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SATELLITE PHOTOGRAPH DIGITAL RECTIFYING

ABSTRACT

Theoretical algorithm basis and block diagram for satellite photograph rectifying are presented, considering and esta - blishing the following items:
- exterior orientation elements of the satellite photographs, using the corresponding points photograph-map;
- geographic grid used in photomap compilation (in stereo - cartographic projection, secant plan);
- projective transform coefficients of the satellite photo- graphs within the stereo-projection plan, using photograph division into section/part, which can be rectified;
- element computations as input data for the photorectifier reading device, which can be rectified during one single stage or more stages, as the case stands.

General features of the computation programme and some consi- derations regarding possibility to apply the above mentioned methodology are, also, given in this paper.

INTRODUCTION

The latest researches concern with the space image uses to de- velop topographic basis, to compile small scale thematic maps, respectively within the various remote sensing programmes re- lated to remotely sense the Earth from space, many recording systems are used, among which the conventional photogrammetric one must be mentioned.
In order to solve various problems related to map updating

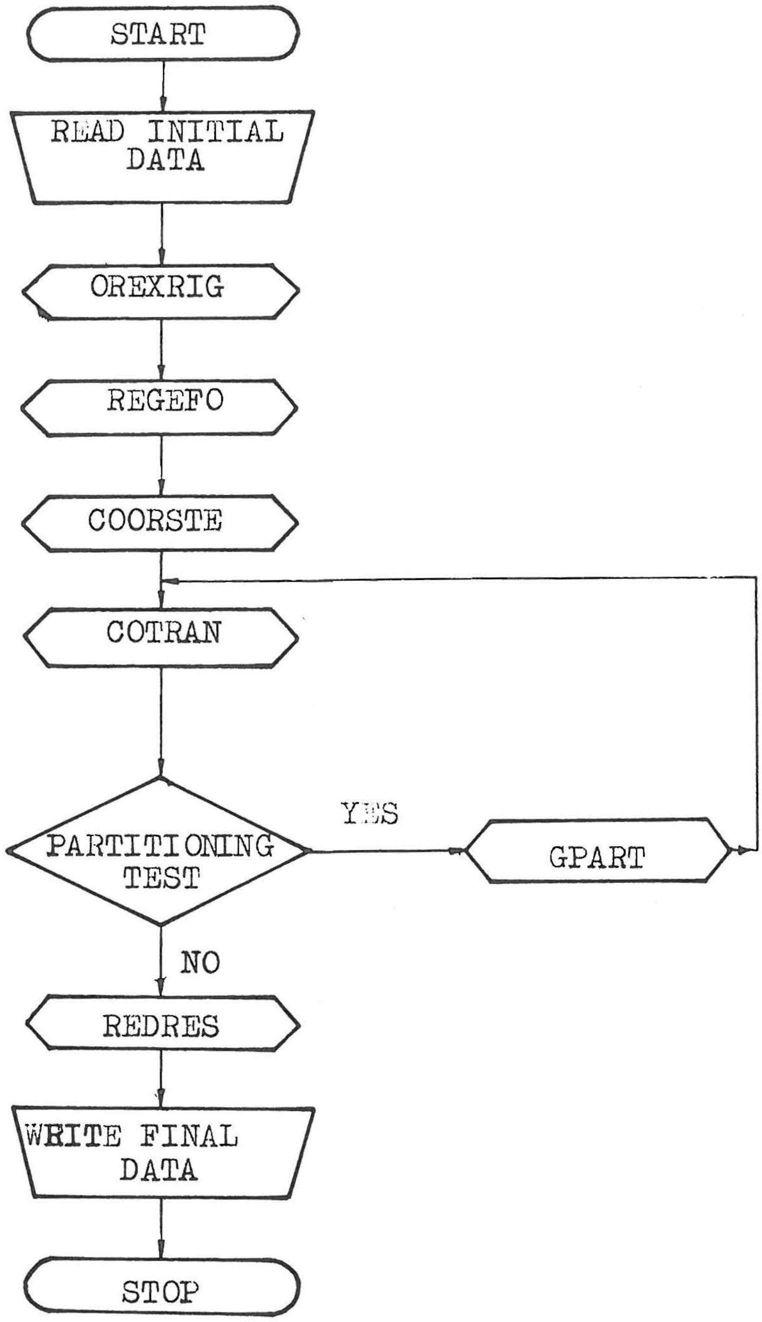

Figure 1. Block diagram of the numerical rectification
Subprograms of the diagram having the following functions

OREXRIG - finds out elements of exterior orientation
REGEFO - works out geographic network on photogram
COORSTE - works out geographic network on photomap
COTRAN - finds out coefficients of projective
GPART - partition of the photogramme
REDRES - calculates elements of rectification

and certain small scale thematic map compilation, photogrammetric equipment and methods suitable to special conditions of taking ground photographs from space can be used,when proper complex systems are not in hand. Such an equipment is a computer - assisted photorectifier,which can make an analytical rectifying;thus,a photomap,satisfying the needed cartographic projection and scale, can be compiled. The block diagram of the technology answering this purpose is shown in Figure 1.

1. ESTABLISHMENT OF THE EXTERIOR ORIENTATION ELEMENTS

Exterior orientation elements of photograph, H, B_N, L_N, α, ω, $\varkappa$, using the corresponding points (photograph-map), are established within the first OREXRIG block of the diagram. Exterior orientation elements (the interior orientation elements are considered as known)are established,using uhe basic central perspective equations, that is collinearity condition equations and taking into account approximate values for H, B_N, L_N, α, ω, $\varkappa$, as well.

Minimum three points, which B, L and h coordinates are taken from an existing map, are required to establish these six unknowns.Having more points, we can solve the problem, using the least squares method and an iterative process.

Because control point coordinates are initially defined by B,L and h geodetic coordinates (point latitude, longitude and height above the reference mean surface),they must be previously transformed into X_G,Y_G,Z_G geocentric coordinates and, then, in X_T, Y_T, Z_T topocentric coordinates.

Transformation of geodetic coordinates into geocentric coordinates is achieved, using the expression below

$$
\begin{aligned}
X_0 &= (N+h)\cos B \cos L\ ;\\
Y_0 &= (N+h)\cos B \sin L\ ;\\
Z_0 &= (N(1-e^2)+h)\sin B\\
N &= a/(1-e^2\sin^2 B)^2
\end{aligned}
\qquad (1)
$$

where

N is the first vertical curvature radius

e is the ellipsoid eccentricity, and

a is its great semi-axis.

The X_G Y_G Z_G geocentric coordinate system is so selected, that X_G axis should cross the zero meridian and Z_G axis should cross the North Pole.

Transformation of geocentric coordinates into topocentric coordinates, having its origin in N nadir point, is achieved using relations:

$$\begin{pmatrix} X_T \\ T_T \\ Z_T \end{pmatrix} = \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \cdot \begin{pmatrix} X_G \\ Y_G \\ Z_G + N_N e^2 \sin B_N \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -N_N \end{pmatrix} \quad (2)$$

where

A_{ik} is the direction cosines of the rotation matrix and

N_N refers to nadir point.

Cosines expressions contain B_N, L_N and A values representing the nadir point longitude and latitude and X_T axis azimuth (the angle between the axis and the meridian passing through the nadir point). Proper to axis system of the considered stereographic cartographic projection, we take A = 0 and the following form of the direction cosines of the rotation matrix from relation (2) has resulted:

$$\begin{pmatrix} \sin B_N \cos L_N & -\sin B_N \sin L_N & \cos BB_N \\ \sin L_N & -\cos L_N & 0 \\ \cos B_N \cos L_N & \cos B_N \sin L_N & \sin B_N \end{pmatrix} \quad (3)$$

After the required matrix processings related to H, B_N, L_N, α ω, $\varkappa$, variables and giving the suitable notations, the correction equations below have resulted:

$$\begin{aligned} S_1^1\, dH + S_1^2\, dB_N + S_1^3\, dL_N + S_1^4\, d\alpha + S_1^5\, d\omega + S_1^6\, d\varkappa + L_x &= V_x \\ S_2^2\, dH + S_2^2\, dB_N + S_2^3\, dL_N + S_2^4\, d\alpha + S_2^5\, d\omega + S_2^6\, d\varkappa + L_x &= V_x \end{aligned} \quad (4)$$

where

S_1^i and S_2^1 ($i = 1,2 \ldots 6$) are the unknown parameter coefficients; L_x, L_y are the corresponding free terms (differences among x and y computed and measured coordinates).

Equation system having the forme below

$$S\,X + L = V \tag{5}$$

and applying the least squares method (that is, indirect observation method), the following normal equation system has resulted

$$S^T \cdot S \cdot X + S^T L = 0 \tag{6}$$

whence

$$X = -\,(S^T S)^{-1}\, SL\,. \tag{7}$$

The $X^T = (dH_N\ dB_N\ dL_N\ d\alpha\ d\omega d\varkappa)$ unknown vectors are established, using the proper subroutines of the mathematical library, which perform various matrix operations related to relation (7).

As we have mentioned, an iterative process is performing these six orientation parameter values, using their approximate values. During each iteration, a new correction set is added to the values previously derived:

$$\begin{aligned} &H^i = H^{i-1} + dH; \quad B^i = B_N^{i-1} + dB_N^i; \quad L^i = L^{i-1} + dL^i \\ &\alpha^i = \alpha^{i-1} + d\alpha; \quad \omega^i = \omega^{i-1} + d\omega^i; \quad \varkappa^i = \varkappa^{i-1} + d\varkappa^i \end{aligned} \tag{8}$$

Experiments on models have proved that only three iterations are required to establish exterior orientation elements of a space photograph, further corrections being meaningless. This problem is solved in three minutes and need 30 K.

2. ESTABLISHING THE GEOGRAPHICAL NETWORK ON PHOTOGRAPH

Within the second block, REGEPO subprogramme carries out the geographical network for space photographs, that is x and y coordinates of the meridian and parallel intersection points, at pre-established intervals, are derived.

To solve this problem, we also use colinearity equations, where f, x_0, y_0 interior orientation elements, $H, B_N, L_N, \alpha, \omega, \varkappa$, exterior orientation elements (derived by the first block),

$\Delta B, \Delta L$ (pre-established)intervals among the adjacent points of the geographical network and h_m mean ground height on the photograph are well known. So, B_j, L_j geodetic coordinates for each intersection of photograph geographic network lines are established, using the relation

$$\begin{aligned} B_j &= B_0 + i \quad B \\ L_j &= L_0 + i \quad B \\ (i &= 1, 2 \ldots . n) \end{aligned} \qquad (9)$$

The x and y values, within the photograph grid coordinate system,for the same intersections, are established using colinearity equations, before transforming geodetic coordinates into geocentric coordinates and, finally, into topocentric coordinates having its origin in the nadir point and keeping A = 0 condition,as we have mentioned above.

The R rotation matrix of passing from the rectangular topocentric coordinatesinto photograph grid coordinates has the form below, as it had in the first block:

$$\begin{pmatrix} \cos\alpha \cos\varkappa - \sin\alpha \sin\omega \sin\varkappa & \cos\omega \sin\varkappa & \sin\alpha \cos\varkappa + \cos\alpha \sin\omega \sin\varkappa \\ -\cos\alpha \sin\varkappa - \sin\alpha \sin\omega \cos\varkappa & \cos\omega \cos\varkappa & -\sin\alpha \sin\varkappa + \cos\alpha \sin\omega \cos\varkappa \\ -\sin\alpha \cos\omega & \sin\omega & \cos\alpha \cos\varkappa \end{pmatrix} \qquad (1o)$$

So obtained x and y coordinates are corrected, using the first photograph increasing ratio and the air refraction ratio in the platform atmosphere where the considered photogrammetric camera is mounted.

A computer needs three minutes and 17 K to make this block computations.

3. ESTABLISHING THE GEOGRAPHICAL NETWORK ON A PHOTOMAP

The third block (COORSTE) derives the photomap geographical network, in cartographic projection and at a given scale. In this case, the cartographic projection secant plan and a given scale are considered.

The transformation of B_j, L_j geodetic coordinates into X_s, Y_s

coordinates is carried out, using expression below:

$$X = 5.10^5 + 0.99975(a_{10} f + a_{20} f^2 + a_{30} f^3 + a_{40} f^4 + a_{50} f^5 + a_{60} f^6 + a_{02} l^2 +$$
$$+ a_{12} f l^2 + a_{22} f^2 l^2 + a_{32} f^3 l^2 + a_{42} f^4 l^2 + a_{04} l^4 + a_{14} f l^4 + a_{24} f^2 l^4 + a_{06} l^6 \tag{11}$$

$$Y = 5.10^5 + 0.99975(B_{01} l + b_{11} f l + b_{21} f^2 l + b_{31} f^3 l + b_{41} f^4 l + b_{51} f^5 l +$$
$$b_{03} l^3 + b_{23} f l^3 + b_{23} f^2 l^3 + b_{33} f^3 l^3 + B_{05} l^5 + b_{15} f l^5)$$

where

a_i, b_i are constant coefficients for the whole considered territory, and f, l are B_j and L_j values given in seconds multiplied by 10^{-4}.

This block computation times to transform 34 points is one minute at least and it requires 20 K.

4. COMPUTATION OF THE PROJECTIVE TRANSFORMATION PARAMETERS

Projective transformation coefficients of the photograph geographical network are established in geographical map network, using COTRAN block.

General formulas of the projective transformation have been used to solve this problem:

$$X_i = (a_{11} X_i + a_{12} Y_i + a_{13})(a_{31} X_i + a_{32} Y_i + 1)$$
$$Y_i = (a_{21} X_i + a_{22} Y_i + a_{23})(a_{31} x_i + a_{32} Y_i + 1)$$
$$(i = 1, 2, \ldots\ldots n)$$

or as matrix form

$$\begin{pmatrix} X_i \\ \vdots \\ X_n \\ Y_i \\ \vdots \\ Y_n \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1 X_1 & -y_1 X_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n & y_n & y_n & 0 & 0 & 0 & -x_n X_n & -x_n X_n \\ 0 & 0 & 0 & x_1 & x_1 & 1 & -x_1 Y_1 & -y_1 Y_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & x_n & y_n & 1 & -x_n Y_n & -y_n Y_n \end{pmatrix} \cdot \begin{pmatrix} a_{11} \\ a_{12} \\ a_{13} \\ a_{21} \\ a_{22} \\ a_{23} \\ a_{31} \\ a_{32} \end{pmatrix} \tag{12}$$

The a_ik projective transformation coefficients are determined interatively.

These approximate parameter values are determined using 4 points (properly spaced), implying an unique solution.

The least squares method is used in the correction equation system to make corrections, having the following form:

$$
\begin{aligned}
x_j \Delta a_{11} + y_j \Delta a_{12} + \Delta a_{13} - x_j X_j \Delta a_{31} - y_j X_j \Delta a_{32} + W_j l_{xj} &= V_{xj} \\
x_j \Delta a_{21} + y_j \Delta a_{22} + \Delta a_{23} - x_j Y_j \Delta a_{31} - y_j Y_j \Delta a_{32} + W_j l_{yj} &= V_{yj}
\end{aligned}
\qquad (13)
$$

where

$$
\begin{aligned}
W_j &= a_{31} X_j + a_{32} Y_j + 1 \\
l_{xj} &= X_j - x_j \\
l_{yj} &= Y_j - y_j \\
(j &= 1, 2 \ldots\ldots n)
\end{aligned}
\qquad (14)
$$

and $X_j^{'}$, $Y_j^{'}$ are coordinates computed using (12) relations for the approximate transformation parameter values.

After each iteration, the corrected values of the $(a_{ik} + \Delta a_{ik})$ transformation parameters are established and the correspon - ding $(X_j^{'}, Y_j^{'}, W_j, l_{x_j}, l_{y_j})$ values are computed.Using these values, a new correction set, based on (13) relations, is obtained. Iterations come to an end when $(\Delta a_{ik} \leq 10^{-5})$ correc - tions are meaningless.

The mean square error is then computed, based on differences among the network intersection points. If the obtained value suits the required tolerance, the next stage should begin (rectifying element establishments).

Otherwise, GPART subprogramme divides photograph into four parts, retaking computations for each part of the photograph. If the required precision condition is not satisfied even in this case, then, the subprogramme should divide photograph into 16 parts and, further, into 64 parts, if necessary.

Experiments on models have shown that the required precision in 1:200,000 scale photomap compilation, using 1:1,000,000 scale photograph, which axis of exposure has a less 10° in - clination, is obtained when photograph is divided into not more than 16 parts. COTRAN subprogramme computes projective transformation coefficients, requiring 91 K and 8 - 10 minutes (depending on number of division).

5. RECTIFYING ELEMENT COMPUTATIONS

REDRES subprogramme is used to establish elements to be introduced into photorectifier for each part which can be rectified. Having this aim in view, firstly, the following parameters are computed using a_{ik} values corresponding to the respective part:

$$
\begin{aligned}
g &= -(v_0/h)(a_{13}-X_0)\sin\psi-(a_{23}-Y_0)\cos\psi) \\
h &= v_0^2\, p \\
\sin\theta &= -a_{31}v_0\ ;\ \cos\theta = a_{32}v_0 \qquad (15)\\
u_0 &= -(v_0/h)((a_{13}-X_0)\cos\psi+(a_{23}-Y_0)\sin\psi) \\
v_0 &= (a_{31}^2+a_{32}^2)^{-1/2}
\end{aligned}
$$

where

$$
\begin{aligned}
p &= (m^2+n^2)^{1/2} \\
m &= a_{12}a_{31}-a_{11}a_{32} \\
u &= a_{22}a_{31}-a_{21}a_{32} \\
\sin\psi &= n/p\ ;\ \cos\psi + m/p \qquad (16)\\
X_0 &= v_0^2(a_{11}a_{31}+a_{12}a_{32}) \\
Y_0 &= v_0^2(a_{21}a_{31}+a_{22}a_{32})
\end{aligned}
$$

These parameters are used to further establish rectifying elements, using formulas below:

$$
\begin{aligned}
\varphi_0 &= \operatorname{arctg}(f_r/(g^2-f_r^2)^{1/2}) \\
d &= (1+t)\,f_r \\
\delta u &= u_0 \qquad (17)\\
\delta v &= v_0 - h/t \\
\varkappa &= \theta
\end{aligned}
$$

where

$$t = ((h^2-f_r^2)/(g^2-f_r^2))^{1/2}$$

previously, cheking up that $g > f_r$ and $h > f_r$.

Because the present day photorectifiers have limitations as regards the screen inclination, the photo-carrier decentration and the scale coefficient, we must ckeck up if the obtained values should allow rectifying process during one single stage. Otherwise, rectifying process should be accomplished during two or more stages. As the affine rectifying ($a_{31} = a_{32} = 0$), this case is not approached here, because we have considered photographs having small tilts.

REDRES subprogramme,computing rectifying elements, needs 6 K and 30 seconds.

So-computed rectifying elements will be introduced into photo-rectifier, in order to rectify the respective parts.

Results obtained when this programme, i.e. its methodology,is used on models are encouraging as regards the economic precision aspect. Further experiments are to be achieved, based on operational materials. Their results should satisfy requirements of topographic map updating and small scale thematic map compilation.

This programme can be also very useful in some repetitive recording processings, based on the same control points.

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