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Presented Paper

The Terrestrial/Photogrammetric (TP) Technique for the Detection and Compensation of Systematic Height Errors in Block Aerial Triangulation

By M.M. Abdel Rahim

Topographic Science, Department of Geography, University of Glasgow, U.K.

1. Accuracy of Aerial Triangulation as derived from Theoretical Considerations

It is well known that the expected accuracy of aerial triangulation can be derived theoretically for different patterns and densities of control points, relying on the principles of propagation of errors. The theoretical investigations which have been carried out with simplified error models can be classified into three main groups (Kubik & Kure 1972) as follows:-

(1) Accuracy of planimetric strip and block triangulation based on the independent model approach (Ackermann 1966).

(2) Accuracy of height strip and block triangulation based on the theory of transfer errors (Jerie 1968).

(3) Accuracy of planimetric and height strip and block triangulation based on the bundle approach (Kunji 1968).

Some conclusions from the height accuracy investigations are as follows:-

(i) If there is no perimeter control, the maximum standard errors in a block with bands of control across the strip will occur at the edges of the block.

(ii) Both the maximum and the mean standard errors in a block are almost independent of the size of the block and depend mainly on the bridging distance between the bands of control.

(iii) The bridging distance between bands of control must be reduced if the overall accuracy of the block is to be improved.

(iv) When auxiliary data is not available, the standard arrangement for the positioning of height control in a block is to provide it in bands across the strips. These control points should be located in (or close to) each lateral overlap, in order to check the lateral tilts of the strips.

The technique which will be described below makes use of these conclusions in detecting and eliminating uncompensated systematic height errors in adjusted photogrammetric blocks.

2. Outline of the Terrestrial/Photogrammetric (TP) Technique

For each of the two blocks to be used as examples (Fig. 1) the horizontal lines define strips of photography, while the vertical lines represent sections across the strips at specific intervals along the strips. Individual models are not shown on the diagram.

ADJUSTMENT PROCEDURES

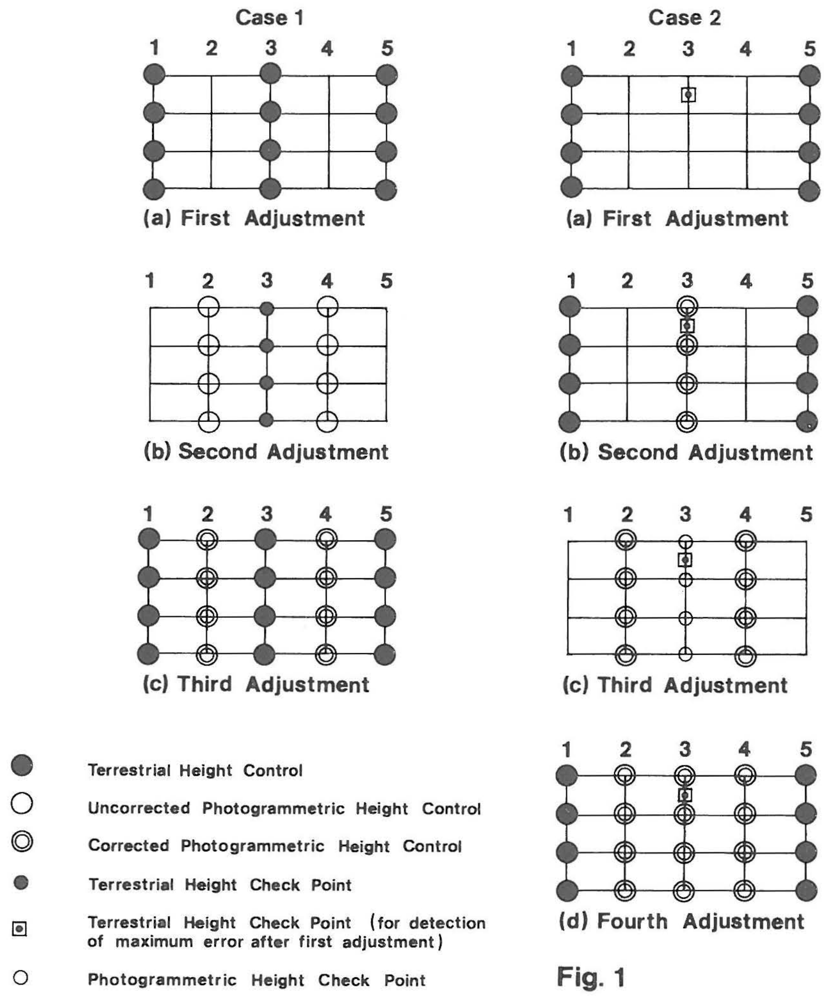

Fig. 1

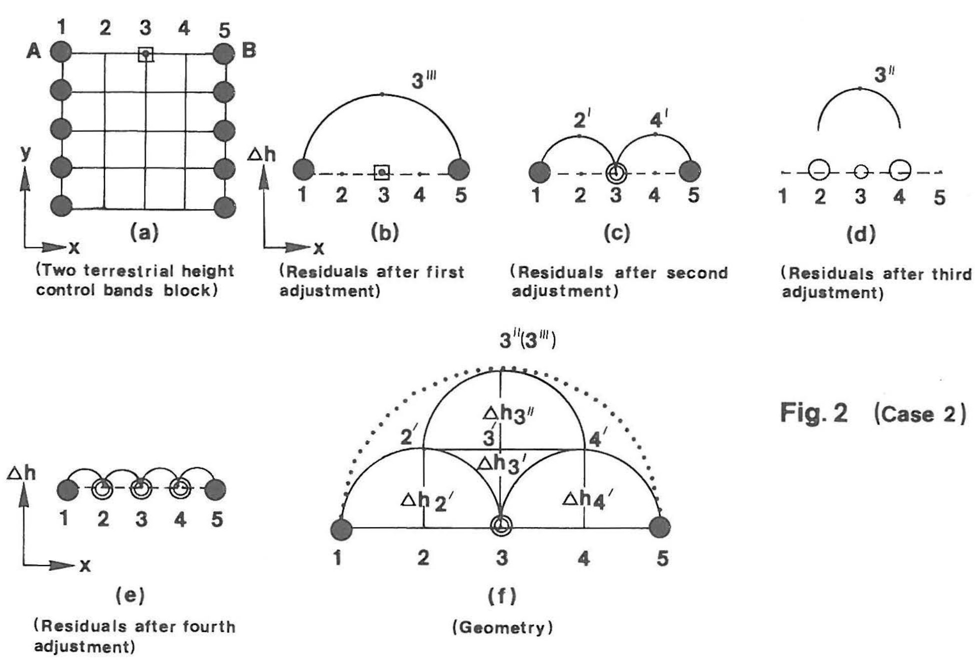

Fig. 2 (Case 2)

In Case 1, the pattern of height control consists of three lines of height control points located at the beginning, the middle and the end of each strip. In Case 2, the pattern consists of two lines located at the beginning and the end of each strip with a single height check point located anywhere in section 3.

Case 1 The procedure starts with the formation of the block and its initial adjustment using the height control points available in Case 1. From this first stage of the adjustment the maximum height errors may be assumed to exist midway between the bands of control. For Case 1, this will be along the vertical sections marked 2 and 4 in Fig. 1.

The next step, i.e. the second stage, is to repeat the adjustment (Case 1(b), Fig. 1) using as height control the photogrammetrically-determined values lying along sections 2 and 4 derived from the first stage of the adjustment procedure. This gives new values for the points lying along section 3, which can be compared with the known values. The difference between the two sets of values for the points lying along section 3 is the basis for correcting the photogrammetric values of the points lying along sections 2 and 4.

Finally, the third stage in the adjustment procedure is to repeat it using as control all five bands of control lying along sections 1 to 5, i.e. the terrestrial values for sections 1, 3 and 5 and the corrected photogrammetric values for sections 2 and 4. The final result, as will be shown later, will be an improvement in the absolute accuracy of the height points throughout the block.

Case 2 This shows the application of the technique to a different pattern of control in which the band of height control points lying along section 3 is replaced by a single height check point. To illustrate the effect of systematic height errors after the first adjustment, consider a horizontal section AB through the block (Fig. 2a). This horizontal section will have systematic height errors after the first adjustment as shown by the curved line in Fig. 2(b). The photogrammetric heights (termed provisional heights) of the tie points located along vertical section 3 will be determined, and the largest height errors may be expected to occur in these points. A single terrestrial check point located in vertical section 3 (coinciding with tie point 3) will have two different values - its provisional height and its terrain value. The difference will form the basis for correcting the provisional values of all height points located along section 3. These corrected values are then used, together with the given terrestrial values along sections 1 and 5 as control points for the second stage of the adjustment (see Case 2(b)) to determine the photogrammetric height values (again termed provisional heights) of points lying along sections 2 and 4.

After this second stage of the adjustment procedure, the remaining errors will be indicated in Fig. 2(c) by the two curved lines. If the third stage of the adjustment procedure (Case 2(c) is then carried out, using as control the provisional heights of these points located along sections 2 and 4, the resulting errors will be as indicated by the curved line in Fig. 2(d). Points located along section 3 will have two different values that will form the basis for correcting the provisional values of points located along sections 2 and 4. The corrected values for the points lying along sections 2 and 4 are then used, together with the given terrestrial values of other points, as control for the fourth stage of the adjustment (see Case 2(d)). The remaining errors after this adjustment will be as indicated by the curved lines in Fig. 2(e). The height errors which will be present at each stage of the adjustment procedure are shown in Fig. 2 (f).

3. Detailed Explanation of the Systematic Errors Detected at Different Stages of the Procedure

This will be carried out in detail for Case 2. Fig. 2(f) can be considered to be an amalgam or summary of the previous diagrams Fig. 2(b) to (e).

The surface 1, 2, 3, 4, 5, is the surface to which systematic errors are referred after the first adjustment. The maximum error will occur at point 3, in this case 3"'. The two curved lines 1, 2', 3 and 3, 4', 5 represent the surfaces interpolated by the second adjustment with the maximum systematic errors occuring at points 2' and 4'. These become the reference surfaces for systematic errors after the third adjustment. The surface represented by the curved line 2', 3", 4' is the surface interpolated by the third adjustment with a maximum error at point 3". The interpolated surface would be 2', 3', 4' if there were no systematic errors after the third adjustment.

As mentioned above, the maximum error after the first adjustment will be at tie point 3. This error is equal to the difference between the two values of terrestrial check point 3 (the reference point). If points 2 and 4 were correct control values used in the third adjustment, the maximum error corresponding to the bridging distance 2, 4 would be $\Delta h_3'$ (equal to $\Delta h_2'$ and $\Delta h_4'$). However point 2 has an actual error $\Delta h_2'$ and point 4 an error $\Delta h_4'$. These two values may be considered as constant errors. Thus, after the third adjustment, in which the bridging distance is 2, 4, the maximum error will be $\Delta h_3''$ (equal to $\Delta h_3'$). The difference between the two values of photogrammetric height check point 3 (the reference point) is then equal to $\Delta h_3'$ plus $\Delta h_3''$. Let this difference be Δh_3(photogrammetric). That is to say,

$$\Delta h_3(\text{photogrammetric}) = \Delta h_{3'} + \Delta h_{3''} = 2 \cdot \Delta h_2' = 2 \cdot \Delta h_{4'} \quad \text{or}$$

$$\boxed{\Delta h_{2'} = \Delta h_{4'} = \tfrac{1}{2} \cdot \Delta h_3(\text{photogrammetric})}$$

4. Test Data and Results

A comprehensive series of tests have been carried out to check the validity of the TP Technique using data from aerial triangulation of several blocks of photography, each with different parameters regarding number of strips, length of strip between control points, photographic scale, pattern of control points, etc. The following sets of data have been used in the test:-

(a) The Durban Test Block Comprising 48 models in 4 strips at a photo scale of 1 : 8,000.

(b) The Pietermaritzburg to Durban Test Strip Comprising 31 models in a single strip at a photo scale of 1: 30,000.

These two sets of data (a) and (b) were provided through the courtesy of the Department of Surveying of the University of Natal, Durban, South Africa.

(c) The DOS Test Block comprising 34 models in 2 strips at a photo scale of 1: 12,500.

This was obtained from the Directorate of Overseas Surveys, Tolworth, England.

(d) The Oberschwaben Test Block comprising 32 models in 4 strips at a photo scale of 1: 28,000.

This was provided through the good offices of the ITC, Enschede, The Netherlands.

These data have been tested extensively using the large (3 megabyte) ICL 2976 mainframe computer of the University of Glasgow. These blocks were tested in a variety of different ways - with different block sizes and different control patterns. The tests made use of the S.B.A.I.M. (Simultaneous Block Adjustment of Independent Models) Program developed by El Maleeh (1976).

Only a short summary table of results can be given here. These are presented in Table No. 1. The first column of the table gives the bridging distance between the bands of control points. The second column gives the expected maximum error as predicted from the theoretical considerations discussed in Section 1. The third and fourth columns give the maximum error values and the standard deviations of the height residuals respectively, first without using the TP technique and then after its use. The fifth column of the Table gives the factor of improvement, comparing the figures for standard deviation before and after the application of the TP technique.

As will be seen, an improvement factor in height accuracy ranging from 2.4 to 14.8 was obtained over bridging distances ranging from 8 to 17 models. The improvement of the height accuracy resulted also in an improvement factor of the planimetric accuracy ranging from 1.05 to 1.53 over bridging distances ranging from 6 to 16 models in other tests.

It will be noted that the largest factors of improvement in the height accuracy are obtained in test blocks (b) and (c) which have the largest bridging distances, and the smallest factor in block (d). This was the Oberschwaben block where the bridging distance is small and the coordinates used in the test had previously been highly corrected for known systematic errors.

5. Comparison with Other Adjustment Techniques.

An attempt has been made to compare the results of the TP technique with other techniques which have been used in recent years using results obtained from various blocks from the well known Oberschwaben test area in each case. These include the additional parameter technique (Bauer & Muller, 1972), self-calibration (Ebner, 1976; Schneider, 1978) and the method of Haugh (1976). Table No. 2 gives the results of this comparison. The accuracy comparison ratio is obvious enough; the cost comparison ratio takes into account the number of control points used in each case.

Comparing it with the other methods, the TP technique appears to be effective, inexpensive and simple to apply. For example, an improvement in the height accuracy from 31.2µm (the poorest accuracy) to 13.2µm (the highest accuracy) was obtained in the Oberschwaben Test Block using the TP technique. It will be noted that the TP technique yields this highest accuracy (13.2µm) while using the least number of terrestrial height points (11 points). The accuracy comparison ratio between the TP technique and the other methods ranges from 1.05 to 1.30. The cost (which in this case is assumed to be proportional to the number of the terrestrial points used) has a comparison ratio ranging from 1.1 to 2.3 corresponding to the range of 12 to 25 terrestrial height points as shown in Table 2.

6. General Discussion and Conclusions

From an analysis of the results of these different projects the following points may be made:-

(i) The results obtained in the various tests have verified well-known conclusions regarding the presence of systematic errors in block triangulation. In particular, the following points may be made:-

   (a) The existence of systematic height errors spoils the overall accuracy of the block.

   (b) Systematic height error is independent of block size and appears to depend to a large extent on the bridging distance between bands of control.

   (c) Maximum systematic height error occurs midway between bands of control.

(ii) A substantial improvement of height accuracy results in a discernible and useful improvement in planimetric accuracy.

(iii) In the various tests carried out using the TP technique, inspection of the systematic height residuals shows that the adjusted photogrammetric heights are almost always larger than the corresponding terrestrial heights for the same points. The explanation for this probably lies in the fact that no account was taken of Earth's curvature and refraction before entering the adjustment phase.

(iv) The results of the various theoretical accuracy studies discussed in Section 1 of this paper can be considered sufficiently realistic since an accuracy better than the expected one could be obtained after elimination of existing systematic height errors. That is to say, the errors of real photographs generally behave in accordance with the theoretical assumptions and the various mathematical models which have been put forward predict reality to a sufficient degree for most purposes.

(v) The TP technique saves the cost of fixing the additional ground control points that would be required to improve the accuracy by an equivalent amount, if the technique had not been used.

(vi) The arrangement of height control which is available for inclusion in the block adjustment affects results. The ideal arrangement of regularly spaced bands of control will result in the optimum improved accuracy being obtained.

(vii) Only two bands of terrestrial height control located at the beginning and end of the block together with an additional terrestrial height check point lying midway between them are sufficient to obtain an excellent height accuracy. Thus if n is the number of parallel strips in a rectangular block, then the terrestrial heights of only (2n + 3) points will be required to obtain this height accuracy when using the TP technique.

(viii) By comparison with other methods (including those of Self-Calibration), the results show that the TP technique is an effective, inexpensive and simple method of removing systematic height errors. On the basis of the present tests it can be strongly recommended for application in practice.

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Projects	(1) Bridging Distance (No. of models)	(2) Expected Maximum Error (‰ H)	(3) Maximum height residuals detected at the terrestrial height check points (‰ H)		(4) The Standard Deviation of the Height Residuals Detected at the Terrestrial Height Check Points (‰ H)		(5) Height Accuracy Improvement Factor
			Before height accuracy improvement	After height accuracy improvement	Before height accuracy improvement	After height accuracy improvement	
Durban Block (a)	12	± 0.725	- 1.420	- 0.417	0.832	0.156	5.3
Pietermaritzburg Strip (b)	16	± 1.28	- 7.378	0.797	5.398	0.418	12.9
DOS Block (c)	17	± 0.94	- 14.920	1.273	11.077	0.747	14.8
Oberschwaben Block (d)	8	± 0.525	- 0.342	0.304	0.204	0.086	2.4

TABLE No. 1 - RESULTS FROM TEST DATA

TABLE No. 2 - Comparison of Results (Oberschwaben Test Area)						
TECHNIQUE No.	DISTANCE	Number of Terrestrial Height Points used for the same block size	m*z / µm /		Comparison Ratio between the TP Technique and other techniques	
			Before the height accuracy improvement	After the height accuracy improvement	Accuracy	Cost
1	8	11	31.2	13.2		
2	8	12	22.2	14.6	1.1	1.1
2	8	12	19.0	17.1	1.30	1.1
2	4	19	14.7	14.1	1.1	1.7
3	5	19	20.0	14.0	1.1	1.7
4	5	19	19.6	16.1	1.2	1.7
4	5	19	19.6	14.7	1.1	1.7
4	2	25	15.7	15.5	1.2	2.3
4	2	25	15.7	14.4	1.1	2.3
4	2	25	15.7	15.8	1.2	2.3
5	2	25	14.9	14.9	1.1	2.3
5	2	25	14.6	14.5	1.1	2.3
6	4	19	15.1	13.8	1.05	1.7

m*z / µm / = The standard deviation of the height residuals at the terrestrial check points (in micrometres (µm) in the negative scale).
Technique No. 1 is the TP technique.

For results of Technique No. 2 (W.A.), See Ebner (1976).
" " " Technique No. 3 (W.A.), See Bauer & Muller (1972).
" " " Technique No. 4 (W.A.), See Bauer (1973).
" " " Technique No. 5 (S.W.A.), See Haugh (1976).
" " " Technique No. 6 (W.A.), See Schneider (1978).