INTERNATIONAL INSTITUTE
FOR
AERIAL SURVEY
AND
EARTH SCIENCES (ITC)
ENSCHEDE,THE NETHERLANDS

# ERROR DETECTION AND RELIABILITY STUDIES IN ANALYTICALLY FORMED STRIPS

T.BOULOUCOS

ISP COMMISSION III, PRESENTED PAPER

itc

FOURTEENTH CONGRESS OF THE INTERNATIONAL SOCIETY FOR PHOTOGRAMMETRY, HAMBURG 1980

Error detection and reliability studies in analytically formed strips

The paper describes a method for gross error detection during the process of strip formation by independent models and a check for the connection of the strips.
The procedure facilitates the detection of gross errors, before block adjustment, using analytically formed strips.
The reliability of the observations is also examined.

L'article décrit une méthode pour la détection de fautes au cours du procédé de formation de bande par modèles indépendents et un contrôle pour la liaison entre bandes.
La procédure facilite la détection de fautes, avant la compensation en bloc, à partir de bandes formées analytiquement.
La fiabilité des observations est également examinée.

Der Beitrag beschreibt ein Verfahren zur Aufdeckung grober Fehler während des Prozesses der rechnerischen Streifenbildung aus unabhängigen Modellen sowie eine Kontrolle des Zusammenschlusses benachbarter Streifen.
Das Verfahren ermöglicht das Auffinden von groben Fehlern vor dem eigentlichen Blockausgleich.
Die Zuverlässigkeit der Messungen wird ebenfalls beurteilt.

## Error detection and reliability studies in analytically formed strips

In recent years the detection of gross errors and the reliability of observations has been one of the main research directions in photogrammetry. There have been many publications on this theme dealing mostly with aerial triangulation systems by independent models or bundles, but aerial triangulation using strips as units, strips formed analytically from independent models or even from comparator measurements via model formation, is a favourite method for photogrammetric organisations, simply because source computer programmes for block adjustment by strips have appeared in photogrammetric publications.

In this paper, a method for gross error detection during the process of strip formation from independent models based on the "Data snooping technique", developed at Delft University, the Netherlands (see Ref. 1,2,3) is described.

Also the planimetric coordinates of the points are checked for errors due to point misidentification between strips. In the last part of the paper a study of the reliability of the observations is carried out.

### Notation

| | |
|---|---|
| $( \ )$ | : Indicates vector or matrix |
| $( \ )^*$ | : Transpose of a vector or matrix |
| $( \ )^{-1}$ | : Inverse of a matrix |
| $\underline{x}$ | : The underscore indicates stochastic variables |
| $\hat{x}$ | : Tilde stands for mathematical expectations e.g. $E\{\underline{x}\} = \hat{x}$ |
| $(g_{xx})$ | : Weight coefficient matrix of the stochastic variables $(\underline{x})$ |
| $g_{x_i x_j}$ | : The i, j element of the matrix $(g_{xx})$ |
| $\sigma_o^2$ | : Variance factor (Variance of unit weight) |
| $\lambda$ | : Scale factor |
| $(R)$ | : Rotation matrix |
| $(S)$ | : Vector of shifts |
| $(\underline{x}_i^I)$ | : Observed coordinates of point i in model I |
| $(\Delta\underline{X})$ | : Corrections to the observations |
| $(\underline{X}_i)$ | : The strip coordinates of point i |
| $\nabla x_i$ | : Error in observation i |

### 2. Mathematical model

#### 2.1. Functional model

The strip coordinates of the points and the transformation parameters are treated as unknown parameters for each connection.

The functional model then is:

$$(\hat{X}_i) - (\hat{x}_i^I) = 0 \qquad (2.1.1)$$

$$(\hat{X}_i) - \hat{\lambda}(\hat{R})(\hat{x}_i^{II}) - (\hat{S}) = 0 \qquad (2.1.2)$$

2.2 Stochastical model

It is assumed that :

a) The model coordinates of the points are uncorrelated.

b) There is no correlation between models

After each connection, the strip coordinates of the points are treated as observations with the weight coefficient matrix obtained from the previous connection. In this way the weight coefficient matrix is generated for the strip.

N.B The assumptions a and b are not of course necessary for the algorithm, but do reduce the computational effort. A study on the weight coefficient matrices for model coordinates and the consequences of assuming uncorrelated model coordinates may be found in Ref. 4.

## 3. Detection of gross errors during the strip formation

### 3.1 Test quantity

Assuming that the only possible source of model errors are gross errors in the observations, the null hypothesis $H_o$ may be formulated as:

$H_o$ : There are no gross errors in the observations

This can be expressed as:

$$E \left\{(\underline{x})/H_o\right\} = (\hat{x}) \qquad (3.1.1)$$

The $H_o$ is tested against a series of alternative hypotheses $H_{a,p}$, assuming one error at a time in the system, then:

$$E \left\{(\underline{x}_p)/H_{a,p}\right\} = (\hat{x}_p) + \nabla x_p \qquad (3.1.2)$$

or

$$E \left\{(x_p)/H_{a,p}\right\} = (\hat{x}_p) + (c)\,\nabla_p \qquad (3.1.3)$$

$p = 1,\ldots,n$ n : number of observations

Hence the only consideration is a possible translation of the probability distribution of the observation $x_p$

(c) : is a column vector with elements

$c_i = 0$ if $i \neq p$

$c_i = 1$ if $i = p$

$\nabla_p$ : a parameter

For testing $H_o$ against $H_{a,p}$ it is possible to derive a one dimensional statistic (see Ref. 2)

$$W_p = \frac{(c)^* (g_{xx})^{-1} (\Delta \underline{X})}{\sigma_o \left((c)^* (g_{xx})^{-1} (g_{\Delta x \Delta x}) (g_{xx})^{-1} (c)\right)^{1/2}} \qquad (3.1.4)$$

The $H_o$ is rejected if

$$|W_p| > F^{1/2}_{1,\infty,a} \qquad (3.1.5)$$

Where : a, is a specified significance level

In such a case the corresponding $H_{a,p}$ is considered as a possible source os gross error.

3.2 Reliability

The quantity $\nabla x_p$ in formula (3.1.2) is not known; but a "Boundary value" $\nabla_o x_p$ can be computed which can just be found by testing with a power $\beta_o$.
The boundary value $\nabla_o x_p$ proves to be (see Ref. 2)

$$\nabla_o x_p = \sigma_o \left( \frac{\lambda_o}{(c)^* (g_{xx})^{-1} (g_{\Delta x \Delta x}) (g_{xx})^{-1} (c)} \right)^{\frac{1}{2}} \qquad (3.2.1)$$

Where : $\lambda_o$ is a function of the significance level a and the power of the test $\beta_o$.

given a, $\beta_o$ ; $\lambda_o$ can be taken from nomogrammes; then $\nabla_o x_p$ can be computed.
It is clear that larger values for $\nabla_o x_p$ can be found with a probability larger than $\beta_o$.
The boundary value, together with the probability $\beta_o$ express the reliability of the observations.

N.B The choice of the significance level a and the power $\beta_o$ is directly related to the statistical concepts of type I and type II errors, and hence with the cost factors of the project for which the measurements have to be carried out.
From experience, a = .001 and $\beta_o$ = .80 are used for gross error detection.
Using the test quantity (3.1.4) a test for gross erorrs is applied for each connected model in the strip. Also the boundary values of the observations are calculated.

## 4. Detection of gross errors for common points between strips

For checking the planimetric coordinates of the common points between the strips, the null hypothesis $H_o$ may be formulated as:

$H_o$ : There are no gross errors in the strip coordinates of the points.

Under $H_o$ it can be stated that, for a triplet of common points between two strips (see Fig. 1), the form elements, i.e. the ratio of lengths and the angle, are equal.

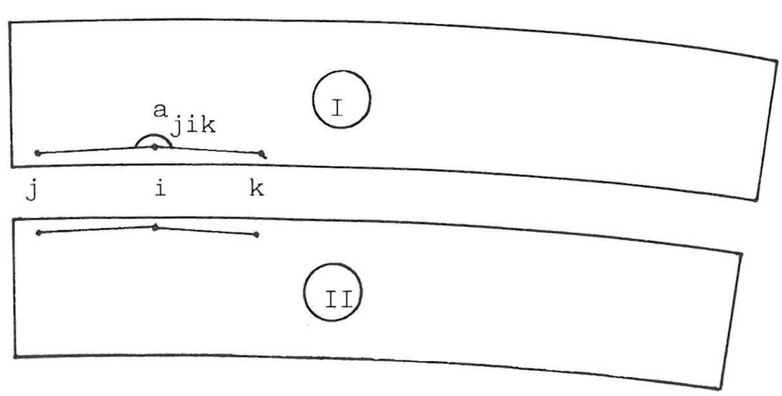



Fig. 1.

This means that:

$$\frac{\hat{l}^{I}_{ik}}{\hat{l}^{I}_{ij}} = \frac{\hat{l}^{II}_{ik}}{\hat{l}^{II}_{ij}} \qquad (4.1)$$

$$\hat{a}^{I}_{jik} = \hat{a}^{II}_{jik} \qquad (4.2)$$

If we define $\hat{V}_{jik} = \frac{\hat{l}_{ik}}{\hat{l}_{ij}}$ and take the naperian logarithms then the equation (4.1) may be written:

$$\hat{t}^{1}_{p} = \ln \hat{V}^{I}_{jik} + \ln \hat{V}^{II}_{kij} = 0 \qquad (4.3)$$

and

$$\hat{t}^{2}_{p} = \hat{a}^{I}_{jik} - \hat{a}^{II}_{jik} = 0 \qquad (4.4)$$

The conditions (4.3), (4.4) are formulated for triplets of common points between adjacent strips.

Substituting the strip coordinates of the points in these conditions, a set of misclosure variate will be produced.

$$\underline{t}^{1}_{p} = \ln \underline{V}^{I}_{jik} + \ln \underline{V}^{II}_{kij} \qquad (4.5)$$

$$\underline{t}^{2}_{p} = \underline{a}^{I}_{jik} - \underline{a}^{II}_{jik} \qquad (4.6)$$

Linearising the relations (4.5), (4.6) (see Ref. 8, 9) and applying the propagation law for the variance, the variance of the misclosures can be calculated (since the variance matrices of the strip coordinates are known from the strip formation).

Now, under $H_o$ we can write

$$E\left\{\underline{t}_p / H_o\right\} = 0 \qquad (4.7)$$

For each misclosure variate we test $H_o$ against the alternative hypothesis $H_a$ :

$$E\left\{\underline{t}_p / H_a\right\} \neq 0 \qquad (4.8)$$

It can be shown (see Ref. 5) that the test quantity is:

$$W_p = \frac{\underline{t}_p}{\sigma_{t_p}} \qquad (4.9)$$

Where $\sigma_{t_p}$ : is the standard deviation of the misclosure $\underline{t}_p$

$H_o$ is rejected if:

$$|W_p| > F^{1/2}_{1,\infty,a} \qquad (4.10)$$

In the case of a rejection, then the conclusion is that there are gross errors in the strip coordinates of the three points which are involved in the calculation of the rejected misclosure.

The boundary value of the coordinates of the points can be calculated by:

$$|\nabla x_i| = \left| \sigma_{t_p} \frac{\sqrt{\lambda_o}}{u_p^i} \right| \qquad (4.11)$$

Where:

$u_p^i$ is the coefficient of the coordinate $\underline{x}_i$ in the linearised expression of the misclosure $\underline{t}_p$.

The boundary values are used for an error location strategy.
An extensive treatment of this approach can be found in Ref. 5.

## 5. Experimental results

A series of experiments were performed with various blocks.
Results concerning the reliability of the observations only are presented here.
The data are extracted from a part of the Oberschwaben test field.
The boundary values have been obtained using:

Significance level $a = .001$
Power $\beta_o = .80$

and they are expressed in units of $\sigma_o$.

In both tables, any boundary value greater than 200 is simply shown as 200.

Table I shows the reliability of the observations of points between connected models.
Five cases have been examined with different configurations of common points between the models.
In this experiment point number 20 is the projection centre.

It is clear that in the case of three common points (case 1), there is no error control for the x coordinate of the points; and errors of considerable magnitude can remain undetected. The addition of point 1312 (case 2) greatly improves the reliability of the x coordinate, while the x coordinate of the projection centre stays unreliable.
For four common points not on the same vertical plane (case 3), there is an improvement in the x coordinate of the projection centre, but a very large boundary value still arises.
In cases 4 and 5 double points are used resulting in a uniform reliability.
In all cases there is no error control for the x coordinate of the projection centre.

In table II the reliability for common points between strips has been calculated.
Here the lengths ratio condition controls the x coordinate of the points (the reliability of the x coordinate is very good). The angles condition controls the y coordinate of the points. It is also noted that the middle point for each triplet of points has the smalles boundary value. In fact the error location strategy (see Ref. 5), is based on these features.

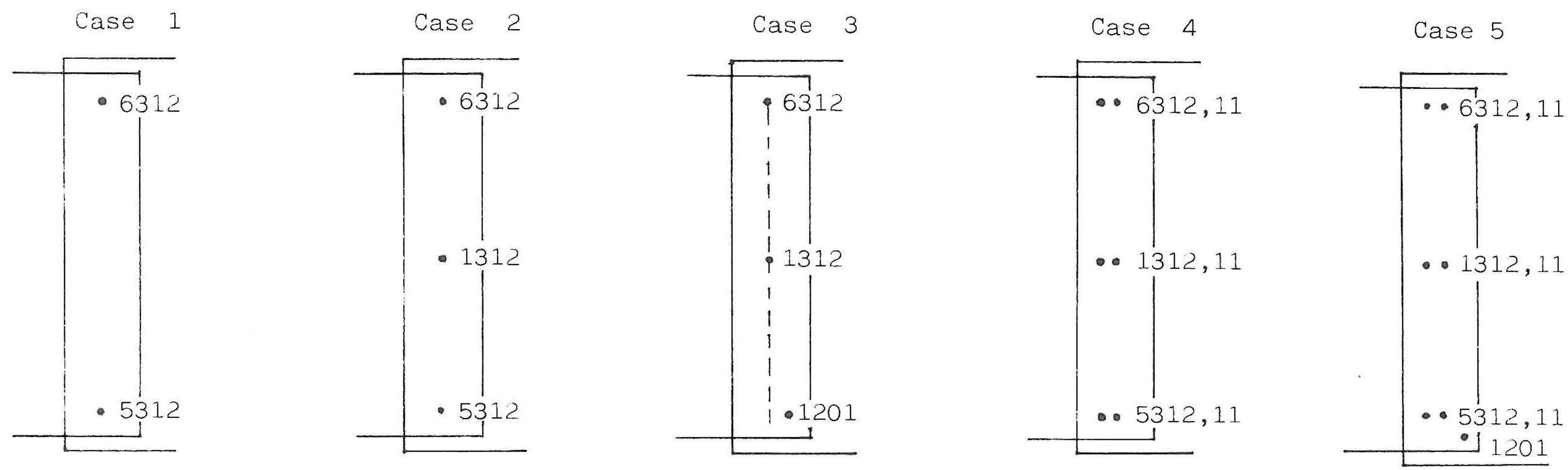
## 6. Conclusions

The method described has proved to be very effective for checking observational data prior to the execution of aerial triangulation using strips as units.

Errors of a magnitude 6 to 9 units of $\sigma_0$ can be readily located.

It is also noted that the x coordinate of the projection centres is difficult to be controled.

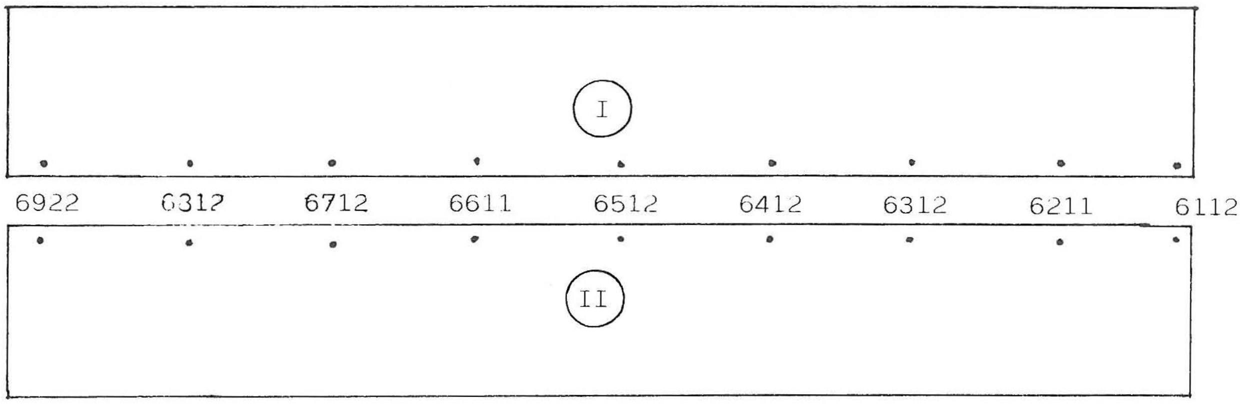
At this stage the computer programme remains in an experimental form and it is hoped that in the near future the computational effort will be reduced to a minimum. This should be achieved from studies of the structure of the weight coefficient matrices during the strip formation.




| | Case 1 | | | Case 2 | | | Case 3 | | | Case 4 | | | Case 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ∇x | ∇y | ∇z | ∇x | ∇y | ∇z | ∇x | ∇y | ∇z | ∇x | ∇y | ∇z | ∇x | ∇y | ∇z |
| 6311 | | ----- | | | ----- | | | ----- | | 7.8 | 7.1 | 7.1 | 7.8 | 7.1 | 7.1 |
| 6312 | 200.0 | 10.2 | 10.2 | 14.5 | 8.6 | 8.6 | 14.6 | 8.6 | 8.6 | 7.8 | 7.1 | 7.1 | 7.8 | 7.1 | 7.1 |
| 1311 | | ----- | | | ----- | | | ----- | | 6.5 | 6.5 | 6.5 | 6.4 | 6.4 | 6.4 |
| 1312 | | ----- | | 7.3 | 7.1 | 7.1 | 7.3 | 7.1 | 7.1 | 7.8 | 7.2 | 7.2 | 6.4 | 6.4 | 6.4 |
| 20 | 200.0 | 10.3 | 10.3 | 200.0 | 10.0 | 10.0 | 68.2 | 10.1 | 10.1 | 200.0 | 8.2 | 8.2 | 52.7 | 7.9 | 7.9 |
| 5311 | | ----- | | | ----- | | | ----- | | 7.8 | 7.2 | 7.2 | 7.1 | 6.8 | 6.8 |
| 5312 | 200.0 | 10.5 | 10.5 | 14.7 | 8.8 | 8.8 | | ----- | | 7.8 | 7.2 | 7.2 | 7.1 | 6.8 | 6.8 |
| 1201 | | ----- | | | ----- | | 14.2 | 8.7 | 8.7 | | ----- | | 7.0 | 6.8 | 6.8 |

Reliability of the observations of points between connected models

Table 1




| Points | | | | Strip I | | | Strip II | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 6922 | 6812 | 6712 | X: | 11.9 | 5.9 | 11.8 | 12.4 | 6.1 | 12.2 | Ratio of lengths |
| | | | Y: | 178.0 | 200.0 | 200.0 | 200.0 | 200.0 | 145.4 | |
| | | | X: | 184.9 | 200.0 | 200.0 | 200.0 | 200.0 | 150.8 | Angles |
| | | | Y: | 12.4 | 6.1 | 12.8 | 12.2 | 6.4 | 12.5 | |
| 6812 | 6712 | 6611 | X: | 11.5 | 5.7 | 11.3 | 12.0 | 5.9 | 11.7 | Ratio of lengths |
| | | | Y: | 200.0 | 150.3 | 95.6 | 142.6 | 200.0 | 150.0 | |
| | | | X: | 200.0 | 158.5 | 100.8 | 150.4 | 200.0 | 158.2 | Angles |
| | | | Y: | 12.2 | 6.0 | 11.9 | 12.6 | 6.3 | 12.4 | |
| 6712 | 6611 | 6512 | X: | 11.1 | 5.6 | 11.2 | 11.4 | 5.8 | 11.7 | Ratio of lengths |
| | | | Y: | 93.3 | 74.1 | 200.0 | 146.4 | 164.4 | 200.0 | |
| | | | X: | 95.1 | 75.5 | 200.0 | 149.2 | 167.4 | 200.0 | Angles |
| | | | Y: | 11.3 | 5.7 | 11.4 | 11.6 | 5.9 | 11.9 | |
| 6611 | 6512 | 6412 | X: | 10.9 | 5.5 | 11.3 | 11.3 | 5.7 | 11.7 | Ratio of lengths |
| | | | Y: | 200.0 | 200.0 | 200.0 | 200.0 | 131.4 | 146.2 | |
| | | | X: | 200.0 | 200.0 | 200.0 | 200.0 | 134.5 | 149.6 | Angles |
| | | | Y: | 11.1 | 5.7 | 11.5 | 11.5 | 5.9 | 12.0 | |
| 6512 | 6412 | 6312 | X: | 11.2 | 5.6 | 11.1 | 11.6 | 5.9 | 11.5 | Ratio of lengths |
| | | | Y: | 200.0 | 200.0 | 200.0 | 145.3 | 138.6 | 200.0 | |
| | | | X: | 200.0 | 200.0 | 200.0 | 150.6 | 143.8 | 200.0 | Angles |
| | | | Y: | 11.6 | 5.8 | 11.5 | 12.1 | 6.0 | 11.9 | |
| 6412 | 6312 | 6211 | X: | 11.2 | 5.6 | 11.1 | 11.6 | 5.8 | 11.5 | Ratio of lengths |
| | | | Y: | 200.0 | 200.0 | 200.0 | 200.0 | 200.0 | 200.0 | |
| | | | X: | 200.0 | 200.0 | 200.0 | 200.0 | 200.0 | 200.0 | Angles |
| | | | Y: | 11.5 | 5.7 | 11.4 | 11.9 | 5.9 | 11.8 | |

Continuation Table 2

| Points | | | | Strip I | | | Strip II | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 6312 | 6211 | 6112 | X: | 11.6 | 5.7 | 11.1 | 12.0 | 5.9 | 11.6 | Ratio of lengths |
| | | | Y: | 200.0 | 200.0 | 200.0 | 200.0 | 110.5 | 183.7 | |
| | | | X: | 200.0 | 200.0 | 200.0 | 200.0 | 115.1 | 191.4 | Angles |
| | | | Y: | 12.1 | 5.9 | 11.6 | 12.5 | 6.1 | 12.1 | |

Reliability for common points between strips

Table 2

REFERENCES


(1) Baarda, W. — Statistical Concepts in Geodesy, Netherlands Geodetic Commission, New Series, Vol. 2, No. 4, 1967

(2) Baarda, W. — A testing procedure for use in Geodetic Networks, Netherlands Geodetic Commission, New Series, Vol. 2, No. 5, 1968

(3) Baarda, W. — Reliability and Precision of Networks, Presented paper to the VII International Course for Engineering Surveys of high Precision, Darmstadt, 1976

(4) Bouloucos, T. contributions from Molenaar, M. — The reliability of the observations of points in planimetric independent model blocks, I.T.C. Journal, 1979-2.

(4a) Bouloucos, T. and Molenaar, M. — A testing procedure for ground control in planimetric independent model blocks, Presented paper to the Symposium of Comm. III of the ISP, Moscow, 1978.

(5) Bouloucos, T. — Preadjustment gross error detection in photogrammetric independent model blocks, Thesis I.T.C., 1978.

(6) Förstner, W. — Statistical test methods for blunder detection in planimetric block triangulation, Presented paper to the Comm. III of the ISP at the XIII Congress, Helsinki, 1976.

(7) Grün, A. — Gross error detection in bundle adjustment, Presented paper at the Aerial triangulation Symposium, Queensland, 1979.

(8) Molenaar, M. — A programme for preadjustment error detection in independent model blocks, Presented paper to the Symposium of Comm. III of the ISP, Moscow, 1978.

(9) Molenaar, M. — Error detection in planimetric block adjustment, Presented paper to the Comm. III of the ISP at the XIII Congress, Helsinki, 1976.