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DIGITAL TERRAIN MODEL, POINT DENSITY, ACCURACY OF MEASUREMENTS, TYPE OF TERRAIN, AND SURVEYING EXPENSES

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Abstract

By means of a stochastic model of the terrain and an economic model of the surveying expenses, it is possible to calculate the connection between the accuracy of the digital terrain model, the terrain, and the surveying strategy.

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Introduction

The accuracy of a digital terrain model depends on several factors. The most important of these are:

- The selection and distribution of points on the natural surface whose coordinates are measured.
- The mean square error of the measuring method.
- The interpolation used to compute the terrain model between the measured points.
- The surface structure of the terrain.

In this paper the influence of the terrain structure will be investigated. Two characteristics of the digital terrain model are examined:

- The standard deviation between heights in the terrain model and heights in the natural terrain surface.
- The amount of work required to map a given area.

The terrain structure will be defined through the frequency spectrum as described by Frederiksen (1980) and Frederiksen, Jacobi, and Justesen (1978).

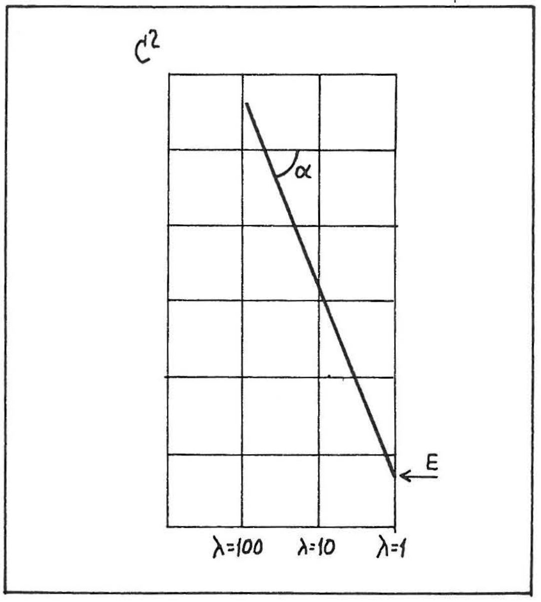
Use of Frequency Spectrum for Estimating the Standard Deviation of the Digital Terrain Model

In order to characterize a terrain type by a frequency spectrum, it is assumed that the height variations in the terrain can be described as a stochastic variable. Given this assumption, the most favorable measuring strategy is to measure points distributed over the terrain surface in a uniform grid of equal-sided triangles. A uniform square grid is more convenient for photogrammetric measurement and has nearly the same qualities as the triangles. In the following, only a square grid with a mesh width of Δx will be considered.

The measuring accuracy will be expressed by the mean square error, m_z, of the height measurements. This is a rough simplification of the surveying method, but it makes it possible to study the connection between measuring accuracy, point density, and terrain type. If a surface with a known frequency spectrum is measured in points distributed in a grid with a mesh width of Δx and with an accuracy of m_z in each point, the standard deviation, s_o, between a digital model made from the measurements and the original terrain surface is

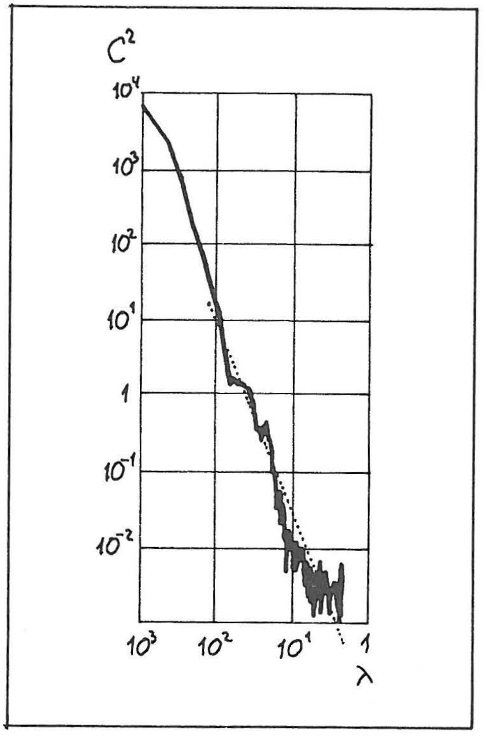
$$s_o^2 = \sum_{\lambda=2\cdot\Delta x}^{\lambda=0} C_\lambda^2 + m_z^2 \quad (1)$$

where C_λ^2 is the spectral value for the wavelength λ. This derivation is shown in (Frederiksen, Jacobi, and Justesen, 1978). In order to simplify the calculations, the frequency spectrum of the terrain is approximated to a straight line in a log/log picture of the spectrum:



$$\log(C_\lambda^2) = \log E + \alpha \log \lambda$$

where α is the slope of the spectrum, and E is the spectral value for the wavelength λ = 1 meter.



As shown by Frederiksen et al. (1978), the standard deviation can be expressed as:

$$s_o^2 = \frac{E \cdot (2 \cdot \Delta x)^{\alpha-1}}{\alpha-1} + m_z^2 \quad (2)$$

Moraine Landscape

The frequency spectrum of a moraine landscape in Denmark is approximated by a straight line:

$$C_\lambda^2 = 10^{-4} \cdot \lambda^{2.5}$$

where $E = 10^{-4}$ and $\alpha = 2.5$. The approximation is valid only for λ smaller than 100 meters, which is the same as Δx smaller than 50 meters. If the values of E and α are entered into (2), the standard deviation s_o for different combinations of Δx and m_z can be calculated as

$$s_o^2 = 1.89 \cdot 10^{-4} \cdot (\Delta x)^{1.5} + m_z^2 \qquad (3)$$

In Table 1 the relationship is calculated for some selected values of Δx and m_z:

Table 1

All units are in meters		
Δx	m_z	s_o
5.0	0.025	0.05
10.0	0.05	0.09
20.0	0.10	0.16
40.0	0.20	0.30
25.5	0.05	0.16
11.4	0.14	0.16

By using different combinations of Δx and m_z, it is possible to get the same standard deviation as the three examples with $s_o = 0.16$ meters show. In order to decide which combination to use, it is necessary in addition to make an economic model.

Economic Model

A simple economic model is constructed for a photogrammetric digital survey. The accuracy of the height measurement is dependent on the altitude of the aerial photographs, which again determines the size of the photogrammetric models and the number of control points. The expense of aerial photography, the measurement of control points, and the setting up of models in the photogrammetric instruments can be connected to the accuracy m_z by an economic factor K_2.

When the photogrammetric models are set up, the task of measuring terrain points is started. The expense of measuring points and the acquisition and storage of data depend on the distance Δx between the measured points by another economic factor K_1. The total cost can now be expressed in the equation

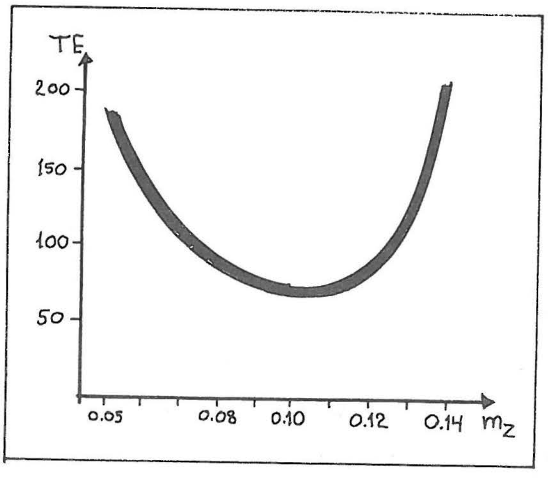
$$TE = \frac{K_1}{\Delta x^2} + \frac{K_2}{m_z^2} \tag{4}$$

TE gives an estimate of the total expense per square kilometer. In order to get an idea of the relationship between these factors, the values of K_1 and K_2 have been set at 7110 and 0.43, respectively. These values are calculated from the cost of a few maps and are not very accurate.

To make a digital model of the moraine landscape with a frequency spectrum approximated by $E = 10^{-4}$ and $\alpha = 2.5$ and a standard deviation $s_o = 0.15$ m, equation (3) is inserted into (4).

Table 2

$s_o = 0.15$ meter		
m_z	Δx	TE
0.05	22.4	186
0.08	19.4	86
0.10	16.4	70
0.12	12.2	77
0.14	6.2	208



There is an economical minimum around $m_z = 0.10$ meter. The minimum can be obtained by inserting equation (3) into (4) and differentiate with regard to m_z

$$\frac{\delta TE}{\delta m_z} = \frac{K_1 \cdot 16}{E} \cdot m_z \cdot \left[\frac{\alpha - 1}{E} \cdot (s_o^2 - m_z^2)\right]^{(1+\alpha)/(1-\alpha)} = 0$$

This gives as a minimum for $s_o = 0.15$ m

$m_z = 0.11$ m

$\Delta x = 15.4$ m

$TE = 69$

For a digital model of the same terrain with a standard deviation of s_o = 0.30 m, the minimum will be

$m_z = 0.22$ m

$\Delta x = 36$ m

TE = 14

Different Terrain Types

The following calculations are made for two different terrain types. The moraine landscape is approximated by $E = 10^{-4}$ and $\alpha = 2.5$ as in the other examples. A mountainous terrain composed of ancient sedimentary rocks in North Greenland is approximated by $E = 10^{-4.38}$ and $\alpha = 3.24$.

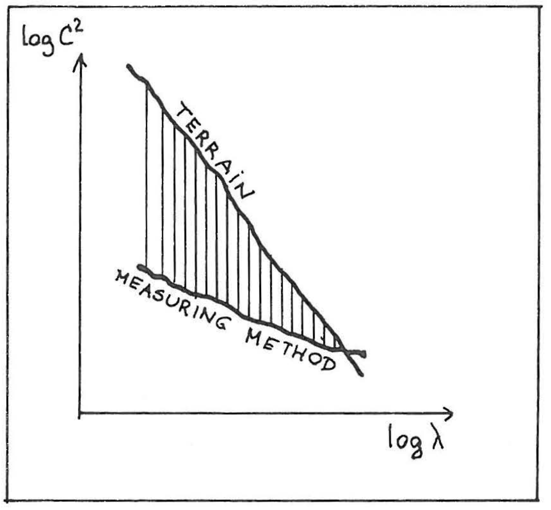
Table 3

s_o	$E = 10^{-4}$, $\alpha = 2.5$			$E = 10^{-4.38}$, $\alpha = 3.24$		
	m_z	Δx	TE	m_z	Δx	TE
0.15	0.15	15	69	0.10	9	127
0.30	0.22	36	14	0.19	18	35
0.50	0.38	67	4.5	0.30	28	13.7
1.00	0.80	156	0.97	0.59	53	3.75
1.50	1.22	254	0.40	0.88	77	1.76
2.00	1.65	358	0.21	1.16	100	1.03

As seen in Table 3, the terrain type has a great impact on the economy of mapping if one wants to reach the same standard deviation. In this example, the price per square kilometer will be from two to five times higher in the mountainous country than in the moraine country. If the same measuring procedure is used, the map of the moraine country will have a standard deviation two times smaller than the map of the mountainous country.

Discussion

In this paper a frequency spectrum has been used to describe the surface roughness of a terrain. It would be obvious to follow this up by using a frequency spectrum to describe the measuring error as well. Some errors, like the absolute orientation error of the photogrammetric models, will influence the long wavelengths, while other errors, like the inaccuracy of setting the floating point on the terrain surface, will introduce short-wave



errors. If such a frequency spectrum of the measuring errors could be established, the accuracy of a digital terrain model or of a map could be more correctly evaluated. The true information about the terrain surface is placed between the spectrum of the terrain and the spectrum of the measuring method.

The terrain type is a factor of great importance in determining the standard deviation of a digital terrain model. By using a frequency spectrum, it is possible to calculate the influence of the terrain. The conclusion of this investigation is the same as the one reached by Kubik and Botman (1976). The accuracy of a digital terrain model depends primarily on the properties of the terrain surface and on the spacing of the measured points.

The choice of interpolation method is important for the cartographic quality of the digital model, but it has only a minor effect on the standard deviation.

References

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