2.5	2.2	2.8	2.0	5.2	4.3	5.2	3.8
			4.1	3.1	4.0	3.1								
60/20		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			3.9	3.8	2.9	2.9	7.3	5.2	4.4	3.4	11.7	9.3	8.2	7.9
			3.4	2.8	3.8	3.0								
60/20		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			5.5	3.8	3.9	2.9	7.4	4.4	5.0	3.2	10.9	9.9	9.1	7.4
			3.6	2.9	4.0	3.0								
60/20		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			6.0	2.8	3.8	2.7	4.9	3.0	3.4	2.7	8.1	7.2	7.5	5.7
			3.9	2.9	4.1	3.0								
60/20		$\mu_x$	without	with	without	with	without	with	without	with				
			add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.	add. p.			
			7.0	3.6	5.3	3.4	3.7	3.0	3.4	2.8	9.9	10.2	9.2	6.9
			3.5	2.9	3.6	2.8								

RMSE of the check points

$$\mu_x = m \cdot \left( \sum_{i=1}^l (x_p - x_g)_i^2 / l \right)^{1/2}$$

$\mu_y$  and  $\mu_z$  correspondingly

$m$  = scale of photography (1:4000)

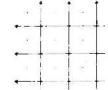
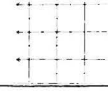



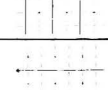
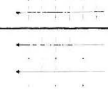
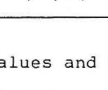
$x_p$  = photogrammetric terrain coordinate of the check point

$x_g$  = geodetic terrain coordinate of the check point

$l$  = number of check points (80-100)

$\sigma_0$  = RMSE of unit weight.

TABEL B

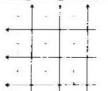


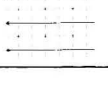
Flight configuration	Control version	Additional parameters (value/RMSE)								
		tg1	tg2	tg3	tg4	ra1	ra2	ra3	af	sh
	A	1.2/0.3	3.9/0.4	-1.2/0.3	5.1/0.3	0.3/1.1	-2.1/1.4	-1.8/0.3	-2.4/0.6	1.6/0.5
	B	1.1/0.3	4.1/0.4	-1.0/0.3	4.9/0.3	0.2/1.1	-1.7/1.4	-1.7/0.3	-2.0/0.6	1.3/0.4
	A	0.6/0.3	3.8/0.4	-0.5/0.3	5.4/0.3	-0.1/1.3	-3.4/1.5	-1.8/0.3	-3.8/0.6	2.1/0.5
	B	0.8/0.3	3.9/0.4	-0.4/0.3	5.2/0.3	-0.5/1.2	-3.5/1.4	-1.6/0.3	-3.2/0.6	1.9/0.5
	A	0.4/0.3	3.3/0.3	-1.0/0.4	5.2/0.3	-2.0/1.2	-4.2/1.5	-2.3/0.3	-2.1/0.8	1.8/0.7
	B	0.5/0.3	3.4/0.3	-0.9/0.3	5.1/0.3	-2.1/1.2	-4.5/1.5	-2.3/0.3	-1.8/0.6	2.3/0.6
	A	1.8/0.3	4.9/0.3	-0.7/0.3	5.0/0.3	1.6/1.1	-1.5/1.3	-1.2/0.3	-1.3/0.8	2.1/0.7
	B	1.6/0.3	4.9/0.3	-0.7/0.3	5.0/0.3	1.5/1.1	-1.4/1.3	-1.2/0.3	-2.2/0.6	1.5/0.5
	A	0.6/0.5	2.5/0.7	-1.7/0.6	4.5/0.7	-0.6/1.8	-1.6/1.9	-2.2/0.5	-0.6/1.7	0.1/1.2
	B	0.6/0.5	3.2/0.7	-1.1/0.6	4.2/0.6	-1.5/1.7	-2.5/2.0	-2.3/0.5	0.4/1.2	2.9/1.0
	A	0.6/0.6	1.6/0.7	-0.7/0.6	6.1/0.7	-0.1/1.6	-3.2/1.9	-2.4/0.5	-4.1/1.7	0.2/1.2
	B	0.7/0.5	2.6/0.6	-0.1/0.5	5.5/0.5	-1.5/1.5	-4.7/1.9	-2.2/0.4	-2.1/1.1	0.2/0.9
	A	2.8/0.5	4.4/0.6	-0.9/0.6	4.7/0.6	0.6/1.4	-2.4/1.7	-1.9/0.4	-0.8/1.4	1.0/1.0
	B	2.6/0.5	4.7/0.6	-0.6/0.5	4.6/0.5	0.7/1.3	-1.5/1.7	-1.6/0.4	-1.4/1.0	0.3/0.8
	A	1.3/0.6	5.3/0.8	-1.0/0.6	5.0/0.8	3.3/1.8	-1.4/2.0	-1.1/0.5	-2.3/1.8	3.8/1.2
	B	1.3/0.5	5.3/0.7	-0.9/0.6	4.4/0.6	2.5/1.6	-0.3/1.8	-0.9/0.5	-1.0/1.2	2.6/0.9

Values and RMSE of additional parameters (with control points)



significant parameters.

TABEL C

Flight configuration	Additional parameters (value/RMSE)								
	tg1	tg2	tg3	tg4	ra1	ra2	ra3	af	sh
	-0.2/0.5	4.9/0.5	-1.1/0.3	5.0/0.3	0.5/2.9	-2.7/1.4	-1.9/0.3	-2.5/0.6	1.7/0.5
	-0.4/0.5	4.1/0.5	-0.5/0.4	5.4/0.3	-0.9/2.9	-3.0/1.6	-1.7/0.3	-3.8/0.7	2.0/0.5
	0.9/2.6	1.6/2.6	-1.0/0.5	4.4/0.5	-1.3/2.9	-4.5/1.6	-2.3/0.3	1.2/2.8	1.3/2.8
	-0.3/2.6	-0.3/2.6	-0.2/0.9	5.9/1.1	-0.7/2.7	-3.4/2.0	-2.6/0.4	-0.0/2.7	-0.3/2.7

Values and RMSE of additional parameters (free adjustment without any control points)



significant parameters.