

XIV. KONGRESS  
DER INTERNATIONALEN GESELLSCHAFT  
FÜR PHOTOGRAMMETRIE

HAMBURG 1980

Kommission: III

Arbeitsgruppe 1

Presented Paper

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CORRELATIONS AND STANDARD ERRORS IN BUNDLE BLOCK ADJUSTMENT  
WITH SOME EMPHASIS ON ADDITIONAL PARAMETERS

Abstract:

The bundle adjustment program BOBUE provides the possibility for computation of the whole standardized inverse of the normal equations or of standard errors only. With this software blocks of synthetic data with generated randomly distributed errors were investigated. Standard errors and correlation coefficients resulting from bundle solutions are presented numerically and graphically. An attempt is made to deal with the influence of point density per photo and variation of parameterization. Some typical findings from real block adjustment are also given.

KORRELATIONEN UND MITTLERE FEHLER IN DER BÜNDEL-BLOCKAUS-  
GLEICHUNG UNTER TEILWEISER BERÜCKSICHTIGUNG ZUSÄTZLICHER  
PARAMETER

Zusammenfassung:

Das Bündelausgleichungsprogramm BOBUE gibt die Möglichkeit, wahlweise die gesamte standardisierte Inverse der Normalgleichungen oder nur die mittleren Fehler der Unbekannten zu berechnen. Mit diesem Programm werden Blocks aus synthetischen Daten mit zufallsverteilten Fehlern untersucht. Mittlere Fehler und Korrelationskoeffizienten aus der Bündellösung werden numerisch und graphisch dargestellt. Es wird versucht, den Einfluß der Punktdichte pro Bild und der Variation der Parameterwahl zu behandeln. Außerdem werden einige typische Ergebnisse aus tatsächlichen Blockausgleichungen mitgeteilt.

# CORRELATION ET ERREURS DANS LA COMPENSATION EN BLOCS DE FAISCEAUX DANS LE CAS CLASSIQUE ET APRÈS INTRODUCTION DE PARAMÈTRES SUPPLÉMENTAIRES

## Résumé:

Le programme de compensation en blocs de faisceaux BOBUE permet de choisir alternativement entre le calcul d'une inversion normé d'équations normales, et le calcul des erreurs moyennes des inconnues. Dans notre cas on a employé ce programme pour l'analyse des blocs se composant des données synthétiques auxquelles sont superposées des erreurs alléatoires. Les erreurs moyennes et les valeurs des corrélations sont listées et représentés graphiquement. On a étudié l'influence de la densité des points par image et du choix des paramètres utilisés. En plus on présente divers résultats typiques des compensations en blocs effectués avec données réelles.

## 1) INTRODUCTION

The first systematical investigations in accuracy after bundle adjustment were published by KUNJI /4/ and TALTS /10/ twelve years ago. Their concern mainly were the standard errors of coordinates of tie points, consequently skipping those of orientation parameters and as a matter of fact all covariances.

Physically induced covariances of image coordinates a posteriori have been dealt with to some extent by ELLENBECK /2/ and KUPFER /5//6//7/, whereas MAUELSHAGEN /8//9/ investigated algebraical correlations, using the bundle adjustment program BOBUE. It was originally written by him and is capable of computing standard errors of unknowns as well as the whole standardized inverse of normal equations.

As there is a lack of insight into the behaviors of correlations within blocks of different sizes and control distribution, this shall be the main object of this paper.

## 2) STANDARD ERRORS AND ALGEBRAICAL CORRELATIONS IN SYNTHETIC BLOCKS.

### 2.1 PATTERN OF BLOCKS COMPUTED

To avoid long CPU times, computations have been restricted to comparatively small blocks. Their pattern is defined in Table 1. All image coordinates and control data have been computed synthetically with random errors attributed to image coordinates with a standard error of  $\pm 5 \mu\text{m}$  at an image scale of 1 : 10,000.

Control point coordinates are assumed to be error free. The blocks are without any height differences.

Table 1 Synthetic Block Patterns

| Strips | Photos per Strip | Control |           | Side Lap                              |
|--------|------------------|---------|-----------|---------------------------------------|
|        |                  | Full    | Elevation |                                       |
| 3      | 7                | 4       | 12        | 20 %<br><br>single<br>or<br>crosswise |
|        |                  | 12      | 4         |                                       |
| 4      | 9                | 4       | 21        |                                       |
|        |                  | 16      | 9         |                                       |
| 5      | 7                | 4       | 12        | 60 %                                  |
|        |                  | 12      | 4         |                                       |
| 7      | 9                | 4       | 21        |                                       |
|        |                  | 16      | 9         |                                       |

Besides this the whole investigation - partly under performance shall cover the following variations (the now available standard version always given first) :

1. Parallel and crosswise strips with 60 % end lap and 20 % side lap.
2. Nine and twentyfive regularly distributed points per fully covered photo.
3. Bundle adjustment without and with use of additional parameters. Additional parameters will however in this paper only be dealt with in context with practically performed flights.
4. WA-, NA- and SWA-photography shall be simulated. In this paper only the first camera type is dealt with.

## 2.2 STANDARD ERRORS

As the simulated flight patterns as well as point and control distributions are fully symmetrical, resulting standard errors need only be given for one quarter of the whole block. They shall be presented in image scale  $M_b$  and referring to a standard error of unit weight  $m_0 = 1 \mu m$ , i.e.

$$\overline{m_x} = \frac{x}{m_0} M_b = \sqrt{Q_x} \cdot M_b$$

According to Table 1 one gets for one block size six sets of normalized standard errors, which will be presented for the

parameters of outer orientation and coordinates of object points separately. It seems however prohibitive to give all data of all variations of procedures in a paper like this. A proper documentation will be given elsewhere. Thus reference is made only to blocks of 4 x 9 strips and flight patterns for this block size. The smaller version is only verbally dealt within comparison to the ones exposed. So is the procedure with others then the standard version.

#### *STANDARD ERRORS OF OUTER ORIENTATION*

It goes without saying, that accuracies of these elements grow with

- increasing number of well distributed control;
- increasing number of intersections of rays per object point, i.e. increasing side lap etc.
- increasing number of points per photo;

alteration of camera type and block size excluded.

Fig.1 thru 3 show the amounts of standard errors with some explanations concerning Fig.1. The uppermost stations are at the ends of the strips, thus standard errors are at a maximum. As the pattern of elevation control is invariant for all flight patterns, there is no large variation of the  $m_{z_0}$ . Crosswise flight pattern gives overall most homogeneous results.

As may be expected, the smaller block size (i.e. 3 x 7 photos) gives numerically better results, at least partly due to the relatively higher density of control.

#### *STANDARD ERRORS OF OBJECT POINTS*

The general statements of the foregoing para hold also true in this context. As far as comparable there is good agreement with the results given by KUNJI /3/ and TALTS /9/. Besides that the homogenizing effect of dense peripheral control is obvious, when comparing Fig.4 thru 6. This is most clearly seen for crosswise flight. Also here standard errors decrease with decrease of block size.

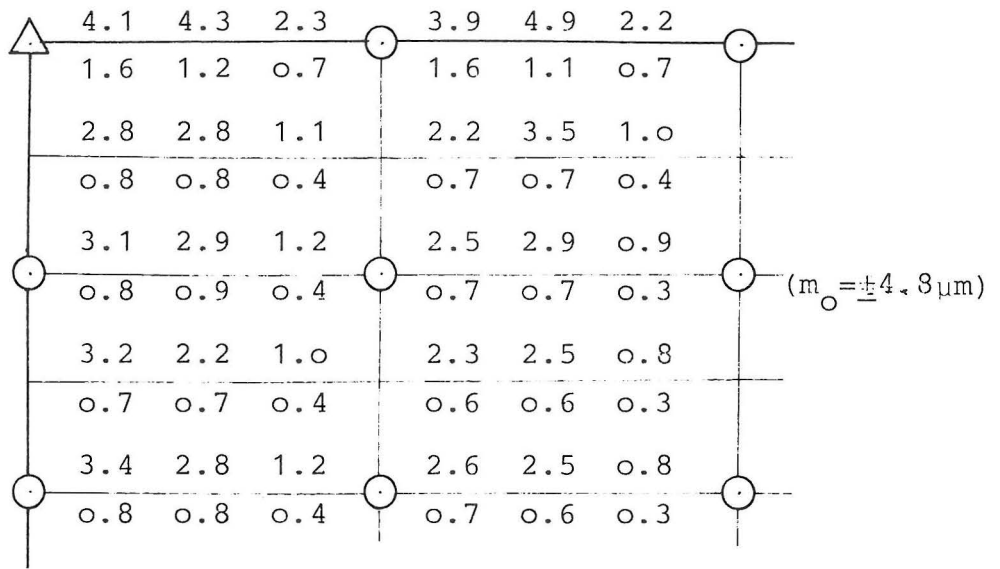
Point densification from 9 to 25 per fully covered photo leads to an increase in accuracy of better than ten percent. The effect of roughly doubling flight lines leads to considerable gains in accuracy (Fig.4 against Fig.5 and 6).

#### 2.3 ALGEBRAICAL CORRELATIONS

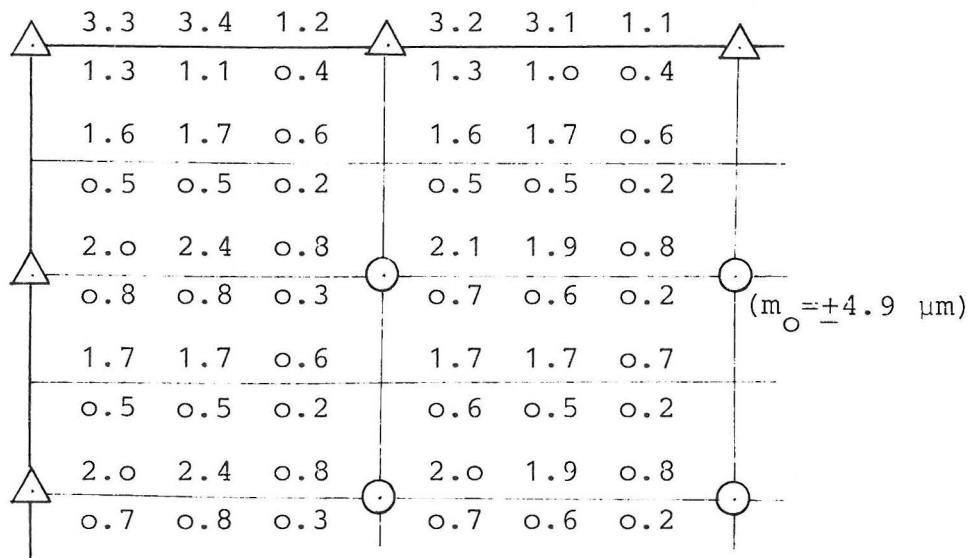
All values are given in per cent as integers.

#### *ELEMENTS OF OUTER ORIENTATION*

Fig.7 thru 9 show for different flying patterns and control distributions at the chosen block size the notorious correlations between  $Y_0$  and  $\omega$  as well as those between  $X_0$  and  $\phi$  over flat terrain. All correlations are quite high. There is however a marked difference between lower values in case of sparse horizontal control and higher amounts with dense control at the block perimeter. Changes of flight patterns do not result in considerable changes of correlations.

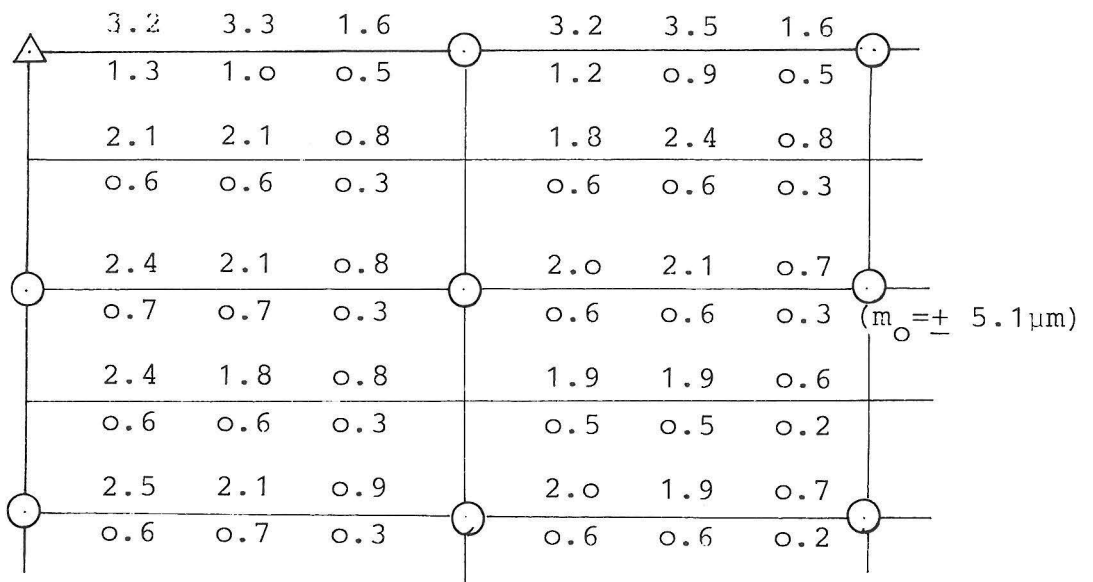


Control: 4 Full, 21 Elevation

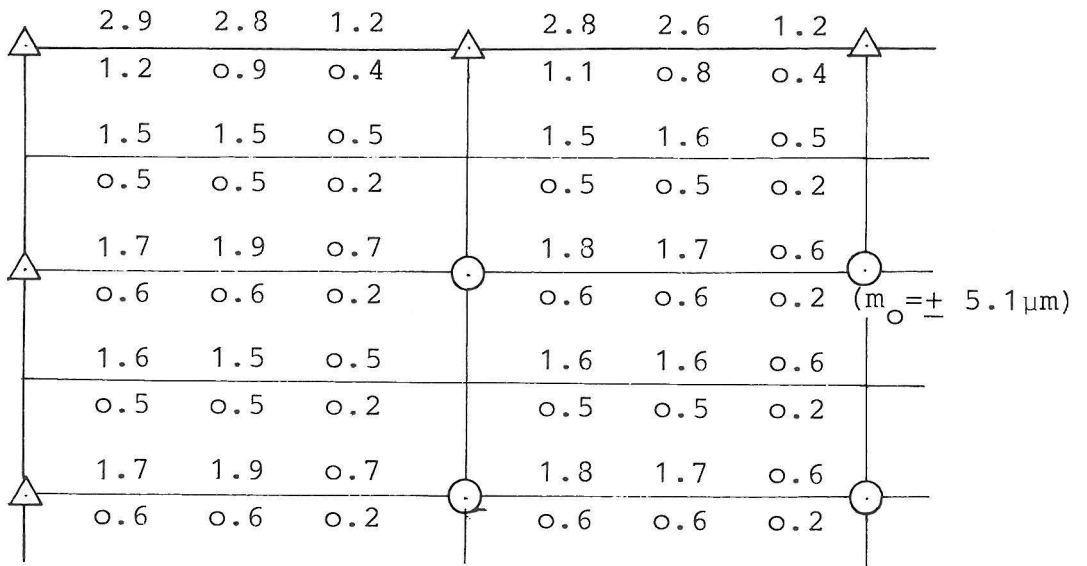


Control : 16 Full, 9 Elevation

Fig. 1 Normalized Standard Errors, Outer Orientation.  
 4 Strips of 9 Photos, 9 Points per fully covered photo,  
 $X_O, Y_O, Z_O$  are given in  $\mu m$  in Photoscale,  
 $\phi, \omega, \kappa$  are given in mgon, arranged in this order at the place of the exposure station.



Control: 4 Full, 21 Elevation

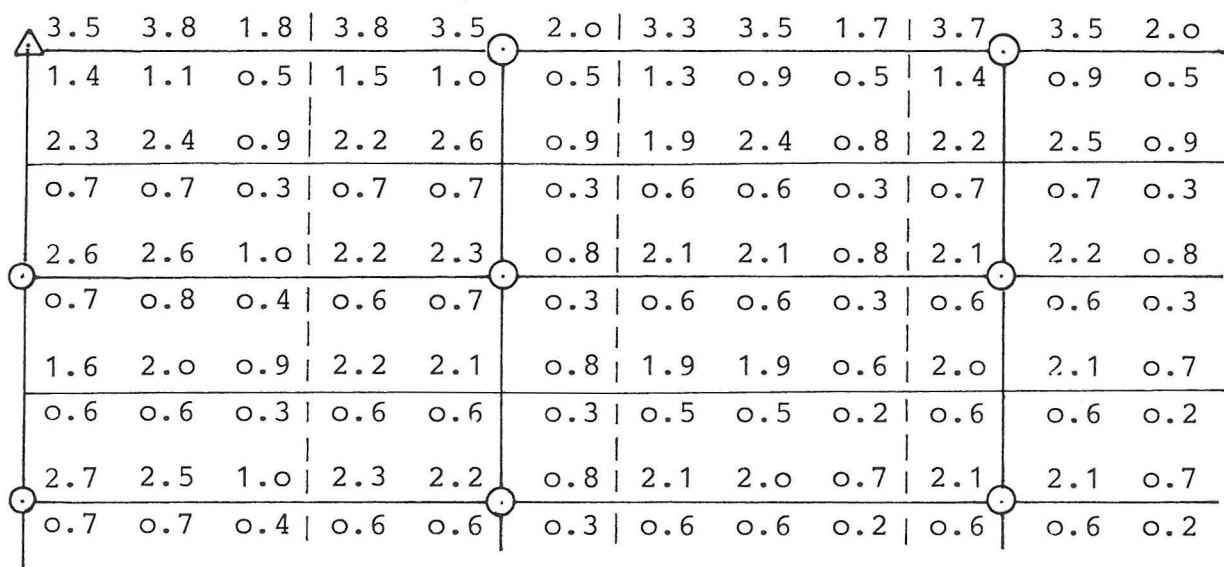


Control: 16 Full, 9 Elevation

Fig.2 Normalized Standard Errors, Outer Orientation  
2 x 4 Strips (Crosswise) of 9 Photos, 9 Points/  
Photo

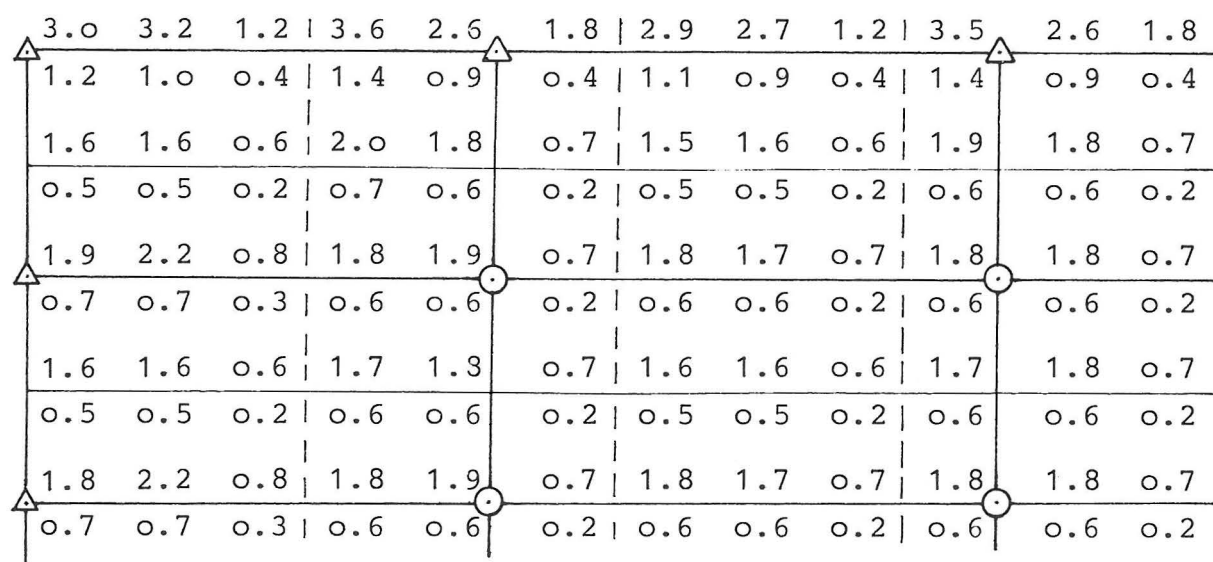
For details c.f. Fig.1

There are only 2 strips displayed, as there  
exists full symmetry.



Control: 4 Full, 21 Elevation

( $m_o = \pm 5.0 \mu\text{m}$ )



Control: 16 Full, 9 Elevation

( $m_o = \pm 5.0 \mu\text{m}$ )

Fig. 3 Normalized Standard Errors, Outer Orientation  
7 Strips (60 % side lap) of 9 Photos, 9 Points/  
Photo

For details c.f. Fig. 1

|     |     |     |     |     |                           |
|-----|-----|-----|-----|-----|---------------------------|
| XXX | 1.6 | 2.2 | 2.4 | 2.6 |                           |
| XXX | 1.7 | 2.4 | 2.8 | 2.9 |                           |
| XXX | 2.7 | XXX | 2.7 | XXX |                           |
| 1.5 | 1.5 | 1.7 | 1.8 | 2.0 |                           |
| 2.1 | 1.6 | 1.9 | 2.2 | 2.2 |                           |
| 2.5 | 1.8 | 1.0 | 1.6 | 1.0 |                           |
| 2.1 | 1.9 | 1.7 | 1.7 | 1.7 |                           |
| 2.2 | 1.8 | 1.8 | 2.0 | 1.9 |                           |
| XXX | 1.4 | XXX | 1.4 | XXX | ( $m_o = \pm 4.8 \mu m$ ) |
| 2.4 | 2.1 | 1.7 | 1.7 | 1.7 |                           |
| 2.9 | 2.0 | 1.9 | 1.9 | 1.8 |                           |
| 2.3 | 1.7 | 0.9 | 1.5 | 0.9 |                           |
| 2.6 | 2.1 | 1.7 | 1.7 | 1.6 |                           |
| 2.7 | 2.1 | 1.9 | 1.9 | 1.8 |                           |
| XXX | 1.4 | XXX | 1.4 | XXX |                           |

Control : 4 Full, 21 Elevation

|     |     |     |     |     |                           |
|-----|-----|-----|-----|-----|---------------------------|
| XXX | 1.2 | XXX | 1.2 | XXX |                           |
| XXX | 1.0 | XXX | 1.0 | XXX |                           |
| XXX | 2.7 | XXX | 2.7 | XXX |                           |
| 0.9 | 0.8 | 0.6 | 0.8 | 0.7 |                           |
| 1.5 | 0.8 | 0.7 | 0.8 | 0.6 |                           |
| 2.4 | 1.8 | 1.0 | 1.6 | 1.0 |                           |
| XXX | 0.7 | 0.6 | 0.8 | 0.7 |                           |
| XXX | 0.8 | 0.7 | 0.8 | 0.7 |                           |
| XXX | 1.4 | XXX | 1.4 | XXX | ( $m_o = \pm 4.9 \mu m$ ) |
| 0.8 | 0.8 | 0.7 | 0.8 | 0.7 |                           |
| 1.4 | 0.8 | 0.7 | 0.8 | 0.7 |                           |
| 2.2 | 1.7 | 0.9 | 1.5 | 0.9 |                           |
| XXX | 0.7 | 0.7 | 0.8 | 0.7 |                           |
| XXX | 0.8 | 0.7 | 0.8 | 0.7 |                           |
| XXX | 1.4 | XXX | 1.4 | XXX |                           |

Control : 16 Full, 9 Elevation

Fig. 4 Normalized Standard Errors of Object Point Coordinates  
 4 Strips of 9 Photos, 9 Points per fully covered photo  
 $m_x, m_y, m_z$  given in  $\mu m$  in photo scale  
 XXX depicts control at the place and coordinate concerned.



|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| XXX | 1.2 | 1.4 | 1.7 | 1.7 |
| XXX | 1.0 | 1.4 | 1.6 | 1.7 |
| XXX | 1.6 | XXX | 1.5 | XXX |
| 1.0 | 1.0 | 1.1 | 1.2 | 1.3 |
| 1.2 | 1.0 | 1.2 | 1.3 | 1.3 |
| 1.6 | 1.1 | 0.8 | 1.0 | 0.8 |
| 1.4 | 1.2 | 1.1 | 1.1 | 1.1 |
| 1.4 | 1.1 | 1.1 | 1.2 | 1.1 |
| XXX | 0.8 | XXX | 0.7 | XXX |
| 1.6 | 1.3 | 1.2 | 1.1 | 1.1 |
| 1.7 | 1.2 | 1.1 | 1.1 | 1.1 |
| 1.5 | 1.0 | 0.7 | 1.0 | 0.7 |
| 1.7 | 1.3 | 1.1 | 1.1 | 1.1 |
| 1.7 | 1.3 | 1.1 | 1.1 | 1.1 |
| XXX | 0.8 | XXX | 0.7 | XXX |

( $m_o = \pm 5.1 \mu\text{m}$ )

Control: 4 Full, 21 Elevation

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| XXX | 0.9 | XXX | 0.9 | XXX |
| XXX | 0.6 | XXX | 0.6 | XXX |
| XXX | 1.6 | XXX | 1.5 | XXX |
| 0.6 | 0.5 | 0.5 | 0.5 | 0.5 |
| 0.9 | 0.5 | 0.5 | 0.5 | 0.5 |
| 1.6 | 1.1 | 0.8 | 1.0 | 0.8 |
| XXX | 0.5 | 0.5 | 0.5 | 0.5 |
| XXX | 0.5 | 0.5 | 0.5 | 0.5 |
| XXX | 0.8 | XXX | 0.7 | XXX |
| 0.6 | 0.5 | 0.5 | 0.5 | 0.5 |
| 0.9 | 0.5 | 0.5 | 0.5 | 0.5 |
| 1.5 | 1.0 | 0.7 | 1.0 | 0.7 |
| XXX | 0.5 | 0.5 | 0.5 | 0.5 |
| XXX | 0.5 | 0.5 | 0.5 | 0.5 |
| XXX | 0.8 | XXX | 0.7 | XXX |

( $m_o = \pm 5.1 \mu\text{m}$ )

Control: 16 Full, 9 Elevation

Fig. 5 Normalized Standard Errors of Object Point Coordinates 2 x 4 Strips (Crosswise) of 9 Photos 9 Points/Photo

For details c.f. Fig. 4

|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| XXX | 1.3 | 1.5 | 1.8 | 1.8 |
| XXX | 1.4 | 1.7 | 1.7 | 1.8 |
| XXX | 1.8 | XXX | 1.4 | XXX |
| 1.3 | 1.2 | 1.3 | 1.3 | 1.4 |
| 1.8 | 1.3 | 1.3 | 1.4 | 1.4 |
| 2.3 | 1.3 | 0.9 | 0.9 | 0.8 |
| 1.7 | 1.3 | 1.2 | 1.3 | 1.3 |
| 1.7 | 1.4 | 1.3 | 1.3 | 1.3 |
| XXX | 1.0 | XXX | 0.8 | XXX |
| 1.8 | 1.5 | 1.3 | 1.2 | 1.2 |
| 2.4 | 1.5 | 1.3 | 1.3 | 1.2 |
| 2.1 | 1.2 | 0.8 | 0.8 | 0.8 |
| 1.9 | 1.5 | 1.3 | 1.2 | 1.2 |
| 2.0 | 1.4 | 1.3 | 1.2 | 1.2 |
| XXX | 1.0 | XXX | 0.8 | XXX |

( $m_o = \pm 5,0 \mu m$ )

Control: 4 Full, 21 Elevation

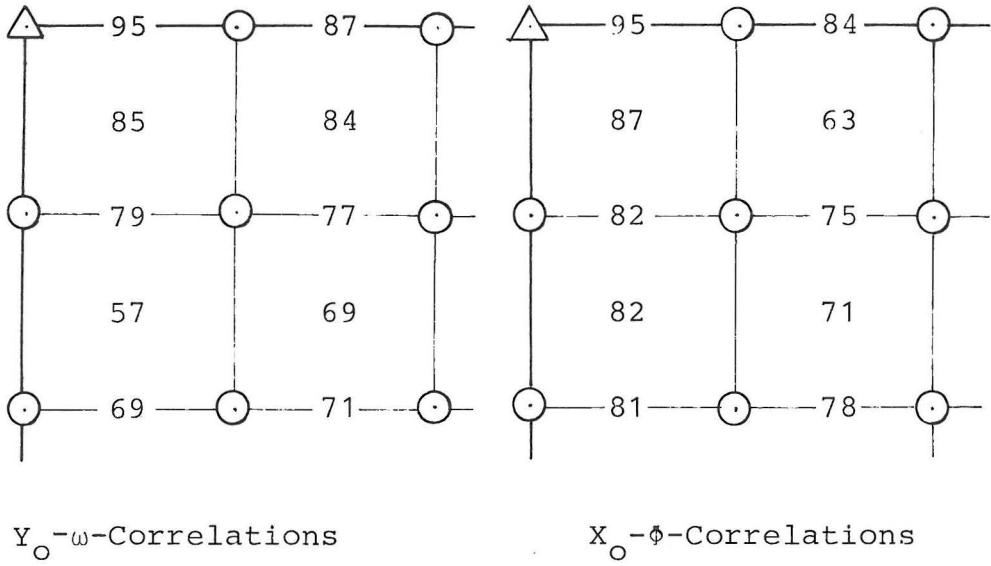
|     |     |     |     |     |
|-----|-----|-----|-----|-----|
| XXX | 0.9 | XXX | 0.7 | XXX |
| XXX | 0.9 | XXX | 0.7 | XXX |
| XXX | 1.8 | XXX | 1.4 | XXX |
| 0.8 | 0.6 | 0.5 | 0.5 | 0.5 |
| 1.4 | 0.7 | 0.5 | 0.5 | 0.5 |
| 2.2 | 1.3 | 0.9 | 0.9 | 0.8 |
| XXX | 0.6 | 0.5 | 0.5 | 0.5 |
| XXX | 0.7 | 0.5 | 0.5 | 0.5 |
| XXX | 1.0 | XXX | 0.8 | XXX |
| 0.8 | 0.6 | 0.5 | 0.5 | 0.5 |
| 1.4 | 0.7 | 0.6 | 0.5 | 0.5 |
| 2.0 | 1.2 | 0.8 | 0.8 | 0.8 |
| XXX | 0.7 | 0.5 | 0.5 | 0.5 |
| XXX | 0.6 | 0.6 | 0.5 | 0.5 |
| XXX | 1.0 | XXX | 0.8 | XXX |

( $m_o = \pm 5,0 \mu m$ )

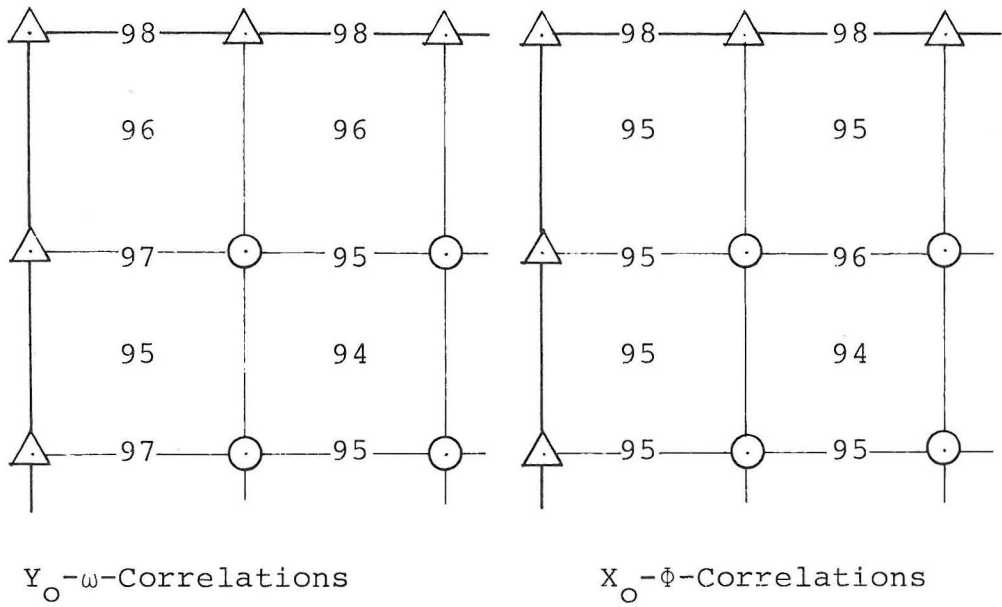
Control: 16 Full, 9 Elevation

Fig.: 6 Normalized Standard Errors of Object Point Coordinates

7 Strips (60 % side lap) of 9 Photos, 9 Points/Photo  
For details c.f. Fig. 4

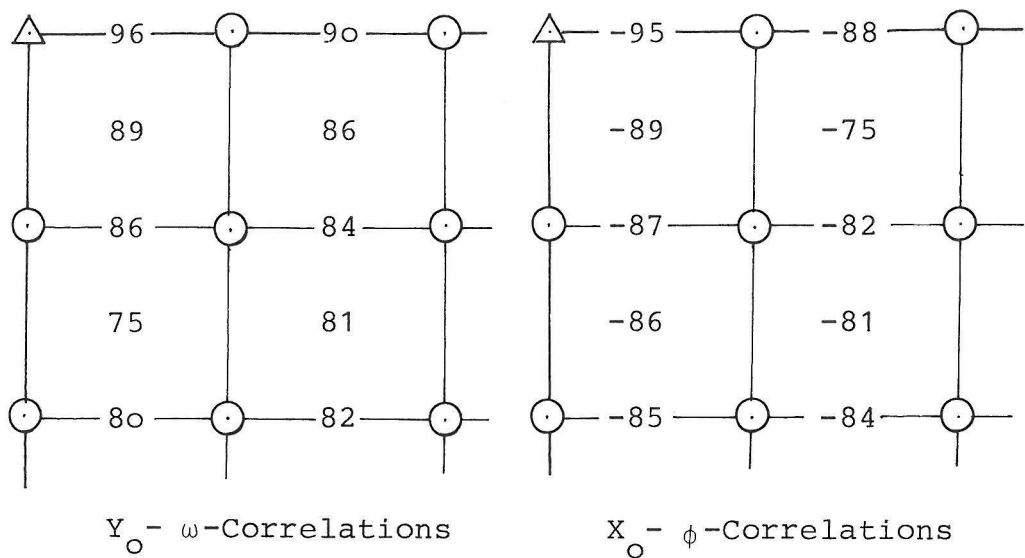


Control : 4 Full, 21 Elevation

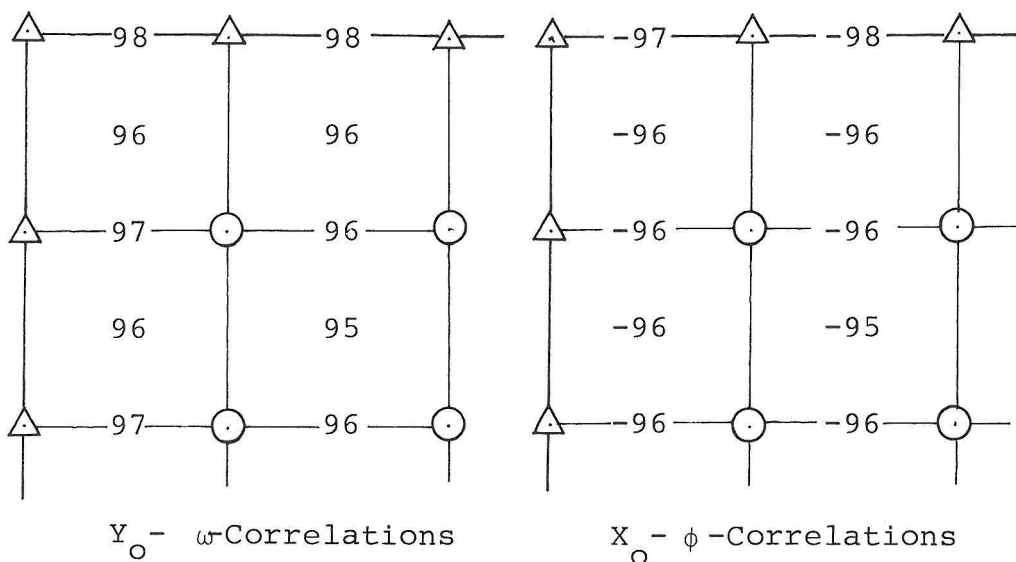


Control : 16 Full, 9 Elevation

Fig. 7    4 Strips With 9 Photos, 9 Points/Photo  
 60 % end lap, 20 % side lap



Control: 4 Full, 21 Elevation

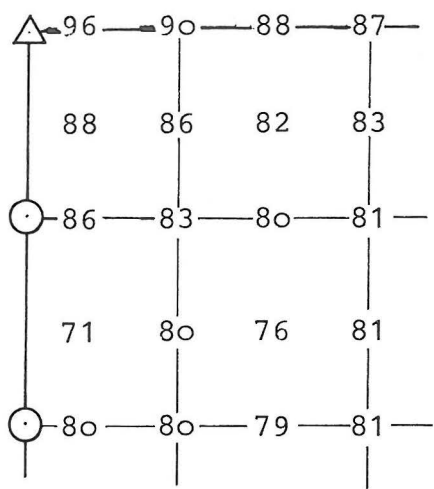


Control: 16 Full, 9 Elevation

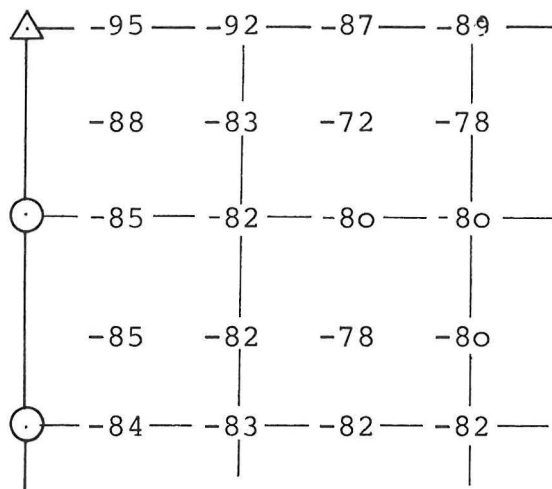
Fig.8 2 x 4 Strips (Crosswise) With 9 Photos,  
9 Points/Photo

60 % end lap, 20 % side lap

As there is full symmetry in both flight directions, only one direction of lines is shown.

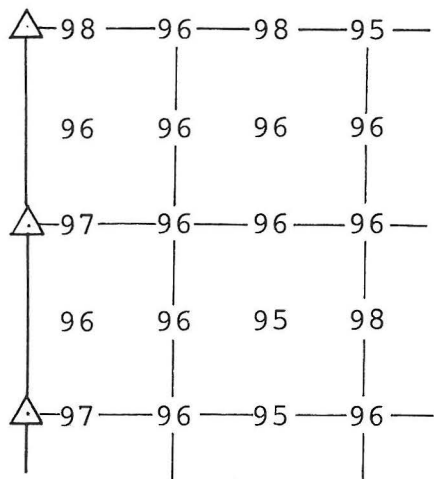


$Y_0 - \omega$ -Correlation

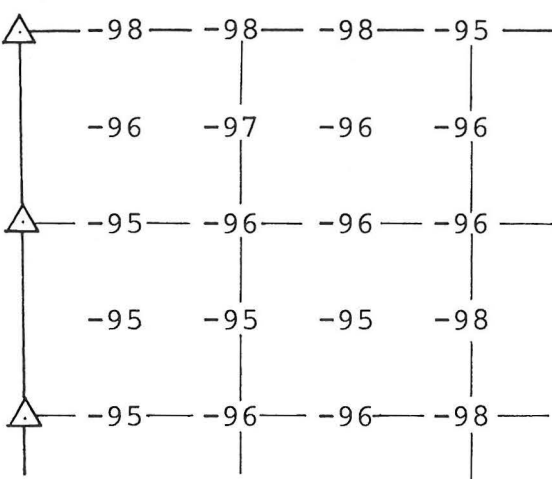


$X_0 - \phi$ -Correlation

Control: 4 Full, 21 Elevation



$Y_0 - \omega$ -Correlation



$X_0 - \phi$ -Correlation

Control: 16 Full, 9 Elevation

Fig.9 7 Strips With 9 Photos (60 % end lap, 60 % side lap)

There are significant correlations between  $Z_0$  and  $\phi$  at the ends of the strips, where only halves of the  $\phi$  photos are covered with points. The amounts are about 70 in case of dense control against 60 in case of sparse control.

#### *OBJECT COORDINATES*

Cross correlations between ground coordinates, that are of any consequence, do not occur after adjustment. Hence one can restrict oneself to auto correlations. These are displayed graphically for typical situations of points within a given block and flight pattern, c.f. Fig.10. Correlations between different reference points and the remaining points are shown by iso lines with 10 % intervals. The heavier lines are those of 0 % and 50 % respectively.

Largest auto correlations occur with reference points which are at largest distances to control points. Thus sparse control leads to strong correlations. In the cases at hand with dense elevation control, X-and Y-correlations are dominant. Both coordinate directions (flight direction is in X-direction) give only small differences in results. For the block center as reference point, half the block size is roughly correlated with larger amounts than 70 %, which is commonly thought to be significant.

Sparse control given, larger blocks show this effect more distinctly.

Auto correlations collapse at once, if at least dense peripheral horizontal control is at hand. Changes of flight patterns do not alter correlation patterns significantly.

The change from 9 to 25 points per fully covered photo diminishes the different amounts only slightly. This effect was however only studied with the 3 x 7 block.

#### *CROSS CORRELATIONS BETWEEN OUTER ORIENTATION AND OBJECT COORDINATES*

Strong correlations as explained in the foregoing chapters do not occur in this domain. There are however typical patterns which are observed in well defined areas of a block. Correlations between  $Z_0$  at the last photo of an outer strip and Y-object-coordinates are found as shown in Fig.11

There is a quite similar correlation between K and X-Object-coordinates.

While these correlations occur typically only in connection with photos at the perimeter of a block, there exist cross-correlations of a similar kind in the middle of a block between  $X_0$ -reference-positions and X-object-coordinates. The same applies to  $y_0$  and Y at this position. Flight patterns with more strips at the same block size lead to a slight decline of these typical correlations.

As was found in other context these cross correlations dwindle with densification of control. Thus a  $Z_0$ -Z-Correlation does not come to any amounts because of the dense control in elevation.

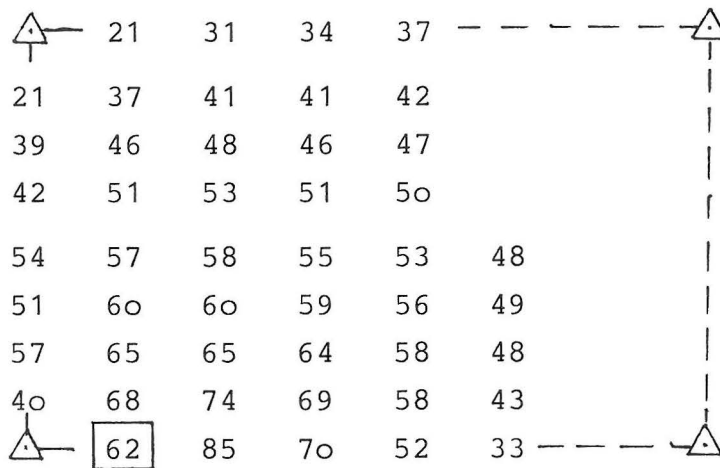


Fig.11 Cross Correlations between  $Z_0$  at Reference Position And Y-Object-Coordinates

4 Strips of 9 Photos, 60 % End Lap, 20 % Side Lap  
9 Points/Photo

Control: 4 Full, 21 Elevation

Cross correlations are not exhaustively described so far. The remaining ones are however of lesser consequence.

#### 2.4 CONCLUSIONS

Dealing only with the algebraical aspect of standard errors and correlations after bundle adjustment, one finds that sparse control gives rise to poor inhomogeneous standard errors as well as to relatively high correlations. An adverse effect is only discernible with cross correlations of elements of outer orientation. Hence dense perimeter control should be recommended as a remedy to these shortcomings. To a certain extent flight patterns with 60 % side lap and - even better - crosswise flights are able to procure homogeneous and fairly good results.

A combined effect will be reached by procuring both, good control distribution as well as a properly chosen flight pattern.

A further enhancement will be achieved by enlarging the number of points per photo.

All findings hold true for normally distributed observations which may however be ascertained by removing systematic errors by field or selfcalibration technics.

#### 3) REAL BLOCK ADJUSTMENTS INCLUDING ADDITIONAL PARAMETERS

The test area of Rheinbach, as discribed in /3/ has been choosen as an example for demonstration of practical circumstances.

The block was flown with a Zeiss RMK A 15/23 WA camera. Four parallel strips of 11 photos each at a scale of  $app. 1 : 11,000$  from one block of  $app. 8 \text{ km} \times 10 \text{ km}$ . The version chosen in this context is with sparse planimetric control at the corners, a densification of elevation control with 2 b distance at the

perimeter and one additional elevation point at the center of the block. Thus there is less control, at least in elevation, as was chosen for the synthetic block. On the other hand the blocks are of similar size.

There exist three computational versions of bundle adjustment:

- one without additional parameters (version 1),
- one with twelve additional parameters (version 2)
- one with a set out of that beforementioned one (version 3)

The additional parameters do ( or should) from orthogonal sets:

$$x = a_{01} y (+ a_{11} xy) + a_{12} xy^2 + a_{21} x^2 y (+ a_2 y^2) (+ a_{22} x^2 y^2)$$

$$y = b_{01} y (+ b_{11} xy) (+ b_{12} xy^2) + b_{21} x^2 y + b_{20} x^2 (+ b_{22} x^2 y^2)$$

The expressions in brackets were removed after a T-test, which proved them to be not significant under any circumstances. As there were about 11 to 16 points per fully covered photo there was theoretically a good reason for choosing the whole parameter-set. Adverse point distributions led to the effect mentioned. The weight zero for all additional parameters may have added to this end.

The following Table 2 shall give at least an insight into correlations which exist between elements of arter orientation themselves and object point coordinates on the other hand. Table 3 shall depict correlations between additional parameters and object point coordinates.

In this case object coordinates directions and flight line directions cut at an angle of appr. 50 gon. Thus coordinates  $X_0$  and  $Y_0$  as well as angles  $\omega$  and  $\phi$  are not handled separately, but mean values for both of them will be shown in Table 2.

It appeared that the term  $b_{12} xy^2$  was not significant in version 3 which it had been in version 2.

A comparison of versions 2 and 3 at Table 2 reveals the following. The full set of additional parameters led to high auto- and cross-correlations between elements of outer orientation. These diminish after deleting the non significant terms. The phenomenon is also to be observed with correlations between outer orientation elements and object point coordinates. These effects are on the other hand restricted to  $X_0/Y_0$  and  $\phi/\omega$  respectively and do not occur with  $Z_0$  and  $K$ .

High correlations between elements of outer orientation and additional parameters do not occur. Root mean square errors at check points prove version 2 to be better than version 3, which may however be quite accidental.

Table 3 at least reveals that high correlations between additional parameters and object coordinates are also in version 2 rather seldom.

The fact that some terms of version 2 are insignificant does not seem to be a good criterium for omission or to avoid an ill conditioned system. Orthogonality of terms (which was not at all the case in version 2) and 3) of parameters is no



radical medicine. Restricted number of terms does not (or at least seems not to) guarantee best results.

Standard errors of all 'unknowns do not clearly alter their amount from one version to an other.

Correlations between object point coordinates remain relatively stable without changing from version to version.

These few remarks show to some extent that we may get good results when applying additional parameters in field or self calibration. There remain however some crucial lacks of insight into the criteria which have us on the safe side of their application.

#### ACKNOWLEDGEMENTS

This paper would not be presented without programming and computational work carried out by Miss Dipl.-Ing. Gertrud Terbrack and Mr. Dipl.-Ing. Harald Schuh concerning synthetic blocks.

All computations were carried out at the IBM 370/168 of the RHRZ Bonn.

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Table 2

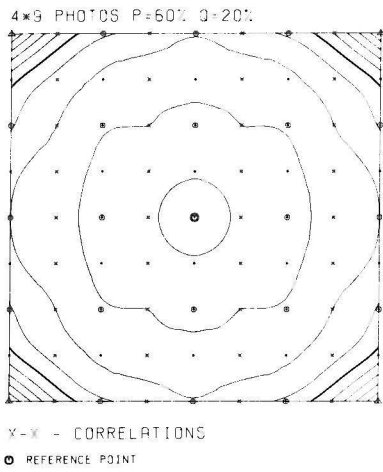
Correlation of Elements of Outer Orientation Between Themselves  
and Between These Elements and Object Point Coordinates  
There are the numbers of Correlation given in per cent which  
surpass a certain amount

| Unknowns                 |                | Correlation Exceed the Magnitude Given Below For |    |    |    |    |                          |    |    |    |    |
|--------------------------|----------------|--|----|----|----|----|--------------------------|----|----|----|----|
|                          |                | Elements of Outer Orientation                    |    |    |    |    | Object Point Coordinates |    |    |    |    |
|                          |                | 90   | 80 | 70 | 60 | 50 | 90                       | 80 | 70 | 60 | 50 |
| Version 2                | $X_O, Y_O$     | 6  | 26 | 98 | 64 | 17 | -                        | 2  | 7  | 28 | 36 |
|                          | $Z_O$          | 2  | 2  | -  | 6  | 4  | -                        | 8  | 12 | 11 | 16 |
|                          | $\phi, \omega$ | 3  | 27 | 88 | 78 | 20 | -                        | -  | 2  | 18 | 43 |
|                          | $K$            | -  | -  | -  | -  | 4  | -                        | -  | -  | -  | 5  |
| Version 3                | $X_O, Y_O$     | 2  | -  | 2  | 2  | 11 | -                        | -  | -  | 6  | 11 |
|                          | $Z_O$          | 2  | 2  | -  | 6  | 8  | -                        | 8  | 12 | 11 | 17 |
|                          | $\phi, \omega$ | -  | -  | 3  | 3  | 14 | -                        | -  | 4  | 6  | 10 |
|                          | $K$            | -  | -  | -  | -  | 2  | -                        | -  | -  | -  | -  |
| Whole Number of Unknowns |                | 264  |    |    |    |    | 435                      |    |    |    |    |

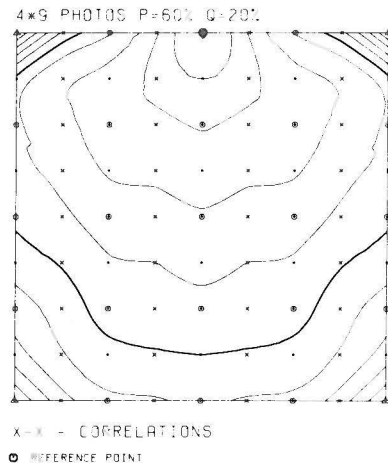
The given values are related to a photo, which is situated within the center of the block, thus avoiding peripheral effects.

Table 3. Correlations Between Additional Parameters and Object Point Coordinates (c.f. Table 2)

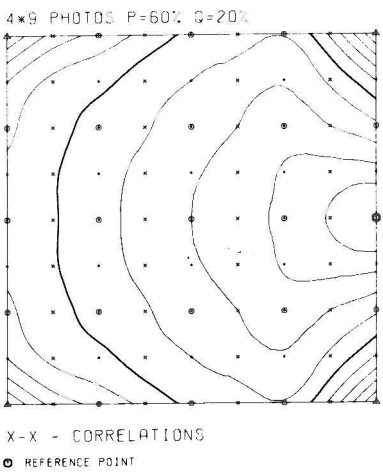
| Term             | Version 2                            |    |    |    |    | Version 3 |    |    |    |    |
|------------------|--------------------------------------|----|----|----|----|-----------|----|----|----|----|
|                  | Correlations Exceed the Magnitude of |    |    |    |    |           |    |    |    |    |
|                  | 90                                   | 80 | 70 | 60 | 50 | 90        | 80 | 70 | 60 | 50 |
| $a_{01} Y$       | -                                    | -  | -  | 39 | 42 | -         | -  | -  | 32 | 24 |
| $a_{11} X Y$     | -                                    | 2  | 27 | 40 | 43 | -         | -  | -  | -  | -  |
| $a_{12} XY^2$    | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |
| $a_{21} X^2 Y$   | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |
| $a_2 Y^2$        | -                                    | 1  | 16 | 28 | 21 | -         | -  | -  | -  | -  |
| $a_{22} X^2 Y^2$ | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |
| $b_{01} Y$       | -                                    | -  | -  | -  | 6  | -         | -  | -  | -  | -  |
| $b_{11} XY$      | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |
| $b_{12} XY^2$    | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |
| $b_{21} X^2 Y$   | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |
| $b_{20} X^2$     | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |
| $b_{22} X^2 Y^2$ | -                                    | -  | -  | -  | -  | -         | -  | -  | -  | -  |



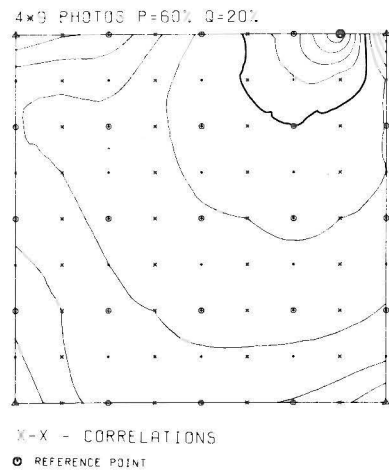
a)



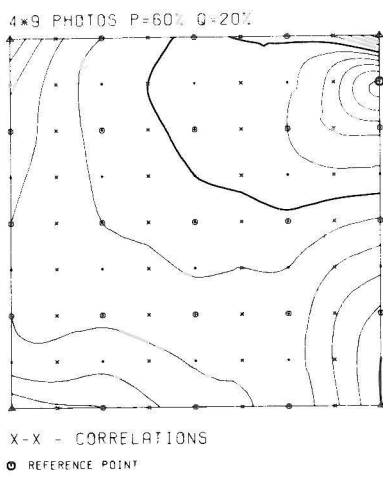
b)



c)



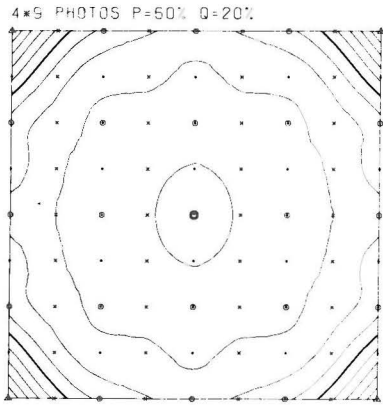
d)



e)

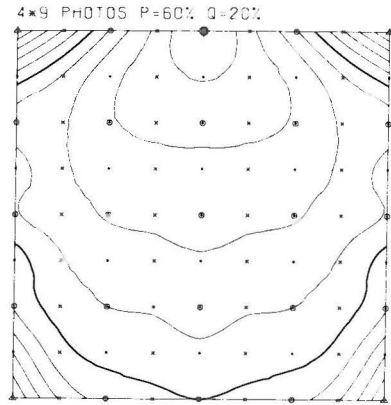
Fig.10

- △ Full Control
- Elevation Control
- × Exposure Station
- Object Point



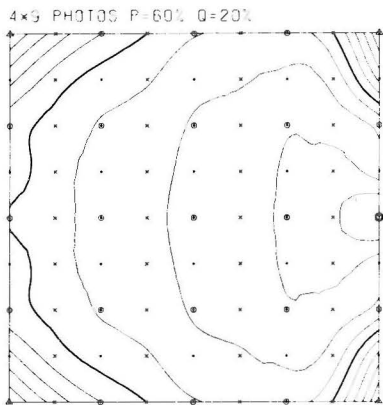
Y-Y - CORRELATIONS  
 ○ REFERENCE POINT

f)



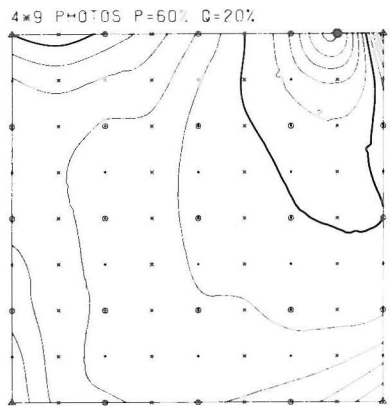
Y-Y - CORRELATIONS  
 ○ REFERENCE POINT

g)



Y-Y - CORRELATIONS  
 ○ REFERENCE POINT

h)

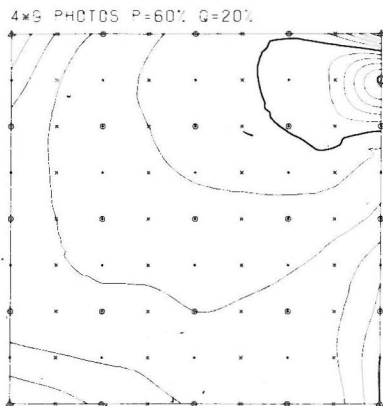


Y-Y - CORRELATIONS  
 ○ REFERENCE POINT

i)

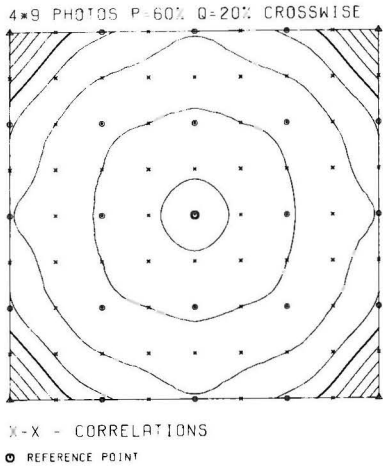
Fig.1o

Continued

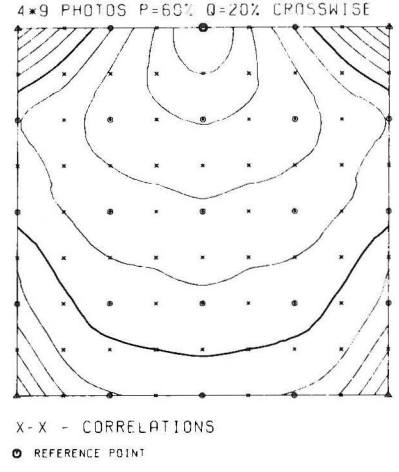


Y-Y - CORRELATIONS  
 ○ REFERENCE POINT

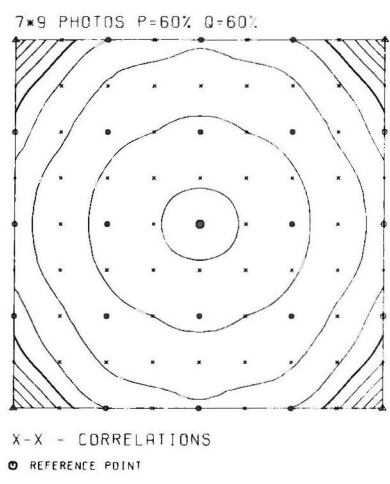
j)



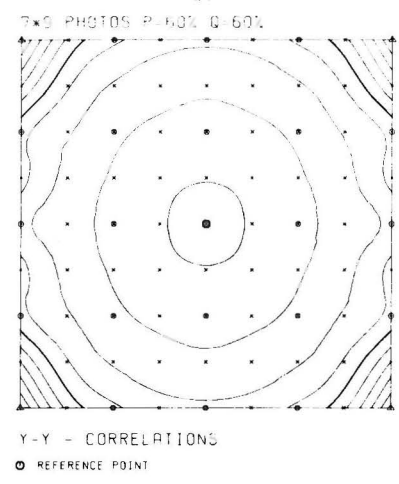
k)



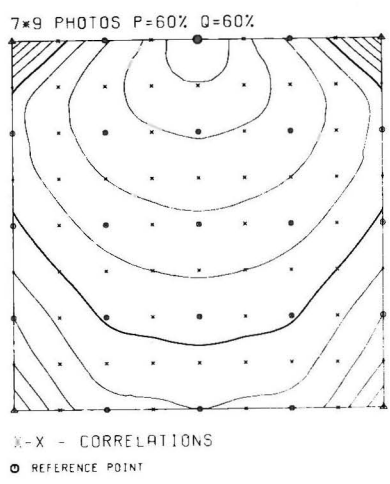
l)



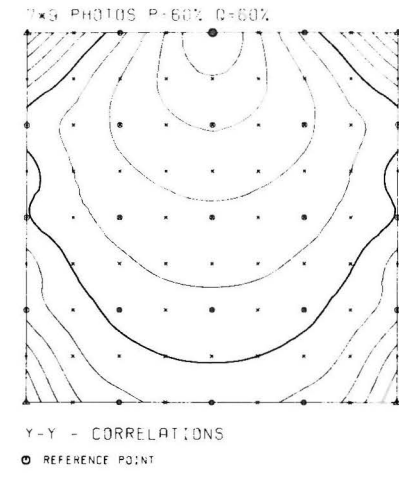
m)



n)



o)



p)

Fig.1o continued