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ERROR AND ACCURACY ANALYSIS IN APPLICATION OF PHOTOGRAMMETRIC METHODS TO LAND SURVEYS

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ABSTRACT

Broadly speaking, land surveys should include not only the position of boundary corners (points) and the area enclosed by the boundaries, but also the inventory of the land resources and its uses. Therefore, remote sensing may also be included in the discussion of photogrammetric methods. However, due to limitations of the application of remote sensing method to the rather refined numerical requirements of the traditional land surveys or cadastral surveys, this paper will emphasize the discussion to the traditional cadastral suveys. The numerical analysis of the errors and accuracy of the photogrammetric methods will consist the examination of original photographic scales and produced map scales, the capability of various analogical and digital, or graphical and analytical instruments, and the geometric variations of the distance, coordinate and aeral values as against the commonly used methods in old land surveys and ground surveys.

Photogrammetric Methods Used in Land Surveys

There are many ways photogrammetric methods could be used to land survey projects. According to the way in which the photographs or its negatives are used, and the planimetric or topographic maps are produced and at the same time obtaining numerical information or after to use the map to obtain the numerical information, the following classifications could be made:

1. Use of photographically products such as single contact or enlarged photos, orthoprojected photos, stereophoto pairs, composite uncontrolled or controlled photo-mosaics, mosaic maps, and orthophoto maps, etc.

2. Use of stereo-mapping plotters to produce land maps and digital land data information. In this category, there are four possible ways of application:

a. Use of analogical plotters.
b. Use of analogical plotters with digitizing devices.
c. Use of analytical plotters with digitizing capabilities.
d. Use of analytical plotters and accompanying computerized land data processing system.

3. Use of analytical photogrammetric methods to obtain numerical information for densification of land survey controls and individual land parcel data. Usually, a block or multiple composit photo models or photo plates with ground control points information are regorously adjusted by the so-called block or bundle adjustment method programmed by several different institutions or consulting photogrammetrists. Land boundary distances and diagonals traditionally form a trilateration network to be taped to compute the area. If these distance values are taken from the photo plates or models directly or computed from coordinates, an adustment can be made by area conditions as proposed by the author in several publications (Reference: LEE, 1964, 1966, 1971). The computer program is also available (LEE, 1970).

4. Use of electronic digitizers with computer to create numerical land data system from existing cadastral maps and/or from photogrammetry produced topographic maps and orthophotomaps.

Errors and Accuracy in the Process of Photogrammetric Methods

The errors in the process of photogrammetric methods used in land surveys discussed in last section are essentially coming from the feature of ground objects, imperfections of the materials and equipments used, the influence of processing environment and limitation of human sensing abilities. Manufacturers have attempted competitively to improve their products in high quality to reduce systematic errors of cemera, film or plate, and map base to a minimum. It is up to the photogrammetrists to control the process of producing working medium, photo/plate and maps, and taking measurements and to eliminate blunders and other systematic errors. If these have been done carefully, the statistical theory of probability of random errors can be applied to the quality contrl of the process of photogrammetric methods and to presenting a criterion of the accuracies applied to land survey.

In order to control the quality and check the produced information of photogrammetric methods in application to land surveys, Table I is prepared to suggest exemplary criteria for use.

In the Table,

$1/K$ = Photo Scale,

= c, Photo/Plate Distance/D, Ground Distance;

$1/s$ = Map Scale or Stereo-Model Scale,

= s, Map or Model Distance/D, Ground Distance.

m_{xy} and m_{XY} are the root mean square errors of grids of photo/plate or map and ground coordinate system respectively. The fiducial marks of the camera or the reseau grids canbe used for photos/plates test. The plane coordinate grids can be used for map test. Using the following formulas to compute:

$$m_{xy} = \pm\sqrt{m_x m_y}, \quad \text{and} \quad m_{XY} = \pm\sqrt{m_X m_Y}.$$

Table I. Exemplary Criteria of Planimetric Errors and Accuracies in Photogrametric Methods Applied to Land Surveys

Photo /Plate or Map Scale $1/K$ or $1/S$	Model or Map Scale $1/S$	Magnification M	Measurements Accuracy Attainable: Plate or Model m_{xy} $\pm$microns	Measurements Accuracy Attainable: Map m_{xy} $\pm$microns	Ground Accuracy Attainable: Point Position m_{XY} $\pm$cm.	Ground Accuracy Attainable: Distance m_D $\pm$cm.	Ground Accuracy Attainable: Area m_A $\pm cm^2$
Map, S=	S=						
2,500	1,000	2.5	-	50	5.0	7.0	100
Plate, K=							
2,500	500	5	-	25	7.5	10.5	225
2,500	250	10	10	-	2.5	3.5	25
2,500	-	-	10	-	2.5	3.5	25
5,000	-	-	10	-	5.0	7.0	100
5,000	1,000	5	-	50	5.0	7.0	100
5,000	500	10	10	-	5.0	7.0	100
10,000	2,000	5	-	25	5.0	7.0	100
10,000	1,000	10	10	-	10.0	14.0	400
10,000	500	20	-	5	5.0	7.0	100
10,000	-	-	5	-	5.0	7.0	100
10,000	-	-	3	3	3.0	4.0	36
20,000	-	-	5	-	10.0	14.0	400
20,000	-	-	3	3	6.0	8.5	144
20,000	2,000	10	5	-	5.0	7.0	100
20,000	1,000	20	5	-	10.0	14.0	400

And then, $m_D = \pm\sqrt{2}\, m_{XY}^2$ for ground distance standard deviation, and

$m_A = \pm 4m_{XY}^2$ for ground area standard deviation.

The maximum allowable errors is three times larger of the values of the attainable accuracies suggested in the table.

Criteria of Errors and Accuracies in Land Surveys

The most serious problem in the land survey is the uncertainty of boundary lines and corners. The boundary lines and corners might have been described in a series of historical deeds and set up by earlier land surveyors. Because change of ground features due to change of time, they could not be identified exactly as described in the deeds by any body or by any means. If the positions of the corners of individual parcels are agreed upon, but not all the corners can be technically and economically signalled for and accurately observable by photogrammetric methods. If we call this source of error as monument identification error and a constant a_3, and the other two sources of errors, the systematic error of distance measurement represented by a_1D, which is proportional to the distance measured, and the random error $a_2\sqrt{D}$, assumed to be varied with the square root of D. In expression, the total error of a boundary distance measurement may be written as:

$$e_D = \pm (a_1D + a_2\sqrt{D} + a_3) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$= \pm (a_1(D^{\frac{1}{2}})^2 + a_2D^{\frac{1}{2}} + a_3) \quad . \quad \ldots\ldots\ldots\ldots\ldots(2)$$

It can be seen that, Equation (2) is a quadratic function of $D^{\frac{1}{2}}$. The least square method can be used to determine the constants a_1 and a_2 emperically in use of various lenths measured in a survey and with various different kind distance measuring device such as tapes, electro-optical devices, graphic scaling on a map, and photogrammetric methods

Another easier method will be proposed by the author in a graph prepared as Figure 1.

In Equation (1), constant a_3 can be assigned a resonable value, e.g., 2 cm for urban flat terrain and 5 cm for rough mountainous areas, such value have been used in German cadastre practices.

If e_D is expressed in cm and D im meter unit, a_1 and a_2 can be considered as contributions of D and $\sqrt{D}$ in percentages.

The graph of Figure 1 consists three portions of e_D. a_3 is represented by the distance between the two vertical ordinate lines of 0.1 of $\sqrt{D(m)}/100$ and 0 of d(m)/100 but not in scale.

The left semi-logarithmic graph is the portion of $a_2D^{\frac{1}{2}}(m)/100$ in cm and the right rectangular graph is the portion of $a_1D(m)/100$, also in cm.

In Germany, $a_1 = 0.03$ and $a_2 = 0.80$ are used in conventional cadastre survey practices for for maximum allowable boundary distance measurement error in normal terrain conditions, and $a_1 = 0.05$ and $a_2 = 1.2$ for rough terrain or survey conditions.

Newer more strict values a_1=0.02 and a_2=0.20 was adopted in the city of Hamburg before the introduction of the computational cadastre.

In the U.S., in the proposed Cadastral Survey Standards in the Conference of MOLDS (Modernization of Land Data System) in 1975, corresponding values of a_1=0.02, a_2=0.00, and a_3=1 cm were specified as the minimum accuracy of land survey in a Committee Report (Chatterton and McLaughlin, 1975).

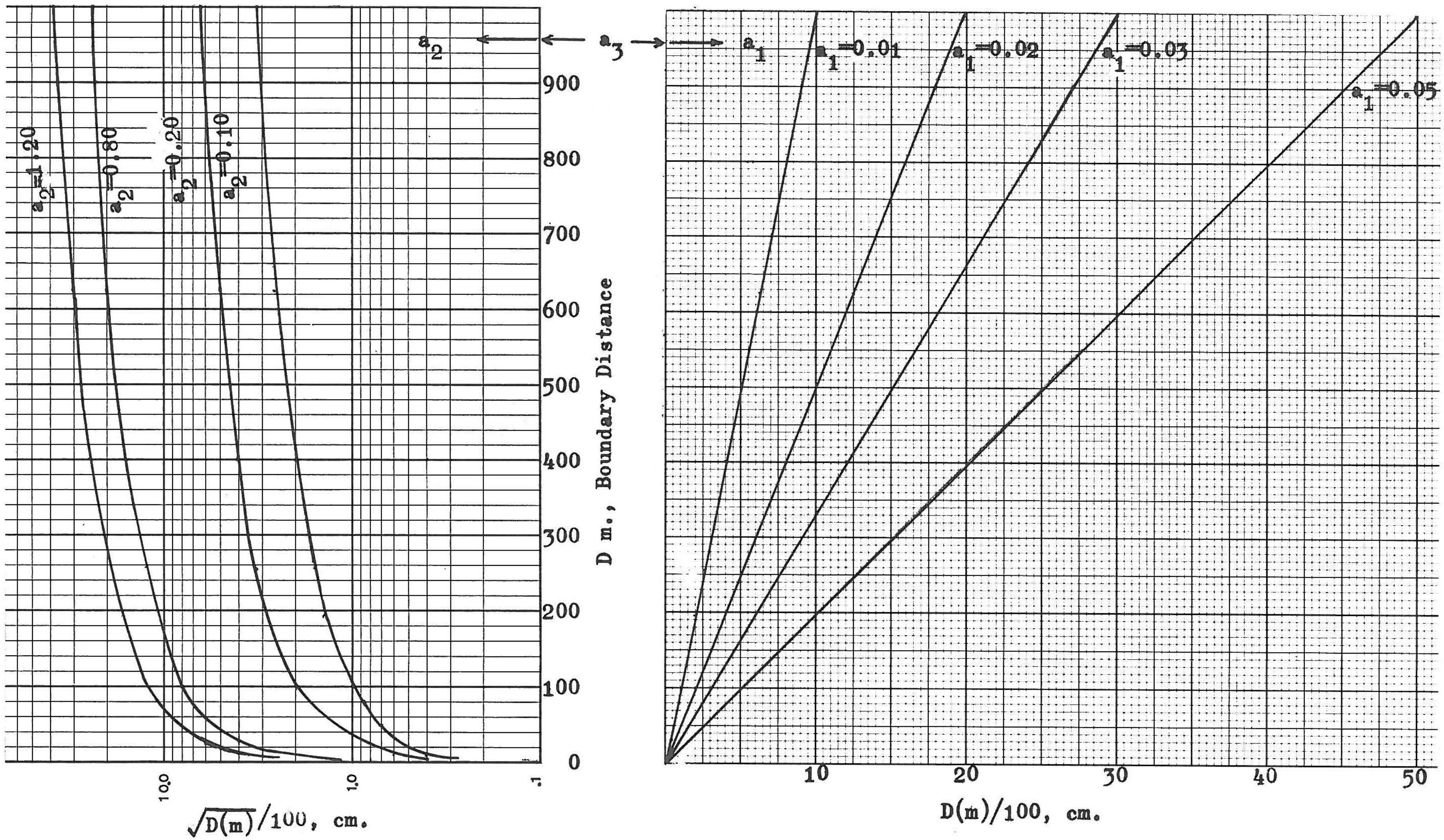

Figure 1. Graphic Evaluation of Boundary Distance Errors

All the different proposals and experiences are presented in Figure 1 in order for comparison and convenience for users. It is up to the users to adopt a set of "a" coefficients suitable for the conditions of the survey area and the requirements of the cadastre to achieve maximum accuracy or give a proper tolerance for the project personnel to judge their results.

Another important quantity in the land survey is the areal value of the individual parcels surveyed. These are more interested to both land owners and administrators than to the surveyors and photogrammetrists.

It can be proved that:

$$m_A = \pm 4m_{XY}^2 = \pm 2A^{\frac{1}{2}}m_{XY} = \pm 2A^{\frac{1}{2}}Sm_{xy}$$

where m_{xy}=0.1 mm, assumed to be the graphic accuracy of the map with scale 1/S. The unit of m_A should be in square meters. The maximum allowable area error should be $3m_A$.

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